1.9 THE COORDINATE PLANE; GRAPHS OF EQUATIONS; CIRCLES

■ The Coordinate Plane ■ The Distance and Midpoint Formulas ■ Graphs of Equations in Two Variables ■ Intercepts ■ Circles ■ Symmetry

The *coordinate plane* is the link between algebra and geometry. In the coordinate plane we can draw graphs of algebraic equations. The graphs, in turn, allow us to "see" the relationship between the variables in the equation. In this section we study the coordinate plane.

■ The Coordinate Plane

The Cartesian plane is named in honor of the French mathematician René Descartes (1596–1650), although another Frenchman, Pierre Fermat (1601–1665), also invented the principles of coordinate geometry at the same time. (See their biographies on pages 201 and 117.)

Just as points on a line can be identified with real numbers to form the coordinate line, points in a plane can be identified with ordered pairs of numbers to form the **coordinate plane** or **Cartesian plane**. To do this, we draw two perpendicular real lines that intersect at 0 on each line. Usually, one line is horizontal with positive direction to the right and is called the ***x*-axis**; the other line is vertical with positive direction upward and is called the ***y*-axis**. The point of intersection of the x-axis and the y-axis is the **origin *O***, and the two axes divide the plane into four **quadrants**, labeled I, II, III, and IV in Figure 1. (The points *on* the coordinate axes are not assigned to any quadrant.)

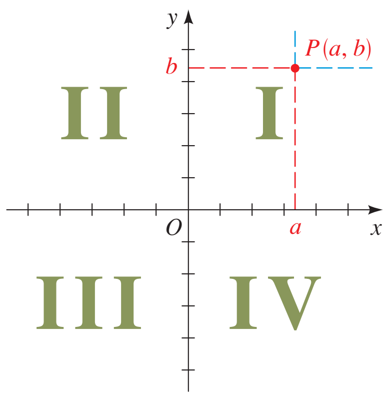


FIGURE 1

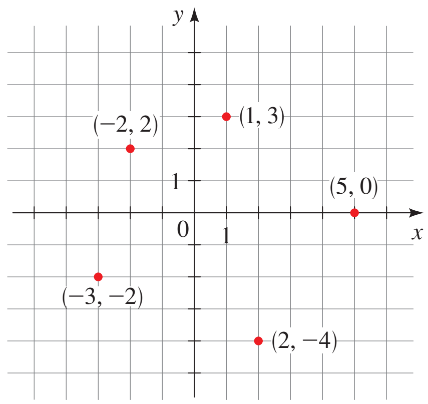


FIGURE 2

Although the notation for a point (a, b) is the same as the notation for an open interval (a, b), the context should make clear which meaning is intended.

Any point P in the coordinate plane can be located by a unique **ordered pair** of numbers (a, b), as shown in Figure 1. The first number a is called the ***x*-coordinate** of P; the second number b is called the ***y*-coordinate** of P. We can think of the coordinates of P as its "address," because they specify its location in the plane. Several points are labeled with their coordinates in Figure 2.

EXAMPLE 1 ■ Graphing Regions in the Coordinate Plane

Describe and sketch the regions given by each set.

(a) $\{(x, y) \mid x \geq 0\}$ **(b)** $\{(x, y) \mid y = 1\}$ **(c)** $\{(x, y) \mid |y| < 1\}$

SOLUTION

(a) The points whose x-coordinates are 0 or positive lie on the y-axis or to the right of it, as shown in Figure 3(a).

(b) The set of all points with y-coordinate 1 is a horizontal line one unit above the x-axis, as shown in Figure 3(b).

Coordinates as Addresses
The coordinates of a point in the *xy*-plane uniquely determine its location. We can think of the coordinates as the "address" of the point. In Salt Lake City, Utah, the addresses of most buildings are in fact expressed as coordinates. The city is divided into quadrants with Main Street as the vertical (North-South) axis and S. Temple Street as the horizontal (East-West) axis. An address such as

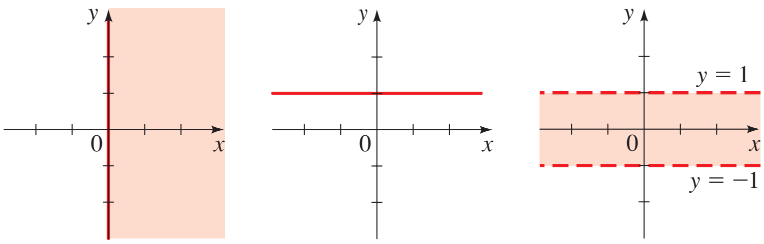
1760 W   2100 S

indicates a location 17.6 blocks west of Main Street and 21 blocks south of S. Temple Street. (This is the address of the main post office in Salt Lake City.) With this logical system it is possible for someone unfamiliar with the city to locate any address immediately, as easily as one locates a point in the coordinate plane.

(c) Recall from Section 1.8 that

$$|y| < 1 \qquad \text{if and only if} \qquad -1 < y < 1$$

So the given region consists of those points in the plane whose y-coordinates lie between -1 and 1. Thus the region consists of all points that lie between (but not on) the horizontal lines $y = 1$ and $y = -1$. These lines are shown as broken lines in Figure 3(c) to indicate that the points on these lines are not in the set.



(a) $x \geq 0$ (b) $y = 1$ (c) $|y| < 1$

FIGURE 3

Now Try Exercises 15 and 17 ■

The Distance and Midpoint Formulas

We now find a formula for the distance $d(A, B)$ between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane. Recall from Section 1.1 that the distance between points a and b on a number line is $d(a, b) = |b - a|$. So from Figure 4 we see that the distance between the points $A(x_1, y_1)$ and $C(x_2, y_1)$ on a horizontal line must be $|x_2 - x_1|$, and the distance between $B(x_2, y_2)$ and $C(x_2, y_1)$ on a vertical line must be $|y_2 - y_1|$.

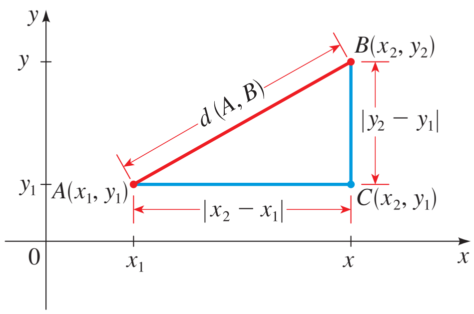


FIGURE 4

Since triangle ABC is a right triangle, the Pythagorean Theorem gives

$$d(A, B) = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

DISTANCE FORMULA

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 2 ■ Applying the Distance Formula

Which of the points $P(1, -2)$ or $Q(8, 9)$ is closer to the point $A(5, 3)$?

SOLUTION By the Distance Formula we have

$$d(P, A) = \sqrt{(5-1)^2 + [3-(-2)]^2} = \sqrt{4^2 + 5^2} = \sqrt{41}$$
$$d(Q, A) = \sqrt{(5-8)^2 + (3-9)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45}$$

This shows that $d(P, A) < d(Q, A)$, so P is closer to A (see Figure 5).

Now Try Exercise 35 ■

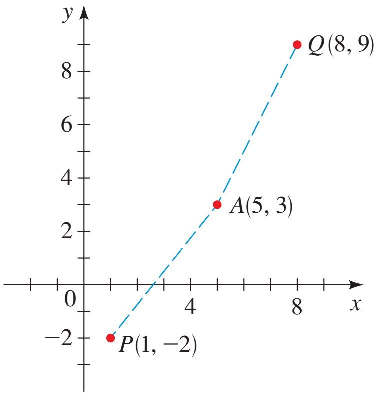


FIGURE 5

Now let's find the coordinates (x, y) of the midpoint M of the line segment that joins the point $A(x_1, y_1)$ to the point $B(x_2, y_2)$. In Figure 6 notice that triangles APM and MQB are congruent because $d(A, M) = d(M, B)$ and the corresponding angles are equal. It follows that $d(A, P) = d(M, Q)$, so

$$x - x_1 = x_2 - x$$

Solving this equation for x, we get $2x = x_1 + x_2$, so $x = \dfrac{x_1 + x_2}{2}$. Similarly, $y = \dfrac{y_1 + y_2}{2}$.

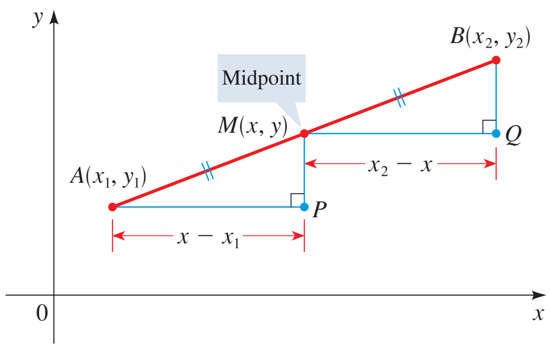


FIGURE 6

MIDPOINT FORMULA

The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

EXAMPLE 3 ■ Applying the Midpoint Formula

Show that the quadrilateral with vertices $P(1, 2)$, $Q(4, 4)$, $R(5, 9)$, and $S(2, 7)$ is a parallelogram by proving that its two diagonals bisect each other.

SOLUTION If the two diagonals have the same midpoint, then they must bisect each other. The midpoint of the diagonal PR is

$$\left(\frac{1+5}{2}, \frac{2+9}{2}\right) = \left(3, \frac{11}{2}\right)$$

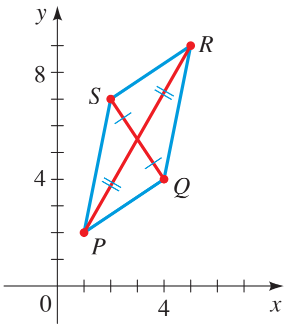


FIGURE 7

Fundamental Principle of Analytic Geometry

A point (x, y) lies on the graph of an equation if and only if its coordinates satisfy the equation.

and the midpoint of the diagonal QS is

$$\left(\frac{4+2}{2}, \frac{4+7}{2}\right) = \left(3, \frac{11}{2}\right)$$

so each diagonal bisects the other, as shown in Figure 7. (A theorem from elementary geometry states that the quadrilateral is therefore a parallelogram.)

Now Try Exercise 49 ■

■ Graphs of Equations in Two Variables

An **equation in two variables**, such as $y = x^2 + 1$, expresses a relationship between two quantities. A point (x, y) **satisfies** the equation if it makes the equation true when the values for x and y are substituted into the equation. For example, the point $(3, 10)$ satisfies the equation $y = x^2 + 1$ because $10 = 3^2 + 1$, but the point $(1, 3)$ does not, because $3 \neq 1^2 + 1$.

THE GRAPH OF AN EQUATION

The **graph** of an equation in x and y is the set of all points (x, y) in the coordinate plane that satisfy the equation.

The graph of an equation is a curve, so to graph an equation, we plot as many points as we can, then connect them by a smooth curve.

EXAMPLE 4 ■ Sketching a Graph by Plotting Points

Sketch the graph of the equation $2x - y = 3$.

SOLUTION We first solve the given equation for y to get

$$y = 2x - 3$$

This helps us calculate the y-coordinates in the following table.

x	$y = 2x - 3$	(x, y)
-1	-5	$(-1, -5)$
0	-3	$(0, -3)$
1	-1	$(1, -1)$
2	1	$(2, 1)$
3	3	$(3, 3)$
4	5	$(4, 5)$

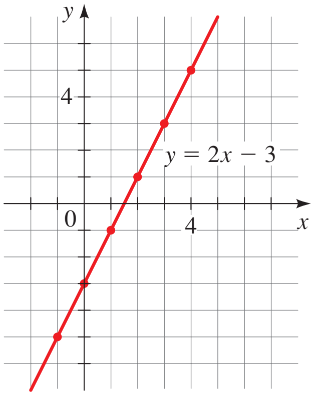


FIGURE 8

Of course, there are infinitely many points on the graph, and it is impossible to plot all of them. But the more points we plot, the better we can imagine what the graph represented by the equation looks like. We plot the points we found in Figure 8; they appear to lie on a line. So we complete the graph by joining the points by a line. (In Section 1.10 we verify that the graph of an equation of this type is indeed a line.)

Now Try Exercise 55 ■

A detailed discussion of parabolas and their geometric properties is presented in Sections 3.1 and 11.1.

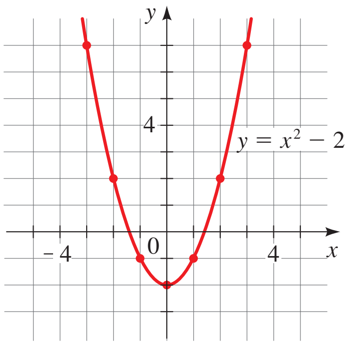


FIGURE 9

EXAMPLE 5 ■ Sketching a Graph by Plotting Points

Sketch the graph of the equation $y = x^2 - 2$.

SOLUTION We find some of the points that satisfy the equation in the following table. In Figure 9 we plot these points and then connect them by a smooth curve. A curve with this shape is called a *parabola*.

x	$y = x^2 - 2$	(x, y)
-3	7	$(-3, 7)$
-2	2	$(-2, 2)$
-1	-1	$(-1, -1)$
0	-2	$(0, -2)$
1	-1	$(1, -1)$
2	2	$(2, 2)$
3	7	$(3, 7)$

Now Try Exercise 57 ■

EXAMPLE 6 ■ Graphing an Absolute Value Equation

Sketch the graph of the equation $y = |x|$.

SOLUTION We make a table of values:

x	$y = \|x\|$	(x, y)
-3	3	$(-3, 3)$
-2	2	$(-2, 2)$
-1	1	$(-1, 1)$
0	0	$(0, 0)$
1	1	$(1, 1)$
2	2	$(2, 2)$
3	3	$(3, 3)$

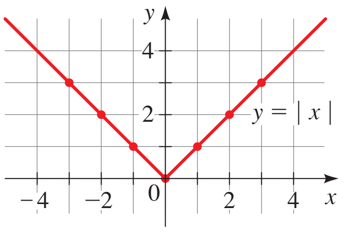


FIGURE 10

In Figure 10 we plot these points and use them to sketch the graph of the equation.

Now Try Exercise 59 ■

We can use a graphing calculator to graph equations. A graphing calculator draws the graph of an equation by plotting points, just as we would do by hand.

See Appendix C, *Graphing with a Graphing Calculator*, for general guidelines on using a graphing calculator. See Appendix D, *Using the TI-83/84 Graphing Calculator*, for specific graphing instructions. Go to **www.stewartmath.com**.

EXAMPLE 7 ■ Graphing an Equation with a Graphing Calculator

Use a graphing calculator to graph the following equation in the viewing rectangle $[-5, 5]$ by $[-1, 2]$.

$$y = \frac{1}{1 + x^2}$$

SOLUTION The graph is shown in Figure 11.

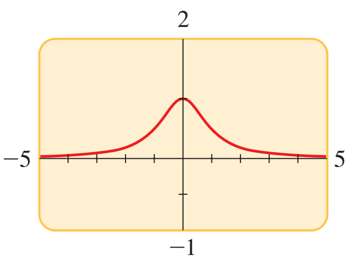


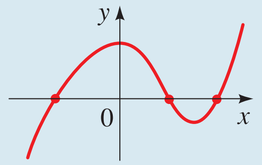
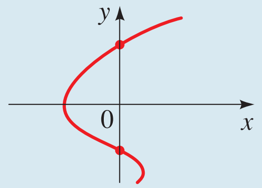
FIGURE 11 Graph of $y = \dfrac{1}{1 + x^2}$

Now Try Exercise 63 ■

■ Intercepts

The x-coordinates of the points where a graph intersects the x-axis are called the ***x*-intercepts** of the graph and are obtained by setting $y = 0$ in the equation of the graph. The y-coordinates of the points where a graph intersects the y-axis are called the ***y*-intercepts** of the graph and are obtained by setting $x = 0$ in the equation of the graph.

DEFINITION OF INTERCEPTS

Intercepts	How to find them	Where they are on the graph
***x*-intercepts:** The x-coordinates of points where the graph of an equation intersects the x-axis	Set $y = 0$ and solve for x	
***y*-intercepts:** The y-coordinates of points where the graph of an equation intersects the y-axis	Set $x = 0$ and solve for y	

EXAMPLE 8 ■ Finding Intercepts

Find the x- and y-intercepts of the graph of the equation $y = x^2 - 2$.

SOLUTION To find the x-intercepts, we set $y = 0$ and solve for x. Thus

$$0 = x^2 - 2 \qquad \text{Set } y = 0$$

$$x^2 = 2 \qquad \text{Add 2 to each side}$$

$$x = \pm\sqrt{2} \qquad \text{Take the square root}$$

The x-intercepts are $\sqrt{2}$ and $-\sqrt{2}$.

To find the y-intercepts, we set $x = 0$ and solve for y. Thus

$$y = 0^2 - 2 \qquad \text{Set } x = 0$$

$$y = -2$$

The y-intercept is -2.

The graph of this equation was sketched in Example 5. It is repeated in Figure 12 with the x- and y-intercepts labeled.

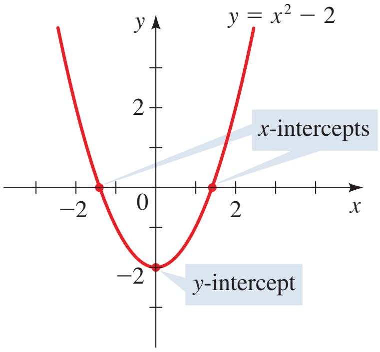


FIGURE 12

Now Try Exercise 71 ■

■ Circles

So far, we have discussed how to find the graph of an equation in x and y. The converse problem is to find an equation of a graph, that is, an equation that represents a given curve in the xy-plane. Such an equation is satisfied by the coordinates of the points on the curve and by no other point. This is the other half of the fundamental principle of analytic geometry as formulated by Descartes and Fermat. The idea is that if a geometric curve can be represented by an algebraic equation, then the rules of algebra can be used to analyze the curve.

As an example of this type of problem, let's find the equation of a circle with radius r and center (h, k). By definition the circle is the set of all points $P(x, y)$ whose

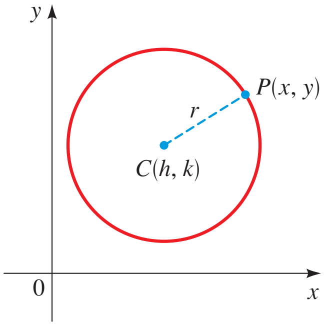


FIGURE 13

distance from the center $C(h, k)$ is r (see Figure 13). Thus P is on the circle if and only if $d(P, C) = r$. From the distance formula we have

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$(x-h)^2 + (y-k)^2 = r^2 \qquad \text{Square each side}$$

This is the desired equation.

> **EQUATION OF A CIRCLE**
>
> An equation of the circle with center (h, k) and radius r is
>
> $$(x-h)^2 + (y-k)^2 = r^2$$
>
> This is called the **standard form** for the equation of the circle. If the center of the circle is the origin $(0, 0)$, then the equation is
>
> $$x^2 + y^2 = r^2$$

EXAMPLE 9 ■ Graphing a Circle

Graph each equation.

(a) $x^2 + y^2 = 25$ **(b)** $(x-2)^2 + (y+1)^2 = 25$

SOLUTION

(a) Rewriting the equation as $x^2 + y^2 = 5^2$, we see that this is an equation of the circle of radius 5 centered at the origin. Its graph is shown in Figure 14.

(b) Rewriting the equation as $(x-2)^2 + (y+1)^2 = 5^2$, we see that this is an equation of the circle of radius 5 centered at $(2, -1)$. Its graph is shown in Figure 15.

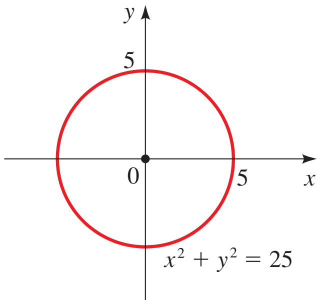


FIGURE 14

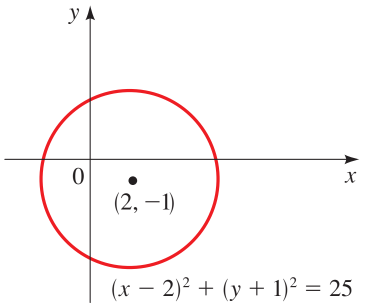


FIGURE 15

Now Try Exercises 83 and 85 ■

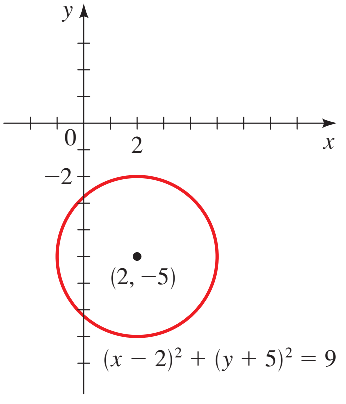


FIGURE 16

EXAMPLE 10 ■ Finding an Equation of a Circle

(a) Find an equation of the circle with radius 3 and center $(2, -5)$.

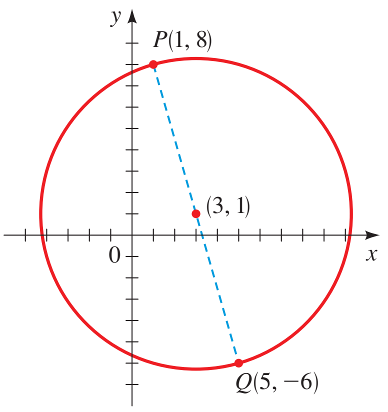
(b) Find an equation of the circle that has the points $P(1, 8)$ and $Q(5, -6)$ as the endpoints of a diameter.

SOLUTION

(a) Using the equation of a circle with $r = 3$, $h = 2$, and $k = -5$, we obtain

$$(x-2)^2 + (y+5)^2 = 9$$

The graph is shown in Figure 16.



$(x-3)^2+(y-1)^2=53$

FIGURE 17

(b) We first observe that the center is the midpoint of the diameter PQ, so by the Midpoint Formula the center is

$$\left(\frac{1+5}{2}, \frac{8-6}{2}\right) = (3, 1)$$

The radius r is the distance from P to the center, so by the Distance Formula

$$r^2 = (3-1)^2 + (1-8)^2 = 2^2 + (-7)^2 = 53$$

Therefore the equation of the circle is

$$(x-3)^2 + (y-1)^2 = 53$$

The graph is shown in Figure 17.

Now Try Exercises 89 and 93 ■

Let's expand the equation of the circle in the preceding example.

$(x-3)^2 + (y-1)^2 = 53$	Standard form
$x^2 - 6x + 9 + y^2 - 2y + 1 = 53$	Expand the squares
$x^2 - 6x + y^2 - 2y = 43$	Subtract 10 to get expanded form

Completing the square is used in many contexts in algebra. In Section 1.5 we used completing the square to solve quadratic equations.

Suppose we are given the equation of a circle in expanded form. Then to find its center and radius, we must put the equation back in standard form. That means that we must reverse the steps in the preceding calculation, and to do that, we need to know what to add to an expression like $x^2 - 6x$ to make it a perfect square—that is, we need to complete the square, as in the next example.

EXAMPLE 11 ■ Identifying an Equation of a Circle

Show that the equation $x^2 + y^2 + 2x - 6y + 7 = 0$ represents a circle, and find the center and radius of the circle.

SOLUTION We first group the x-terms and y-terms. Then we complete the square within each grouping. That is, we complete the square for $x^2 + 2x$ by adding $\left(\frac{1}{2}\cdot 2\right)^2 = 1$, and we complete the square for $y^2 - 6y$ by adding $\left[\frac{1}{2}\cdot(-6)\right]^2 = 9$.

We must add the same numbers to *each side* to maintain equality.

$(x^2 + 2x \quad) + (y^2 - 6y \quad) = -7$	Group terms
$(x^2 + 2x + 1) + (y^2 - 6y + 9) = -7 + 1 + 9$	Complete the square by adding 1 and 9 to each side
$(x+1)^2 + (y-3)^2 = 3$	Factor and simplify

Comparing this equation with the standard equation of a circle, we see that $h = -1$, $k = 3$, and $r = \sqrt{3}$, so the given equation represents a circle with center $(-1, 3)$ and radius $\sqrt{3}$.

Now Try Exercise 99 ■

■ Symmetry

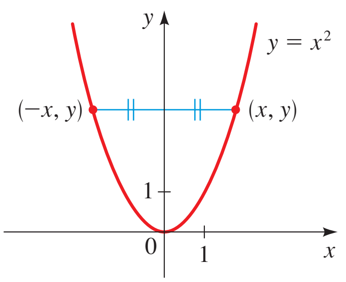
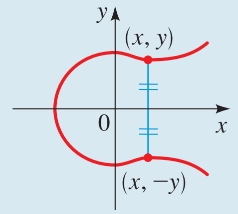
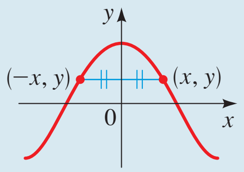
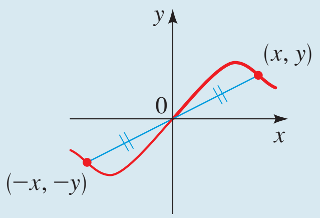


FIGURE 18

Figure 18 shows the graph of $y = x^2$. Notice that the part of the graph to the left of the y-axis is the mirror image of the part to the right of the y-axis. The reason is that if the point (x, y) is on the graph, then so is $(-x, y)$, and these points are reflections of each other about the y-axis. In this situation we say that the graph is **symmetric with respect to the y-axis**. Similarly, we say that a graph is **symmetric with respect to the x-axis** if whenever the point (x, y) is on the graph, then so is $(x, -y)$. A graph is **symmetric with respect to the origin** if whenever (x, y) is on the graph, so is $(-x, -y)$. (We often say symmetric "about" instead of "with respect to.")

TYPES OF SYMMETRY

Symmetry	Test	Graph	Property of Graph
With respect to the *x*-axis	Replace y by $-y$. The resulting equation is equivalent to the original one.		Graph is unchanged when reflected about the x-axis. See Figures 14 and 19.
With respect to the *y*-axis	Replace x by $-x$. The resulting equation is equivalent to the original one.		Graph is unchanged when reflected about the y-axis. See Figures 9, 10, 11, 12, 14, and 18.
With respect to the origin	Replace x by $-x$ and y by $-y$. The resulting equation is equivalent to the original one.		Graph is unchanged when rotated 180° about the origin. See Figures 14 and 20.

The remaining examples in this section show how symmetry helps us to sketch the graphs of equations.

EXAMPLE 12 ■ Using Symmetry to Sketch a Graph

Test the equation $x = y^2$ for symmetry and sketch the graph.

SOLUTION If y is replaced by $-y$ in the equation $x = y^2$, we get

$$x = (-y)^2 \qquad \text{Replace } y \text{ by } -y$$

$$x = y^2 \qquad \text{Simplify}$$

and so the equation is equivalent to the original one. Therefore the graph is symmetric about the x-axis. But changing x to $-x$ gives the equation $-x = y^2$, which is not equivalent to the original equation, so the graph is not symmetric about the y-axis.

We use the symmetry about the x-axis to sketch the graph by first plotting points just for $y > 0$ and then reflecting the graph about the x-axis, as shown in Figure 19.

y	$x = y^2$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	4	(4, 2)
3	9	(9, 3)

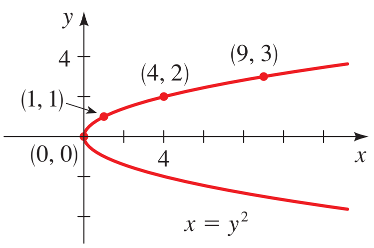


FIGURE 19

Now Try Exercises 105 and 111 ■

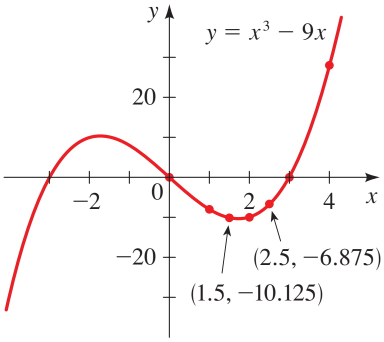


FIGURE 20

EXAMPLE 13 ■ Testing an Equation for Symmetry

Test the equation $y = x^3 - 9x$ for symmetry.

SOLUTION If we replace x by $-x$ and y by $-y$ in the equation, we get

$$-y = (-x)^3 - 9(-x) \quad \text{Replace } x \text{ by } -x \text{ and } y \text{ by } -y$$
$$-y = -x^3 + 9x \quad \text{Simplify}$$
$$y = x^3 - 9x \quad \text{Multiply by } -1$$

and so the equation is equivalent to the original one. This means that the graph is symmetric with respect to the origin, as shown in Figure 20.

Now Try Exercise 107 ■

1.9 EXERCISES

CONCEPTS

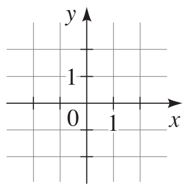
1. The point that is 3 units to the right of the y-axis and 5 units below the x-axis has coordinates (____, ____).

2. The distance between the points (a, b) and (c, d) is ________________. So the distance between $(1, 2)$ and $(7, 10)$ is ________.

3. The point midway between (a, b) and (c, d) is ________. So the point midway between $(1, 2)$ and $(7, 10)$ is ________.

4. If the point $(2, 3)$ is on the graph of an equation in x and y, then the equation is satisfied when we replace x by ________ and y by ________. Is the point $(2, 3)$ on the graph of the equation $2y = x + 1$? Complete the table, and sketch a graph.

x	y	(x, y)
−2		
−1		
0		
1		
2		



5. (a) To find the x-intercept(s) of the graph of an equation, we set ________ equal to 0 and solve for ________. So the x-intercept of $2y = x + 1$ is ________.

(b) To find the y-intercept(s) of the graph of an equation, we set ________ equal to 0 and solve for ________. So the y-intercept of $2y = x + 1$ is ________.

6. The graph of the equation $(x - 1)^2 + (y - 2)^2 = 9$ is a circle with center (____, ____) and radius ________.

7. (a) If a graph is symmetric with respect to the x-axis and (a, b) is on the graph, then (____, ____) is also on the graph.

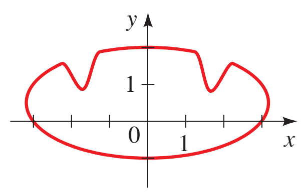
(b) If a graph is symmetric with respect to the y-axis and (a, b) is on the graph, then (____, ____) is also on the graph.

(c) If a graph is symmetric about the origin and (a, b) is on the graph, then (____, ____) is also on the graph.

8. The graph of an equation is shown below.

(a) The x-intercept(s) are ________, and the y-intercept(s) are ________.

(b) The graph is symmetric about the ________ (x-axis/y-axis/origin).



9–10 ■ *Yes or No*? If *No*, give a reason.

9. If the graph of an equation is symmetric with respect to both the x- and y-axes, is it necessarily symmetric with respect to the origin?

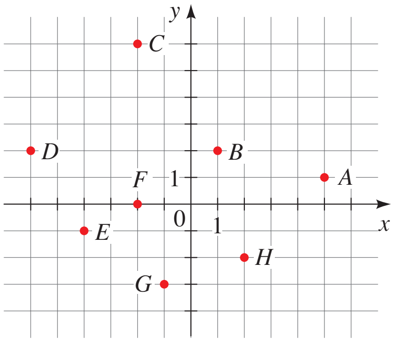
10. If the graph of an equation is symmetric with respect to the origin, is it necessarily symmetric with respect to the x- or y-axes?

SKILLS

11–12 ■ Points in a Coordinate Plane Refer to the figure below.

11. Find the coordinates of the points shown.

12. List the points that lie in Quadrants I and III.



13–14 ■ Points in a Coordinate Plane Plot the given points in a coordinate plane.

13. $(0, 5)$, $(-1, 0)$, $(-1, -2)$, $\left(\frac{1}{2}, \frac{2}{3}\right)$

14. $(-5, 0)$, $(2, 0)$, $(2.6, -1.3)$, $(-2.5, 3.5)$

15–20 ■ Sketching Regions Sketch the region given by the set.

15. **(a)** $\{(x, y) \mid x \geq 2\}$ **(b)** $\{(x, y) \mid y = 2\}$

16. **(a)** $\{(x, y) \mid y < 3\}$ **(b)** $\{(x, y) \mid x = -4\}$

17. **(a)** $\{(x, y) \mid -3 < x < 3\}$ **(b)** $\{(x, y) \mid |x| \leq 2\}$

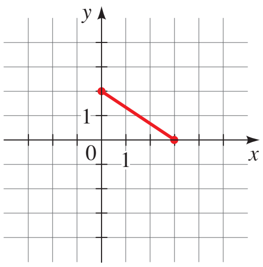
18. **(a)** $\{(x, y) \mid 0 \leq y \leq 2\}$ **(b)** $\{(x, y) \mid |y| > 2\}$

19. **(a)** $\{(x, y) \mid -2 < x < 2 \text{ and } y \geq 1\}$
(b) $\{(x, y) \mid xy < 0\}$

20. **(a)** $\{(x, y) \mid |x| \leq 1 \text{ and } |y| \leq 3\}$
(b) $\{(x, y) \mid xy > 0\}$

21–24 ■ Distance and Midpoint A pair of points is graphed. **(a)** Find the distance between them. **(b)** Find the midpoint of the segment that joins them.

21.



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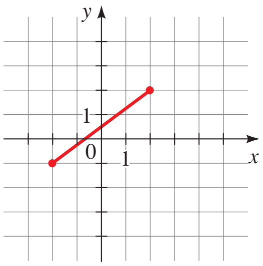

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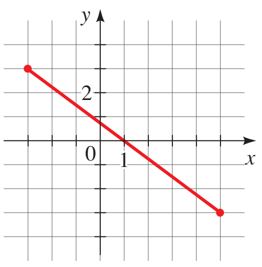

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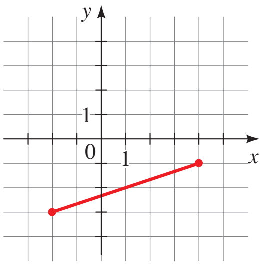


25–30 ■ Distance and Midpoint A pair of points is given. **(a)** Plot the points in a coordinate plane. **(b)** Find the distance between them. **(c)** Find the midpoint of the segment that joins them.

25. $(0, 8)$, $(6, 16)$ **26.** $(-2, 5)$, $(10, 0)$

27. $(3, -2)$, $(-4, 5)$ **28.** $(-1, 1)$, $(-6, -3)$

29. $(6, -2)$, $(-6, 2)$ **30.** $(0, -6)$, $(5, 0)$

31–34 ■ Area In these exercises we find the areas of plane figures.

31. Draw the rectangle with vertices $A(1, 3)$, $B(5, 3)$, $C(1, -3)$, and $D(5, -3)$ on a coordinate plane. Find the area of the rectangle.

32. Draw the parallelogram with vertices $A(1, 2)$, $B(5, 2)$, $C(3, 6)$, and $D(7, 6)$ on a coordinate plane. Find the area of the parallelogram.

33. Plot the points $A(1, 0)$, $B(5, 0)$, $C(4, 3)$, and $D(2, 3)$ on a coordinate plane. Draw the segments AB, BC, CD, and DA. What kind of quadrilateral is $ABCD$, and what is its area?

34. Plot the points $P(5, 1)$, $Q(0, 6)$, and $R(-5, 1)$ on a coordinate plane. Where must the point S be located so that the quadrilateral $PQRS$ is a square? Find the area of this square.

35–39 ■ Distance Formula In these exercises we use the Distance Formula.

35. Which of the points $A(6, 7)$ or $B(-5, 8)$ is closer to the origin?

36. Which of the points $C(-6, 3)$ or $D(3, 0)$ is closer to the point $E(-2, 1)$?

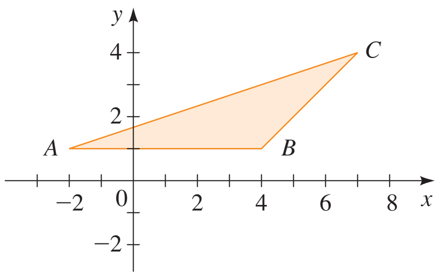
37. Which of the points $P(3, 1)$ or $Q(-1, 3)$ is closer to the point $R(-1, -1)$?

38. **(a)** Show that the points $(7, 3)$ and $(3, 7)$ are the same distance from the origin.

(b) Show that the points (a, b) and (b, a) are the same distance from the origin.

39. Show that the triangle with vertices $A(0, 2)$, $B(-3, -1)$, and $C(-4, 3)$ is isosceles.

40. **Area of Triangle** Find the area of the triangle shown in the figure.

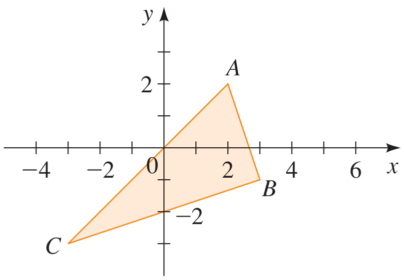


41–42 ■ Pythagorean Theorem In these exercises we use the converse of the Pythagorean Theorem (Appendix A) to show that the given triangle is a right triangle.

41. Refer to triangle ABC in the figure below.

(a) Show that triangle ABC is a right triangle by using the converse of the Pythagorean Theorem.

(b) Find the area of triangle ABC.



42. Show that the triangle with vertices $A(6, -7)$, $B(11, -3)$, and $C(2, -2)$ is a right triangle by using the converse of the Pythagorean Theorem. Find the area of the triangle.

43–45 ■ Distance Formula In these exercises we use the Distance Formula.

43. Show that the points $A(-2, 9)$, $B(4, 6)$, $C(1, 0)$, and $D(-5, 3)$ are the vertices of a square.

44. Show that the points $A(-1, 3)$, $B(3, 11)$, and $C(5, 15)$ are collinear by showing that $d(A, B) + d(B, C) = d(A, C)$.

45. Find a point on the y-axis that is equidistant from the points $(5, -5)$ and $(1, 1)$.

46–50 ■ Distance and Midpoint Formulas In these exercises we use the Distance Formula and the Midpoint Formula.

46. Find the lengths of the medians of the triangle with vertices $A(1, 0)$, $B(3, 6)$, and $C(8, 2)$. (A *median* is a line segment from a vertex to the midpoint of the opposite side.)

47. Plot the points $P(-1, -4)$, $Q(1, 1)$, and $R(4, 2)$ on a coordinate plane. Where should the point S be located so that the figure $PQRS$ is a parallelogram?

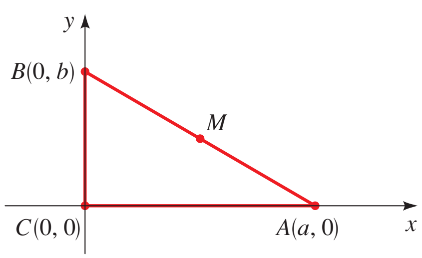
48. If $M(6, 8)$ is the midpoint of the line segment AB and if A has coordinates $(2, 3)$, find the coordinates of B.

49. **(a)** Sketch the parallelogram with vertices $A(-2, -1)$, $B(4, 2)$, $C(7, 7)$, and $D(1, 4)$.

(b) Find the midpoints of the diagonals of this parallelogram.

(c) From part (b) show that the diagonals bisect each other.

50. The point M in the figure is the midpoint of the line segment AB. Show that M is equidistant from the vertices of triangle ABC.



51–54 ■ Points on a Graph? Determine whether the given points are on the graph of the equation.

51. $x - 2y - 1 = 0$; $(0, 0), (1, 0), (-1, -1)$

52. $y(x^2 + 1) = 1$; $(1, 1), \left(1, \frac{1}{2}\right), \left(-1, \frac{1}{2}\right)$

53. $x^2 + xy + y^2 = 4$; $(0, -2), (1, -2), (2, -2)$

54. $x^2 + y^2 = 1$; $(0, 1), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

55–60 ■ Graphing Equations Make a table of values, and sketch the graph of the equation.

55. $4x + 5y = 40$ **56.** $3x - 5y = 30$

57. $y = x^2 + 4$ **58.** $y = 3 - x^2$

59. $y = |x| - 1$ **60.** $y = |x + 1|$

61–64 ■ Graphing Equations Use a graphing calculator to graph the equation in the given viewing rectangle.

61. $y = 0.01x^3 - x^2 + 5$; $[-100, 150]$ by $[-2000, 2000]$

62. $y = \sqrt{12x - 17}$; $[0, 10]$ by $[0, 20]$

63. $y = \dfrac{x}{x^2 + 25}$; $[-50, 50]$ by $[-0.2, 0.2]$

64. $y = x^4 - 4x^3$; $[-4, 6]$ by $[-50, 100]$

65–70 ■ Graphing Equations Make a table of values, and sketch the graph of the equation. Find the x- and y-intercepts, and test for symmetry.

65. **(a)** $2x - y = 6$ **(b)** $y = -(x + 1)^2$

66. **(a)** $x - 4y = 8$ **(b)** $y = -x^2 + 3$

67. **(a)** $y = \sqrt{x + 1}$ **(b)** $y = -|x|$

68. **(a)** $y = 3 - \sqrt{x}$ **(b)** $x = |y|$

69. **(a)** $y = \sqrt{4 - x^2}$ **(b)** $x = y^3 + 2y$

70. **(a)** $y = -\sqrt{4 - x^2}$ **(b)** $x = y^3$

71–74 ■ Intercepts Find the x- and y-intercepts of the graph of the equation.

71. **(a)** $y = x + 6$ **(b)** $y = x^2 - 5$

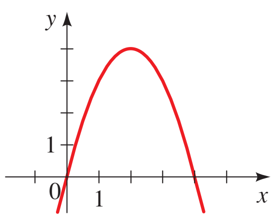
72. **(a)** $4x^2 + 25y^2 = 100$ **(b)** $x^2 - xy + 3y = 1$

73. **(a)** $9x^2 - 4y^2 = 36$ **(b)** $y - 2xy + 4x = 1$

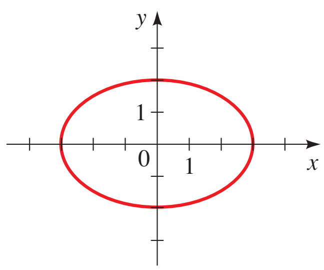
74. **(a)** $y = \sqrt{x^2 - 16}$ **(b)** $y = \sqrt{64 - x^3}$

75–78 ■ Intercepts An equation and its graph are given. Find the x- and y-intercepts.

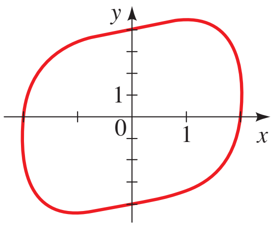
75. $y = 4x - x^2$



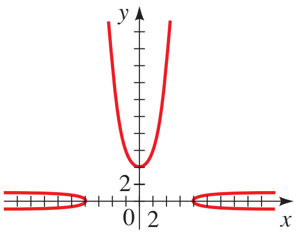
76. $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$



77. $x^4 + y^2 - xy = 16$



78. $x^2 + y^3 - x^2y^2 = 64$



79–82 ■ Graphing Equations An equation is given. **(a)** Use a graphing calculator to graph the equation in the given viewing rectangle. **(b)** Find the x- and y-intercepts from the graph. **(c)** Verify your answers to part (b) algebraically (from the equation).

79. $y = x^3 - x^2$; $[-2, 2]$ by $[-1, 1]$

80. $y = x^4 - 2x^3$; $[-2, 3]$ by $[-3, 3]$

81. $y = -\dfrac{2}{x^2 + 1}$; $[-5, 5]$ by $[-3, 1]$

82. $y = \sqrt[3]{1 - x^2}$; $[-5, 5]$ by $[-5, 3]$

83–88 ■ Graphing Circles Find the center and radius of the circle, and sketch its graph.

83. $x^2 + y^2 = 9$

84. $x^2 + y^2 = 5$

85. $x^2 + (y - 4)^2 = 1$

86. $(x + 1)^2 + y^2 = 9$

87. $(x + 3)^2 + (y - 4)^2 = 25$

88. $(x + 1)^2 + (y + 2)^2 = 36$

89–96 ■ Equations of Circles Find an equation of the circle that satisfies the given conditions.

89. Center $(2, -1)$; radius 3

90. Center $(-1, -4)$; radius 8

91. Center at the origin; passes through $(4, 7)$

92. Center $(-1, 5)$; passes through $(-4, -6)$

93. Endpoints of a diameter are $P(-1, 1)$ and $Q(5, 9)$

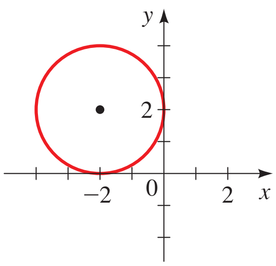
94. Endpoints of a diameter are $P(-1, 3)$ and $Q(7, -5)$

95. Center $(7, -3)$; tangent to the x-axis

96. Circle lies in the first quadrant, tangent to both x- and y-axes; radius 5

97–98 ■ Equations of Circles Find the equation of the circle shown in the figure.

97.



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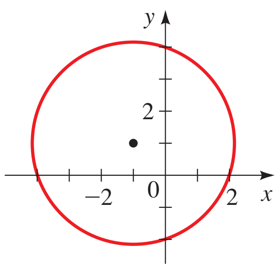


99–104 ■ Equations of Circles Show that the equation represents a circle, and find the center and radius of the circle.

99. $x^2 + y^2 + 4x - 6y + 12 = 0$

100. $x^2 + y^2 + 6y + 2 = 0$

101. $x^2 + y^2 - \frac{1}{2}x + \frac{1}{2}y = \frac{1}{8}$

102. $x^2 + y^2 + \frac{1}{2}x + 2y + \frac{1}{16} = 0$

103. $2x^2 + 2y^2 - 3x = 0$

104. $3x^2 + 3y^2 + 6x - y = 0$

105–110 ■ Symmetry Test the equation for symmetry.

105. $y = x^4 + x^2$

106. $x = y^4 - y^2$

107. $x^2y^2 + xy = 1$

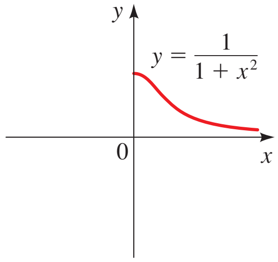
108. $x^4y^4 + x^2y^2 = 1$

109. $y = x^3 + 10x$

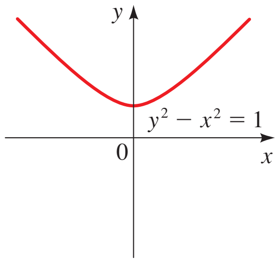
110. $y = x^2 + |x|$

111–114 ■ Symmetry Complete the graph using the given symmetry property.

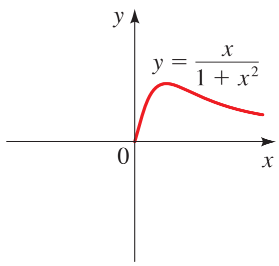
111. Symmetric with respect to the y-axis



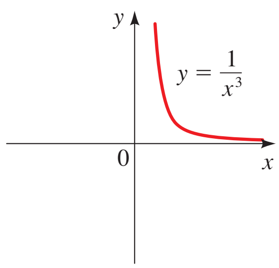
112. Symmetric with respect to the x-axis



113. Symmetric with respect to the origin



114. Symmetric with respect to the origin



SKILLS Plus

115–116 ■ Graphing Regions Sketch the region given by the set.

115. $\{(x, y) \mid x^2 + y^2 \le 1\}$

116. $\{(x, y) \mid x^2 + y^2 > 4\}$

117. Area of a Region Find the area of the region that lies outside the circle $x^2 + y^2 = 4$ but inside the circle

$$x^2 + y^2 - 4y - 12 = 0$$

118. Area of a Region Sketch the region in the coordinate plane that satisfies both the inequalities $x^2 + y^2 \le 9$ and $y \ge |x|$. What is the area of this region?

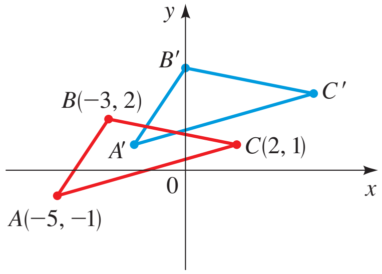
119. Shifting the Coordinate Plane Suppose that each point in the coordinate plane is shifted 3 units to the right and 2 units upward.

(a) The point $(5, 3)$ is shifted to what new point?

(b) The point (a, b) is shifted to what new point?

(c) What point is shifted to $(3, 4)$?

(d) Triangle ABC in the figure has been shifted to triangle $A'B'C'$. Find the coordinates of the points A', B', and C'.



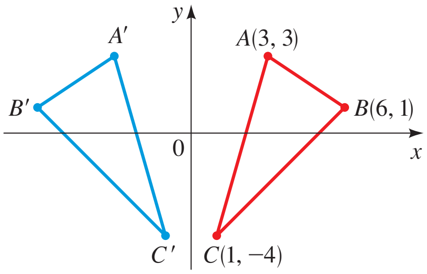
120. Reflecting in the Coordinate Plane Suppose that the y-axis acts as a mirror that reflects each point to the right of it into a point to the left of it.

(a) The point $(3, 7)$ is reflected to what point?

(b) The point (a, b) is reflected to what point?

(c) What point is reflected to $(-4, -1)$?

(d) Triangle ABC in the figure is reflected to triangle $A'B'C'$. Find the coordinates of the points A', B', and C'.

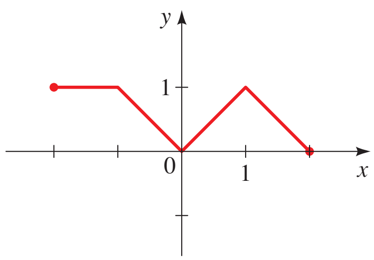


121. Making a Graph Symmetric The graph shown in the figure is not symmetric about the x-axis, the y-axis, or the origin. Add more line segments to the graph so that it exhibits the indicated symmetry. In each case, add as little as possible.

(a) Symmetry about the x-axis

(b) Symmetry about the y-axis

(c) Symmetry about the origin



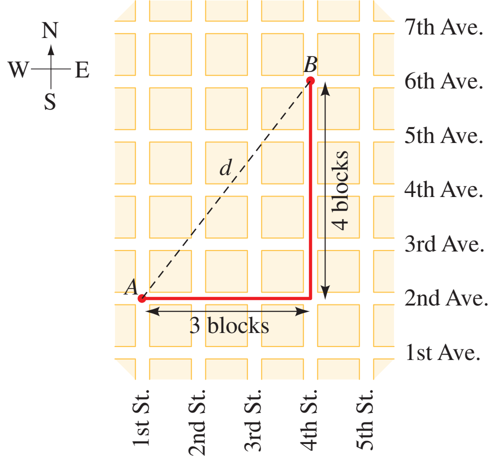
APPLICATIONS

122. Distances in a City A city has streets that run north and south and avenues that run east and west, all equally spaced. Streets and avenues are numbered sequentially, as shown in the figure. The *walking* distance between points A and B is 7 blocks—that is, 3 blocks east and 4 blocks north. To find the *straight-line* distance d, we must use the Distance Formula.

(a) Find the straight-line distance (in blocks) between A and B.

(b) Find the walking distance and the straight-line distance between the corner of 4th St. and 2nd Ave. and the corner of 11th St. and 26th Ave.

(c) What must be true about the points P and Q if the walking distance between P and Q equals the straight-line distance between P and Q?



123. Halfway Point Two friends live in the city described in Exercise 122, one at the corner of 3rd St. and 7th Ave., the other at the corner of 27th St. and 17th Ave. They frequently meet at a coffee shop halfway between their homes.

(a) At what intersection is the coffee shop located?

(b) How far must each of them walk to get to the coffee shop?

124. Orbit of a Satellite A satellite is in orbit around the moon. A coordinate plane containing the orbit is set up with the center of the moon at the origin, as shown in the graph, with distances measured in megameters (Mm). The equation of the satellite's orbit is

$$\frac{(x-3)^2}{25} + \frac{y^2}{16} = 1$$

(a) From the graph, determine the closest and the farthest that the satellite gets to the center of the moon.

(b) There are two points in the orbit with y-coordinates 2. Find the x-coordinates of these points, and determine their distances to the center of the moon.