MATH 195

§3.6 RATIONAL FUNCTIONS

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#

1. $\infty, -\infty$

2. 2

3. $-1, 2$

4. $r(0) = \frac{-2}{-6} = \boxed{\frac{1}{3}}$

5. $-2, 3$

6. $r(x) = \frac{x^2 - x - 2}{x^2 - x - 6} \to \boxed{1}$ AS $x \to \pm\infty$

7. $r(x) = \frac{x^2 + x}{(x+1)(2x-4)} = \frac{x(x+1)}{2(x+1)(x-2)}$

(a) FALSE (b) TRUE (c) FALSE (d) TRUE

8. TRUE

9. $r(x) = \frac{x}{x-2}$

(b) $y \to -\infty$ AS $x \to 2^-$

$y \to \infty$ AS $x \to 2^+$

$y \to 1$ AS $x \to \infty$

$y \to 1$ AS $x \to -\infty$

(c) $y = 1$

Table 1	
x	r(x)
1.5	-3
1.9	-19
1.99	-199
1.999	-1999

Table 2	
x	r(x)
2.5	5
2.1	21
2.01	201
2.001	2001

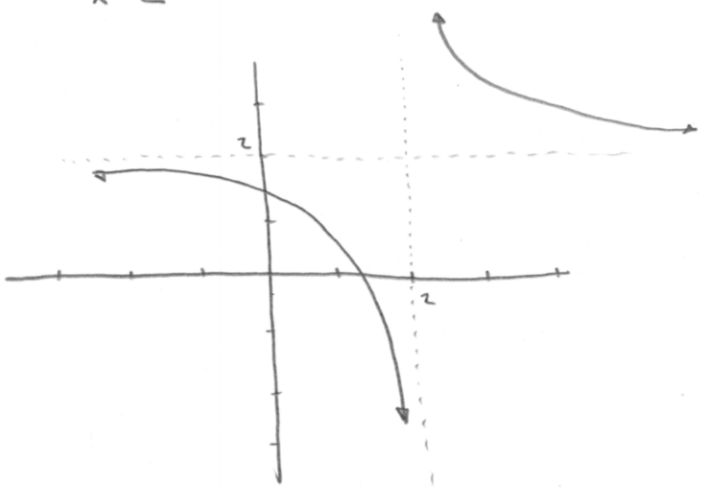
Table 3	
x	r(x)
10	1.25
50	1.04166667
100	1.02040816
1000	1.00200401

Table 4	
x	r(x)
-10	0.83333333
-50	0.96153846
-100	0.98039216
-1000	0.99800399

17. $t(x) = \dfrac{2x-3}{x-2} = \dfrac{2x-4+1}{x-2} = \dfrac{2(x-2)}{x-2} + \dfrac{1}{x-2}$

$= 2 + \dfrac{1}{x-2}$

$y = 2 + \dfrac{1}{x-2}$ IS TRANSFORMED $y = \frac{1}{x}$, 2 RIGHT & 2 UP



29. x-INTERCEPTS : $-1, 1$ VERTICAL ASYMPTOTES : $x = -2, x = 2$

y-INTERCEPT : $= \frac{1}{4}$ HORIZONTAL ASYMPTOTE : $y = 1$

30. x-INTERCEPTS : $-2, 2$ VERTICAL ASYMPTOTES : NONE

y-INTERCEPT : -6 HORIZONTAL ASYMPTOTE : $y = 2$.

35. $s(x) = \dfrac{6x^2+1}{2x^2+x-1} = \dfrac{6x^2+1}{(2x-1)(x+1)}$

TOP & BOTTOM BOTH DEGREE 2 : HORIZONTAL ASYMPTOTE $y = \frac{6}{2} = 3$

DENOMINATOR : $(2x-1)(x+1) = 0 \longrightarrow$ $x = \frac{1}{2}$, $x = -1$ VERTICAL ASYMPTOTES

45. $r(x) = \dfrac{3x^2 - 12x + 13}{x^2 - 4x + 4}$

x-INTERCEPT(S): $3x^2 - 12x + 13 = 0$

$$x = \frac{12 \pm \sqrt{12^2 - 4(3)(13)}}{2(3)} \rightarrow \text{NO SOLNS}$$

$\therefore$ NONE

y-INTERCEPT: $r(0) = \dfrac{13}{4}$

VERTICAL ASYMPTOTE: $x^2 - 4x + 4 = 0$

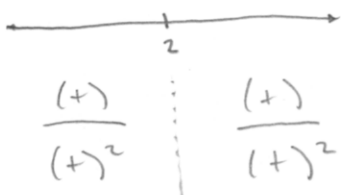
$(x-2)^2 = 0 \qquad x = 2$

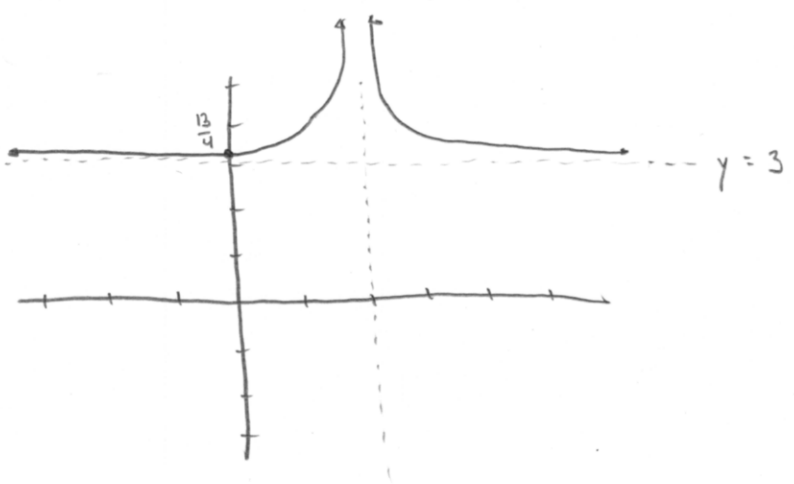
HORIZONTAL ASYMPTOTE:

$$r(x) = \frac{3x^2 - 12 + 13}{x^2 - 4x + 4} \rightarrow 3 \text{ AS } x \rightarrow \pm\infty$$

SAME DEGREE $\qquad y = 3$

$\dfrac{3x^2 - 12x + 13}{(x-2)^2}$





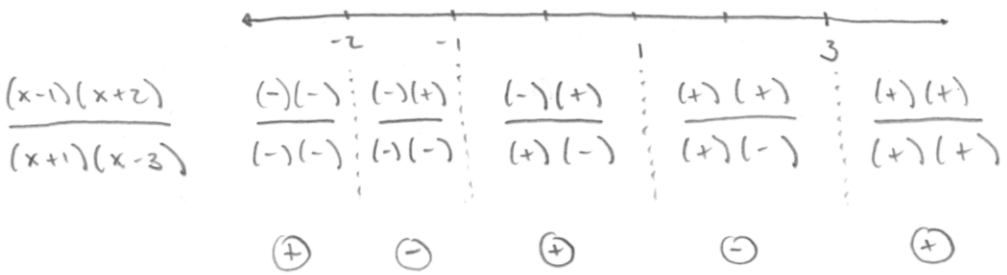
53. $r(x) = \dfrac{(x-1)(x+2)}{(x+1)(x-3)}$ → x-INT: $-2, 1$

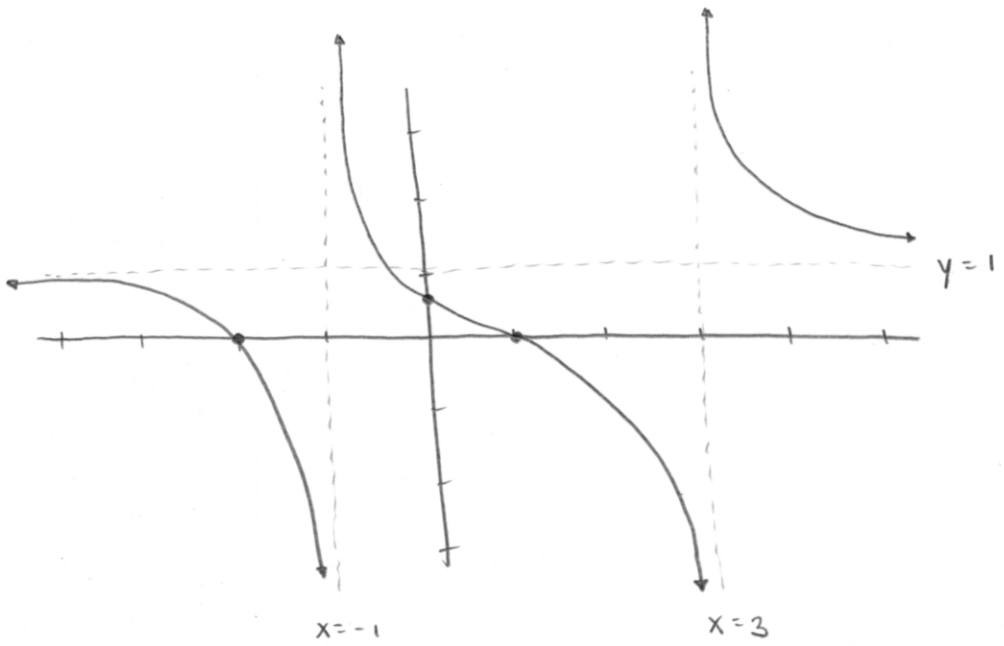
→ VERTICAL ASYMPTOTES: $x = -1, x = 3$

y-INT: $r(0) = \dfrac{(-1)(2)}{(1)(-3)} = \dfrac{-2}{-3} = \dfrac{2}{3}$

$r(x) = \dfrac{x^2 + x - 2}{x^2 - 2x - 3}$ → 1 AS $x \to \pm\infty$.

HORIZONTAL ASYMPTOTE: $y = 1$





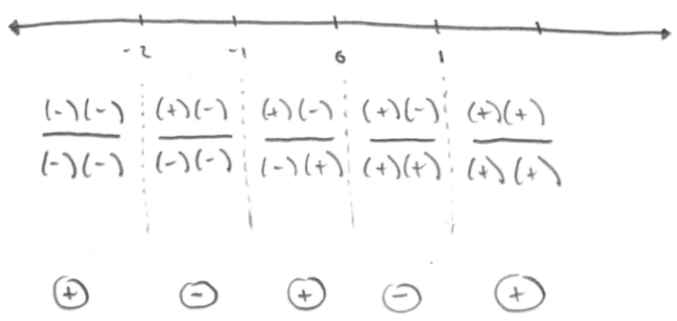
55. $r(x) = \dfrac{2x^2 + 2x - 4}{x^2 + x} = \dfrac{2(x+2)(x-1)}{x(x+1)}$ → x-INT: $-2, 1$

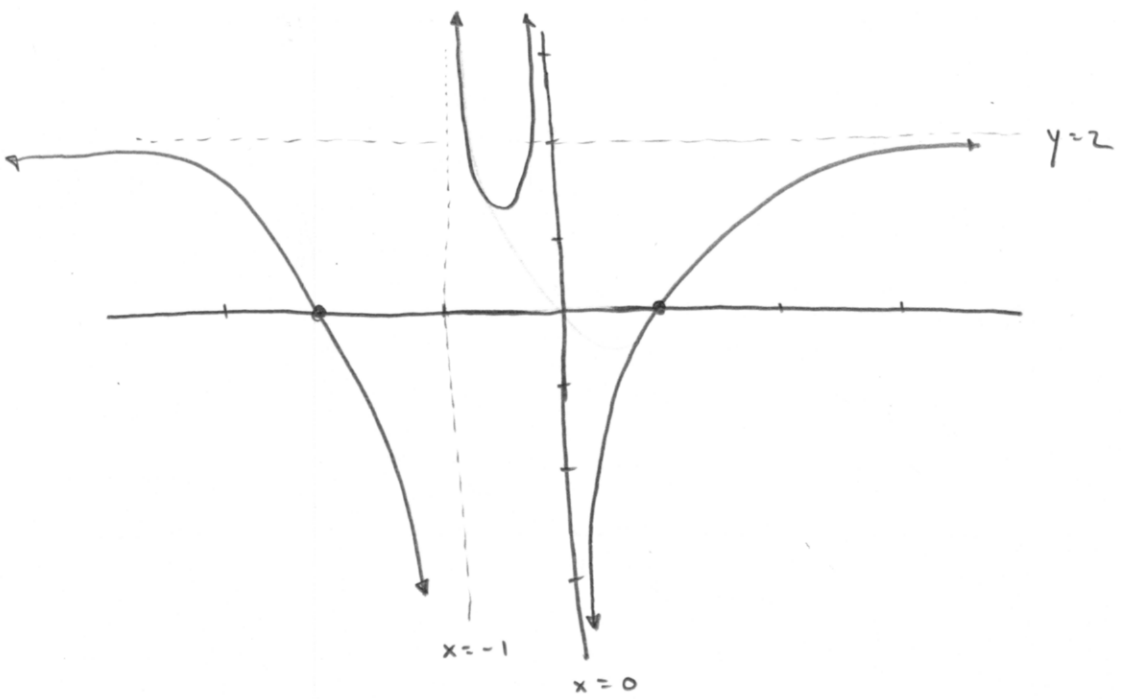
→ VERTICAL ASYMPTOTES: $x = 0, \; x = -1$

y-INT: $r(0)$ = UNDEFINED (NONE)

HORIZONTAL ASYMPTOTE $y = 2$ (RATIO OF LEAD COEFF OF TOP & BOTTOM, SINCE TOP & BOTTOM HAVE SAME DEGREE)

$\dfrac{2(x+2)(x-1)}{x(x+1)}$





57. $r(x) = \dfrac{x^2 - 2x + 1}{x^3 - 3x^2} = \dfrac{(x-1)^2}{x^2(x-3)}$

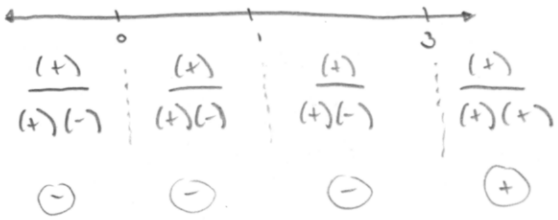
x-INT: 1

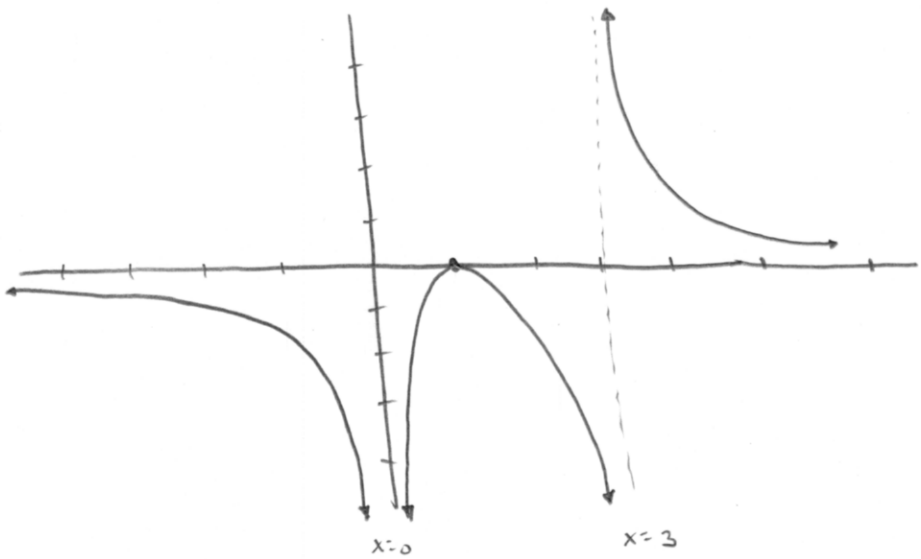
y-INT: NONE

HOR. ASYMPTOTE: $y = 0$ (DEGREE BOTTOM > DEGREE TOP)

VERT. ASYMPTOTE: $x = 0, x = 3$

$\dfrac{(x-1)^2}{x^2(x-3)}$





63. $r(x) = \dfrac{x^2 + 4x - 5}{x^2 + x - 2} = \dfrac{(x+5)(x-1)}{(x+2)(x-1)} = \dfrac{x+5}{x+2}$ UNLESS $x = 1$ WHERE r IS UNDEFINED

NOTE: $\dfrac{x+5}{x+2} = \dfrac{x+2+3}{x+2} = \dfrac{x+2}{x+2} + \dfrac{3}{x+2} = 1 + \dfrac{3}{x+2}$