MATH 195

§ 4.6 MODELING WITH EXPONENTIAL FUNCTIONS

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3-23 ODD

3. (a) IT DOUBLES EVERY 6 YEARS, SO IN THE LAST 30 YEARS, IT HAS DOUBLE 5 TIMES.

INITIAL POPULATION IS $100{,}000 \times \left(\frac{1}{2}\right)^5 = \frac{100{,}000}{32} = \boxed{3{,}125}$

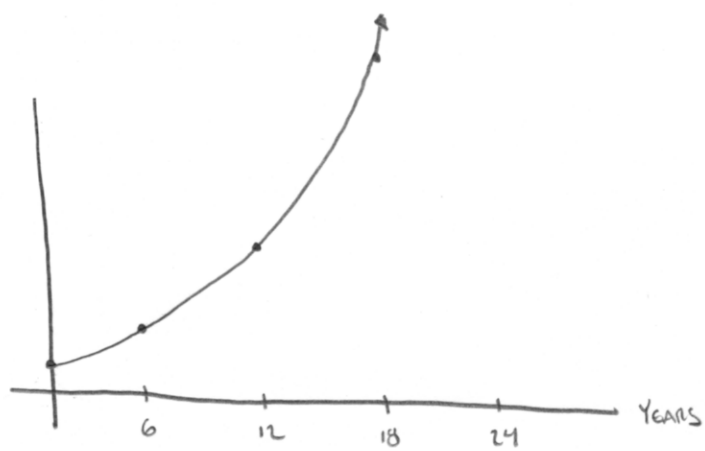
(b) POPULATION GETS MULTIPLIED BY 2 EVERY 6 YEARS

$\Rightarrow$ MULTIPLIED BY $\sqrt[6]{2} = 2^{1/6}$ EVERY 1 YEAR.

IN 10 YEARS, POP. GETS MULTIPLIED BY $\left(2^{1/6}\right)^{10} = 2^{10/6}$

$100{,}000 \times 2^{10/6} \approx \boxed{317{,}480}$

(c)



5. (a) $n(t) = n_0 e^{rt}$. GIVEN $n_0 = 18000$, $r = .08$

$n(t) = 18000\, e^{.08t}$

(b) $n(8) = 18000\, e^{.64} \approx 34{,}137$

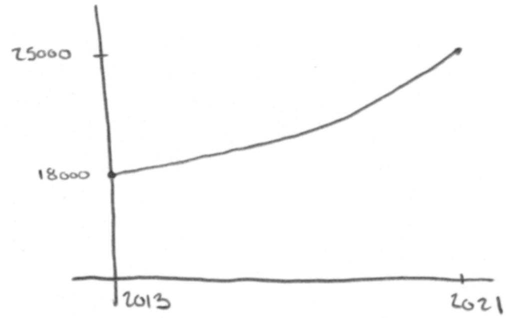
(c) $n(t) = 18000\, e^{.08t} = 25000$

$e^{.08t} = \frac{25}{18}$

$.08t = \ln \frac{25}{18}$

$t = \boxed{\frac{\ln \frac{25}{18}}{.08} \left(\approx 4.1 \text{ YEARS}\right)}$

(d)



<u>7.</u> LET $P(t)$ = POPULATION t YEARS AFTER 2011.

(a) $P(25) = 110\,e^{.03(25)}$

UNIT

$= \boxed{110\,e^{.75} \approx 232.87 \text{ MILLION}}$

(b) $P(25) = 110\,e^{.02(25)}$

$= \boxed{110\,e^{.5} \approx 181.36 \text{ MILLION}}$

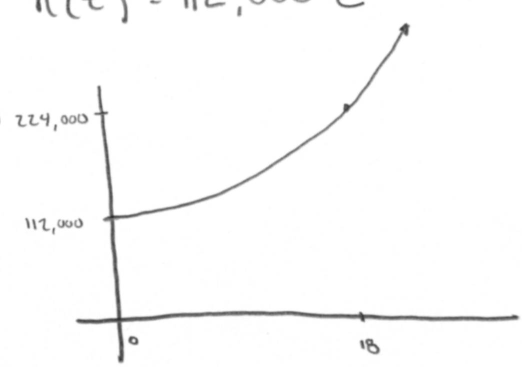
<u>9.</u> (a) $n(t) = 112{,}000 \cdot 2^{t/18}$

(b) SET $e^r = 2^{1/18}$

$r = \frac{1}{18} \ln 2$

$n(t) = 112{,}000\, e^{\frac{1}{18}\ln(2)\, t}$

(c)



(d) $n(t) = 112000\, 2^{t/18} = 500{,}000$

$2^{t/18} = \frac{500}{112} = \frac{125}{28}$

$\frac{t}{18} \ln(2) = \ln\left(\frac{125}{28}\right)$

$$\boxed{t = \frac{18 \ln\left(\frac{125}{28}\right)}{\ln(2)} \quad (\approx 38.85 \text{ YEARS})}$$

11. (a) 20,000

(b) $P(t) = 20000\,K^t$

$$31000 = P(4) = 20000\,K^4$$

$$\frac{31}{20} = K^4$$

$$K = \left(\frac{31}{20}\right)^{1/4}$$

$$\therefore \boxed{P(t) = 20000\left(\frac{31}{20}\right)^{t/4}}$$

(c) $P(8) = 20000\left(\frac{31}{20}\right)^{8/4}$

$$\boxed{= 20000\left(\frac{31}{20}\right)^2 \left(= 48050\right)}$$

13. (a) EVERY HOUR, POP IS MULTIPLIED BY

$$\frac{10000}{8600} = \frac{100}{86} = \frac{50}{43}$$

$$\therefore \boxed{n(t) = 8600\left(\frac{50}{43}\right)^t}$$

(b) $\boxed{n(2) = 8600\left(\frac{50}{43}\right)^2 \quad (\approx 11{,}628 \text{ BACT.})}$

(c) $n(t) = 8600\left(\frac{50}{43}\right)^t = 17200$

$$\left(\frac{50}{43}\right)^t = 2$$

$$t \ln\left(\frac{50}{43}\right) = \ln(2) \rightarrow t = \boxed{\frac{\ln 2}{\ln\left(\frac{50}{43}\right)} \left(\approx 4.6 \text{ HOURS}\right)}$$

15. (a) $P(t) = P_0 K^t$, $P_0 = 29.76$

$$33.87 = P(10) = 29.76 K^{10}$$

$$\frac{3387}{2976} = K^{10} \to K = \left(\frac{3387}{2976}\right)^{1/10}$$

$$\boxed{P(t) = 29.76 \left(\frac{3387}{2976}\right)^{t/10}}$$

(b) $$\left(\frac{3387}{2976}\right)^{t/10} = 2$$

$$\frac{t}{10} \ln\left(\frac{3387}{2976}\right) = \ln 2$$

$$t = \boxed{\frac{10 \ln(2)}{\ln\left(\frac{3387}{2976}\right)} \left(\approx 53.58 \text{ YEARS}\right)}$$

(c) $$P(20) = \boxed{29.76 \left(\frac{3387}{2976}\right)^{20/10}}$$

(≈ 38.55 MILLION
(ACTUAL POP OF CA IN 2010 : 37.35 MILLION))

17. (a) $M(t) = 22 \cdot 2^{-t/1600}$

(b) $2^{-1/1600} = e^r$

$$r = -\frac{1}{1600} \ln 2$$

$$\therefore M(t) = 22 e^{-\frac{1}{1600} \ln(2) t}$$

(c) $M(4000) = 22 (2)^{-4000/1600}$

$$\boxed{= 22 (2)^{-5/2} \left(\approx 3.89 \text{ mg}\right)}$$

(d) $22(2)^{-t/1600} = 18$

$$\boxed{t = \frac{-1600 \ln\left(\frac{9}{11}\right)}{\ln(2)} \left(\approx 463 \text{ YEARS}\right)}$$

19. $M(t) = 50 \cdot 2^{-t/28} = 32$

$$2^{-t/28} = \frac{32}{50} = \frac{16}{25}$$

$$-\frac{t}{28}\ln(2) = \ln\left(\frac{16}{25}\right)$$

$$\boxed{t = \frac{-28\ln\left(\frac{16}{25}\right)}{\ln(2)} \quad (\approx 18.03 \text{ YEARS})}$$

21. $M(t) = 250\left(\frac{200}{250}\right)^{t/48} = 125$

$$\left(\frac{200}{250}\right)^{t/48} = \frac{1}{2}$$

$$\frac{t}{48}\ln\left(\frac{4}{5}\right) = \ln\left(\frac{1}{2}\right)$$

$$\boxed{t = \frac{48\ln 0.5}{\ln 0.8} \quad (\approx 149.1 \text{ HOURS})}$$

23. $2^{-t/5730} = .65$

$$\frac{-t}{5730}\ln(2) = \ln(.65)$$

$$\boxed{t = \frac{-5730\ln(.65)}{\ln(2)} \quad (\approx 3561 \text{ YEARS OLD})}$$