107. DISCOVER: Limiting Behavior of Powers Complete the following tables. What happens to the nth root of 2 as n gets large? What about the nth root of $\frac{1}{2}$?

n	$2^{1/n}$
1	
2	
5	
10	
100	

n	$\left(\frac{1}{2}\right)^{1/n}$
1	
2	
5	
10	
100	

Construct a similar table for $n^{1/n}$. What happens to the nth root of n as n gets large?

108. PROVE: Laws of Exponents Prove the following Laws of Exponents for the case in which m and n are positive integers and $m > n$.

(a) Law 2: $\dfrac{a^m}{a^n} = a^{m-n}$ **(b)** Law 5: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

109. PROVE: Laws of Exponents Prove the following Laws of Exponents.

(a) Law 6: $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$ **(b)** Law 7: $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$

1.3 ALGEBRAIC EXPRESSIONS

■ Adding and Subtracting Polynomials ■ Multiplying Algebraic Expressions ■ Special Product Formulas ■ Factoring Common Factors ■ Factoring Trinomials ■ Special Factoring Formulas ■ Factoring by Grouping Terms

A **variable** is a letter that can represent any number from a given set of numbers. If we start with variables, such as x, y, and z, and some real numbers and combine them using addition, subtraction, multiplication, division, powers, and roots, we obtain an **algebraic expression**. Here are some examples:

$$2x^2 - 3x + 4 \qquad \sqrt{x} + 10 \qquad \frac{y - 2z}{y^2 + 4}$$

A **monomial** is an expression of the form ax^k, where a is a real number and k is a nonnegative integer. A **binomial** is a sum of two monomials and a **trinomial** is a sum of three monomials. In general, a sum of monomials is called a *polynomial*. For example, the first expression listed above is a polynomial, but the other two are not.

POLYNOMIALS

A **polynomial** in the variable x is an expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

where $a_0, a_1, \ldots, a_n$ are real numbers, and n is a nonnegative integer. If $a_n \neq 0$, then the polynomial has **degree n**. The monomials a_kx^k that make up the polynomial are called the **terms** of the polynomial.

Note that the degree of a polynomial is the highest power of the variable that appears in the polynomial.

Polynomial	Type	Terms	Degree
$2x^2 - 3x + 4$	trinomial	$2x^2, -3x, 4$	2
$x^8 + 5x$	binomial	$x^8, 5x$	8
$8 - x + x^2 - \frac{1}{2}x^3$	four terms	$-\frac{1}{2}x^3, x^2, -x, 3$	3
$5x + 1$	binomial	$5x, 1$	1
$9x^5$	monomial	$9x^5$	5
6	monomial	6	0

Adding and Subtracting Polynomials

Distributive Property

$ac + bc = (a + b)c$

We **add** and **subtract** polynomials using the properties of real numbers that were discussed in Section 1.1. The idea is to combine **like terms** (that is, terms with the same variables raised to the same powers) using the Distributive Property. For instance,

$$5x^7 + 3x^7 = (5 + 3)x^7 = 8x^7$$

In subtracting polynomials, we have to remember that if a minus sign precedes an expression in parentheses, then the sign of every term within the parentheses is changed when we remove the parentheses:

$$-(b + c) = -b - c$$

[This is simply a case of the Distributive Property, $a(b + c) = ab + ac$, with $a = -1$.]

EXAMPLE 1 ■ Adding and Subtracting Polynomials

(a) Find the sum $(x^3 - 6x^2 + 2x + 4) + (x^3 + 5x^2 - 7x)$.

(b) Find the difference $(x^3 - 6x^2 + 2x + 4) - (x^3 + 5x^2 - 7x)$.

SOLUTION

(a)
$$\begin{aligned}
&(x^3 - 6x^2 + 2x + 4) + (x^3 + 5x^2 - 7x) \\
&\quad = (x^3 + x^3) + (-6x^2 + 5x^2) + (2x - 7x) + 4 && \text{Group like terms} \\
&\quad = 2x^3 - x^2 - 5x + 4 && \text{Combine like terms}
\end{aligned}$$

(b)
$$\begin{aligned}
&(x^3 - 6x^2 + 2x + 4) - (x^3 + 5x^2 - 7x) \\
&\quad = x^3 - 6x^2 + 2x + 4 - x^3 - 5x^2 + 7x && \text{Distributive Property} \\
&\quad = (x^3 - x^3) + (-6x^2 - 5x^2) + (2x + 7x) + 4 && \text{Group like terms} \\
&\quad = -11x^2 + 9x + 4 && \text{Combine like terms}
\end{aligned}$$

Now Try Exercises 17 and 19 ■

Multiplying Algebraic Expressions

To find the **product** of polynomials or other algebraic expressions, we need to use the Distributive Property repeatedly. In particular, using it three times on the product of two binomials, we get

$$(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$$

The acronym **FOIL** helps us remember that the product of two binomials is the sum of the products of the **F**irst terms, the **O**uter terms, the **I**nner terms, and the **L**ast terms.

This says that we multiply the two factors by multiplying each term in one factor by each term in the other factor and adding these products. Schematically, we have

$$(a + b)(c + d) = \underset{\substack{\uparrow \\ \text{F}}}{ac} + \underset{\substack{\uparrow \\ \text{O}}}{ad} + \underset{\substack{\uparrow \\ \text{I}}}{bc} + \underset{\substack{\uparrow \\ \text{L}}}{bd}$$

In general, we can multiply two algebraic expressions by using the Distributive Property and the Laws of Exponents.

EXAMPLE 2 ■ Multiplying Binomials Using FOIL

$$\begin{aligned}
(2x + 1)(3x - 5) &= \underset{\substack{\uparrow \\ \text{F}}}{6x^2} - \underset{\substack{\uparrow \\ \text{O}}}{10x} + \underset{\substack{\uparrow \\ \text{I}}}{3x} - \underset{\substack{\uparrow \\ \text{L}}}{5} && \text{Distributive Property} \\
&= 6x^2 - 7x - 5 && \text{Combine like terms}
\end{aligned}$$

Now Try Exercise 25 ■

When we multiply trinomials or other polynomials with more terms, we use the Distributive Property. It is also helpful to arrange our work in table form. The next example illustrates both methods.

EXAMPLE 3 ■ Multiplying Polynomials

Find the product: $(2x + 3)(x^2 - 5x + 4)$

SOLUTION 1: Using the Distributive Property

$$\begin{aligned}(2x + 3)(x^2 - 5x + 4) &= 2x(x^2 - 5x + 4) + 3(x^2 - 5x + 4) && \text{Distributive Property}\\ &= (2x \cdot x^2 - 2x \cdot 5x + 2x \cdot 4) + (3 \cdot x^2 - 3 \cdot 5x + 3 \cdot 4) && \text{Distributive Property}\\ &= (2x^3 - 10x^2 + 8x) + (3x^2 - 15x + 12) && \text{Laws of Exponents}\\ &= 2x^3 - 7x^2 - 7x + 12 && \text{Combine like terms}\end{aligned}$$

SOLUTION 2: Using Table Form

$$\begin{array}{rrrrl} & x^2 & -\ 5x & +\ 4 & \\ & & 2x & +\ 3 & \\ \hline & 3x^2 & -\ 15x & +\ 12 & \text{Multiply } x^2 - 5x + 4 \text{ by } 3\\ 2x^3 & -\ 10x^2 & +\ 8x & & \text{Multiply } x^2 - 5x + 4 \text{ by } 2x\\ \hline 2x^3 & -\ 7x^2 & -\ 7x & +\ 12 & \text{Add like terms}\end{array}$$

Now Try Exercise 47 ■

Special Product Formulas

Certain types of products occur so frequently that you should memorize them. You can verify the following formulas by performing the multiplications.

SPECIAL PRODUCT FORMULAS

If A and B are any real numbers or algebraic expressions, then

1. $(A + B)(A - B) = A^2 - B^2$ — Sum and difference of same terms
2. $(A + B)^2 = A^2 + 2AB + B^2$ — Square of a sum
3. $(A - B)^2 = A^2 - 2AB + B^2$ — Square of a difference
4. $(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$ — Cube of a sum
5. $(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$ — Cube of a difference

The key idea in using these formulas (or any other formula in algebra) is the **Principle of Substitution**: We may substitute any algebraic expression for any letter in a formula. For example, to find $(x^2 + y^3)^2$ we use Product Formula 2, substituting x^2 for A and y^3 for B, to get

$$(x^2 + y^3)^2 = (x^2)^2 + 2(x^2)(y^3) + (y^3)^2$$

$$(A + B)^2 = A^2 + 2AB + B^2$$

EXAMPLE 4 ■ Using the Special Product Formulas

Use the Special Product Formulas to find each product.

(a) $(3x + 5)^2$ **(b)** $(x^2 - 2)^3$

SOLUTION

(a) Substituting $A = 3x$ and $B = 5$ in Product Formula 2, we get

$$(3x + 5)^2 = (3x)^2 + 2(3x)(5) + 5^2 = 9x^2 + 30x + 25$$

(b) Substituting $A = x^2$ and $B = 2$ in Product Formula 5, we get

$$\begin{aligned}(x^2 - 2)^3 &= (x^2)^3 - 3(x^2)^2(2) + 3(x^2)(2)^2 - 2^3 \\ &= x^6 - 6x^4 + 12x^2 - 8\end{aligned}$$

Now Try Exercises 31 and 43 ■

EXAMPLE 5 ■ Using the Special Product Formulas

Find each product.

(a) $(2x - \sqrt{y})(2x + \sqrt{y})$ **(b)** $(x + y - 1)(x + y + 1)$

SOLUTION

(a) Substituting $A = 2x$ and $B = \sqrt{y}$ in Product Formula 1, we get

$$(2x - \sqrt{y})(2x + \sqrt{y}) = (2x)^2 - (\sqrt{y})^2 = 4x^2 - y$$

(b) If we group $x + y$ together and think of this as one algebraic expression, we can use Product Formula 1 with $A = x + y$ and $B = 1$.

$$\begin{aligned}(x + y - 1)(x + y + 1) &= [(x + y) - 1][(x + y) + 1] \\ &= (x + y)^2 - 1^2 && \text{Product Formula 1} \\ &= x^2 + 2xy + y^2 - 1 && \text{Product Formula 2}\end{aligned}$$

Now Try Exercises 57 and 61 ■

Mathematics in the Modern World

Changing Words, Sound, and Pictures into Numbers

Pictures, sound, and text are routinely transmitted from one place to another via the Internet, fax machines, or modems. How can such things be transmitted through telephone wires? The key to doing this is to change them into numbers or bits (the digits 0 or 1). It's easy to see how to change text to numbers. For example, we could use the correspondence A = 00000001, B = 00000010, C = 00000011, D = 00000100, E = 00000101, and so on. The word "BED" then becomes 000000100000010100000100. By reading the digits in groups of eight, it is possible to translate this number back to the word "BED."

Changing sound to bits is more complicated. A sound wave can be graphed on an oscilloscope or a computer. The graph is then broken down mathematically into simpler components corresponding to the different frequencies of the original sound. (A branch of mathematics called Fourier analysis is used here.) The intensity of each component is a number, and the original sound can be reconstructed from these numbers. For example, music is stored on a CD as a sequence of bits; it may look like 101010001010010100101010 10000010 11110101000101011. . . . (One second of music requires 1.5 million bits!) The CD player reconstructs the music from the numbers on the CD.

Changing pictures into numbers involves expressing the color and brightness of each dot (or pixel) into a number. This is done very efficiently using a branch of mathematics called wavelet theory. The FBI uses wavelets as a compact way to store the millions of fingerprints they need on file.

■ Factoring Common Factors

We use the Distributive Property to expand algebraic expressions. We sometimes need to reverse this process (again using the Distributive Property) by **factoring** an expression as a product of simpler ones. For example, we can write

FACTORING →

$$x^2 - 4 = (x - 2)(x + 2)$$

← EXPANDING

We say that $x - 2$ and $x + 2$ are **factors** of $x^2 - 4$.

The easiest type of factoring occurs when the terms have a common factor.

EXAMPLE 6 ■ Factoring Out Common Factors

Factor each expression.

(a) $3x^2 - 6x$ **(b)** $8x^4y^2 + 6x^3y^3 - 2xy^4$ **(c)** $(2x + 4)(x - 3) - 5(x - 3)$

SOLUTION

(a) The greatest common factor of the terms $3x^2$ and $-6x$ is $3x$, so we have

$$3x^2 - 6x = 3x(x - 2)$$

CHECK YOUR ANSWER

Multiplying gives

$$3x(x - 2) = 3x^2 - 6x \quad ✓$$

(b) We note that

$$8, 6, \text{ and } -2 \text{ have the greatest common factor } 2$$
$$x^4, x^3, \text{ and } x \text{ have the greatest common factor } x$$
$$y^2, y^3, \text{ and } y^4 \text{ have the greatest common factor } y^2$$

So the greatest common factor of the three terms in the polynomial is $2xy^2$, and we have

$$\begin{aligned} 8x^4y^2 + 6x^3y^3 - 2xy^4 &= (2xy^2)(4x^3) + (2xy^2)(3x^2y) + (2xy^2)(-y^2) \\ &= 2xy^2(4x^3 + 3x^2y - y^2) \end{aligned}$$

CHECK YOUR ANSWER

Multiplying gives

$$\begin{aligned} &2xy^2(4x^3 + 3x^2y - y^2) \\ &\quad = 8x^4y^2 + 6x^3y^3 - 2xy^4 \checkmark \end{aligned}$$

(c) The two terms have the common factor $x - 3$.

$$\begin{aligned} (2x + 4)(x - 3) - 5(x - 3) &= [(2x + 4) - 5](x - 3) && \text{Distributive Property} \\ &= (2x - 1)(x - 3) && \text{Simplify} \end{aligned}$$

Now Try Exercises 63, 65, and 67 ■

Factoring Trinomials

To factor a trinomial of the form $x^2 + bx + c$, we note that

$$(x + r)(x + s) = x^2 + (r + s)x + rs$$

so we need to choose numbers r and s so that $r + s = b$ and $rs = c$.

EXAMPLE 7 ■ Factoring $x^2 + bx + c$ by Trial and Error

Factor: $x^2 + 7x + 12$

SOLUTION We need to find two integers whose product is 12 and whose sum is 7. By trial and error we find that the two integers are 3 and 4. Thus the factorization is

$$x^2 + 7x + 12 = (x + \underbrace{3)(x + 4}_{\text{factors of } 12})$$

CHECK YOUR ANSWER

Multiplying gives

$$(x + 3)(x + 4) = x^2 + 7x + 12 \checkmark$$

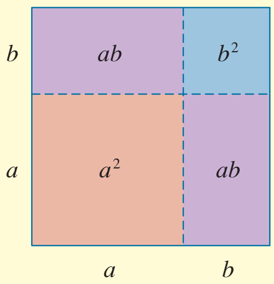
Now Try Exercise 69 ■

To factor a trinomial of the form $ax^2 + bx + c$ with $a \neq 1$, we look for factors of the form $px + r$ and $qx + s$:

$$ax^2 + bx + c = (px + r)(qx + s) = pqx^2 + (ps + qr)x + rs$$

$$ax^2 + bx + c = (\overset{\text{factors of } a}{\overbrace{p}}x + \underset{\text{factors of } c}{\underbrace{r)(q}}x + s)$$

Therefore we try to find numbers p, q, r, and s such that $pq = a$, $rs = c$, $ps + qr = b$. If these numbers are all integers, then we will have a limited number of possibilities to try for p, q, r, and s.



DISCOVERY PROJECT

Visualizing a Formula

Many of the Special Product Formulas in this section can be "seen" as geometrical facts about length, area, and volume. For example, the formula about the square of a sum can be interpreted to be about areas of squares and rectangles. The ancient Greeks always interpreted algebraic formulas in terms of geometric figures. Such figures give us special insight into how these formulas work. You can find the project at **www.stewartmath.com**.

EXAMPLE 8 ■ Factoring $ax^2 + bx + c$ by Trial and Error

Factor: $6x^2 + 7x - 5$

SOLUTION We can factor 6 as $6 \cdot 1$ or $3 \cdot 2$, and -5 as $-5 \cdot 1$ or $5 \cdot (-1)$. By trying these possibilities, we arrive at the factorization

factors of 6

$$6x^2 + 7x - 5 = (3x + 5)(2x - 1)$$

factors of -5

CHECK YOUR ANSWER

Multiplying gives

$(3x + 5)(2x - 1) = 6x^2 + 7x - 5$ ✓

Now Try Exercise 71 ■

EXAMPLE 9 ■ Recognizing the Form of an Expression

Factor each expression.

(a) $x^2 - 2x - 3$ **(b)** $(5a + 1)^2 - 2(5a + 1) - 3$

SOLUTION

(a) $x^2 - 2x - 3 = (x - 3)(x + 1)$ Trial and error

(b) This expression is of the form

$$\square^2 - 2\square - 3$$

where $\square$ represents $5a + 1$. This is the same form as the expression in part (a), so it will factor as $(\square - 3)(\square + 1)$.

$$(5a + 1)^2 - 2(5a + 1) - 3 = [(5a + 1) - 3][(5a + 1) + 1]$$
$$= (5a - 2)(5a + 2)$$

Now Try Exercise 75 ■

■ Special Factoring Formulas

Some special algebraic expressions can be factored by using the following formulas. The first three are simply Special Product Formulas written backward.

SPECIAL FACTORING FORMULAS

Formula	Name
1. $A^2 - B^2 = (A - B)(A + B)$	Difference of squares
2. $A^2 + 2AB + B^2 = (A + B)^2$	Perfect square
3. $A^2 - 2AB + B^2 = (A - B)^2$	Perfect square
4. $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$	Difference of cubes
5. $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$	Sum of cubes

EXAMPLE 10 ■ Factoring Differences of Squares

Factor each expression.

(a) $4x^2 - 25$ **(b)** $(x + y)^2 - z^2$

Terms and Factors

When we multiply two numbers together, each of the numbers is called a **factor** of the product. When we add two numbers together, each number is called a **term** of the sum.

$$\underbrace{2 \times 3}_{\text{Factors}} \qquad \underbrace{2 + 3}_{\text{Terms}}$$

If a factor is common to each term of an expression we can factor it out. The following expression has two terms.

$$\underbrace{ax + 2ay}_{a\text{ is a factor of each term}}$$

Each term contains the factor a, so we can factor a out and write the expression as

$$ax + 2ay = a(x + 2y)$$

SOLUTION

(a) Using the Difference of Squares Formula with $A = 2x$ and $B = 5$, we have

$$4x^2 - 25 = (2x)^2 - 5^2 = (2x - 5)(2x + 5)$$
$$A^2 - B^2 = (A - B)(A + B)$$

(b) We use the Difference of Squares Formula with $A = x + y$ and $B = z$.

$$(x + y)^2 - z^2 = (x + y - z)(x + y + z)$$

Now Try Exercises 77 and 111 ■

A trinomial is a perfect square if it is of the form

$$A^2 + 2AB + B^2 \qquad \text{or} \qquad A^2 - 2AB + B^2$$

So we **recognize a perfect square** if the middle term ($2AB$ or $-2AB$) is plus or minus twice the product of the square roots of the outer two terms.

EXAMPLE 11 ■ Recognizing Perfect Squares

Factor each trinomial.

(a) $x^2 + 6x + 9$ **(b)** $4x^2 - 4xy + y^2$

SOLUTION

(a) Here $A = x$ and $B = 3$, so $2AB = 2 \cdot x \cdot 3 = 6x$. Since the middle term is $6x$, the trinomial is a perfect square. By the Perfect Square Formula we have

$$x^2 + 6x + 9 = (x + 3)^2$$

(b) Here $A = 2x$ and $B = y$, so $2AB = 2 \cdot 2x \cdot y = 4xy$. Since the middle term is $-4xy$, the trinomial is a perfect square. By the Perfect Square Formula we have

$$4x^2 - 4xy + y^2 = (2x - y)^2$$

Now Try Exercises 107 and 109 ■

EXAMPLE 12 ■ Factoring Differences and Sums of Cubes

Factor each polynomial.

(a) $27x^3 - 1$ **(b)** $x^6 + 8$

SOLUTION

(a) Using the Difference of Cubes Formula with $A = 3x$ and $B = 1$, we get

$$\begin{aligned} 27x^3 - 1 &= (3x)^3 - 1^3 = (3x - 1)[(3x)^2 + (3x)(1) + 1^2] \\ &= (3x - 1)(9x^2 + 3x + 1) \end{aligned}$$

(b) Using the Sum of Cubes Formula with $A = x^2$ and $B = 2$, we have

$$x^6 + 8 = (x^2)^3 + 2^3 = (x^2 + 2)(x^4 - 2x^2 + 4)$$

Now Try Exercises 79 and 81 ■

When we factor an expression, the result can sometimes be factored further. In general, *we first factor out common factors*, then inspect the result to see whether it can be factored by any of the other methods of this section. We repeat this process until we have factored the expression completely.

EXAMPLE 13 ■ Factoring an Expression Completely

Factor each expression completely.

(a) $2x^4 - 8x^2$ **(b)** $x^5y^2 - xy^6$

SOLUTION

(a) We first factor out the power of x with the smallest exponent.

$$2x^4 - 8x^2 = 2x^2(x^2 - 4) \qquad \text{Common factor is } 2x^2$$
$$= 2x^2(x - 2)(x + 2) \qquad \text{Factor } x^2 - 4 \text{ as a difference of squares}$$

(b) We first factor out the powers of x and y with the smallest exponents.

$$x^5y^2 - xy^6 = xy^2(x^4 - y^4) \qquad \text{Common factor is } xy^2$$
$$= xy^2(x^2 + y^2)(x^2 - y^2) \qquad \text{Factor } x^4 - y^4 \text{ as a difference of squares}$$
$$= xy^2(x^2 + y^2)(x + y)(x - y) \qquad \text{Factor } x^2 - y^2 \text{ as a difference of squares}$$

Now Try Exercises 117 and 119 ■

In the next example we factor out variables with fractional exponents. This type of factoring occurs in calculus.

EXAMPLE 14 ■ Factoring Expressions with Fractional Exponents

Factor each expression.

(a) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ **(b)** $(2 + x)^{-2/3}x + (2 + x)^{1/3}$

SOLUTION

(a) Factor out the power of x with the *smallest exponent*, that is, $x^{-1/2}$.

$$3x^{3/2} - 9x^{1/2} + 6x^{-1/2} = 3x^{-1/2}(x^2 - 3x + 2) \qquad \text{Factor out } 3x^{-1/2}$$
$$= 3x^{-1/2}(x - 1)(x - 2) \qquad \text{Factor the quadratic } x^2 - 3x + 2$$

To factor out $x^{-1/2}$ from $x^{3/2}$, we *subtract* exponents:

$$x^{3/2} = x^{-1/2}(x^{3/2-(-1/2)})$$
$$= x^{-1/2}(x^{3/2+1/2})$$
$$= x^{-1/2}(x^2)$$

(b) Factor out the power of $2 + x$ with the *smallest exponent*, that is, $(2 + x)^{-2/3}$.

$$(2 + x)^{-2/3}x + (2 + x)^{1/3} = (2 + x)^{-2/3}[x + (2 + x)] \qquad \text{Factor out } (2 + x)^{-2/3}$$
$$= (2 + x)^{-2/3}(2 + 2x) \qquad \text{Simplify}$$
$$= 2(2 + x)^{-2/3}(1 + x) \qquad \text{Factor out 2}$$

CHECK YOUR ANSWERS

To see that you have factored correctly, multiply using the Laws of Exponents.

(a) $3x^{-1/2}(x^2 - 3x + 2)$
$= 3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ ✓

(b) $(2 + x)^{-2/3}[x + (2 + x)]$
$= (2 + x)^{-2/3}x + (2 + x)^{1/3}$ ✓

Now Try Exercises 93 and 95 ■

■ Factoring by Grouping Terms

Polynomials with at least four terms can sometimes be factored by grouping terms. The following example illustrates the idea.

EXAMPLE 15 ■ Factoring by Grouping

Factor each polynomial.

(a) $x^3 + x^2 + 4x + 4$ **(b)** $x^3 - 2x^2 - 9x + 18$

SOLUTION

(a) $x^3 + x^2 + 4x + 4 = (x^3 + x^2) + (4x + 4)$ Group terms

$= x^2(x + 1) + 4(x + 1)$ Factor out common factors

$= (x^2 + 4)(x + 1)$ Factor $x + 1$ from each term

(b) $x^3 - 2x^2 - 9x + 18 = (x^3 - 2x^2) - (9x - 18)$ Group terms

$= x^2(x - 2) - 9(x - 2)$ Factor common factors

$= (x^2 - 9)(x - 2)$ Factor $(x - 2)$ from each term

$= (x - 3)(x + 3)(x - 2)$ Factor completely

Now Try Exercises 85 and 121 ■

1.3 EXERCISES

CONCEPTS

1. Consider the polynomial $2x^5 + 6x^4 + 4x^3$.

(a) How many terms does this polynomial have? ________

List the terms: ________________.

(b) What factor is common to each term? ________

Factor the polynomial: $2x^5 + 6x^4 + 4x^3 =$ ________.

2. To factor the trinomial $x^2 + 7x + 10$, we look for two integers whose product is ________ and whose sum is ________. These integers are ________ and ________, so the trinomial factors as ________________.

3. The greatest common factor in the expression $3x^3 + x^2$ is ________, and the expression factors as ☐ (☐ + ☐).

4. The Special Product Formula for the "square of a sum" is $(A + B)^2 =$ ________________.

So $(2x + 3)^2 =$ ________________.

5. The Special Product Formula for the "product of the sum and difference of terms" is $(A + B)(A - B) =$ ________________.

So $(5 + x)(5 - x) =$ ________________.

6. The Special Factoring Formula for the "difference of squares" is $A^2 - B^2 =$ ________________. So $4x^2 - 25$ factors as ________________.

7. The Special Factoring Formula for a "perfect square" is $A^2 + 2AB + B^2 =$ ________________. So $x^2 + 10x + 25$ factors as ________________.

8. *Yes or No*? If *No*, give a reason.

(a) Is the expression $(x + 5)^2$ equal to $x^2 + 25$?

(b) When you expand $(x + a)^2$, where $a \neq 0$, do you get three terms?

(c) Is the expression $(x + 5)(x - 5)$ equal to $x^2 - 25$?

(d) When you expand $(x + a)(x - a)$, where $a \neq 0$, do you get two terms?

SKILLS

9–14 ■ Polynomials Complete the following table by stating whether the polynomial is a monomial, binomial, or trinomial; then list its terms and state its degree.

Polynomial	Type	Terms	Degree
9. $5x^3 + 6$			
10. $-2x^2 + 5x - 3$			
11. -8			
12. $\frac{1}{2}x^7$			
13. $x - x^2 + x^3 - x^4$			
14. $\sqrt{2}x - \sqrt{3}$			

15–24 ■ Polynomials Find the sum, difference, or product.

15. $(12x - 7) - (5x - 12)$

16. $(5 - 3x) + (2x - 8)$

17. $(-2x^2 - 3x + 1) + (3x^2 + 5x - 4)$

18. $(3x^2 + x + 1) - (2x^2 - 3x - 5)$

19. $(5x^3 + 4x^2 - 3x) - (x^2 + 7x + 2)$

20. $3(x - 1) + 4(x + 2)$

21. $8(2x + 5) - 7(x - 9)$

22. $4(x^2 - 3x + 5) - 3(x^2 - 2x + 1)$

23. $2(2 - 5t) + t^2(t - 1) - (t^4 - 1)$

24. $5(3t - 4) - (t^2 + 2) - 2t(t - 3)$

25–30 ■ Using FOIL Multiply the algebraic expressions using the FOIL method and simplify.

25. $(3t - 2)(7t - 4)$

26. $(4s - 1)(2s + 5)$

27. $(3x + 5)(2x - 1)$

28. $(7y - 3)(2y - 1)$

29. $(x + 3y)(2x - y)$

30. $(4x - 5y)(3x - y)$

31–46 ■ Using Special Product Formulas Multiply the algebraic expressions using a Special Product Formula and simplify.

31. $(5x + 1)^2$ **32.** $(2 - 7y)^2$

33. $(2u + v)^2$ **34.** $(x - 3y)^2$

35. $(2x + 3y)^2$ **36.** $(r - 2s)^2$

37. $(x + 6)(x - 6)$ **38.** $(5 - y)(5 + y)$

39. $(3x - 4)(3x + 4)$ **40.** $(2y + 5)(2y - 5)$

41. $(\sqrt{x} + 2)(\sqrt{x} - 2)$ **42.** $(\sqrt{y} + \sqrt{2})(\sqrt{y} - \sqrt{2})$

43. $(y + 2)^3$ **44.** $(x - 3)^3$

45. $(1 - 2r)^3$ **46.** $(3 + 2y)^3$

47–62 ■ Multiplying Algebraic Expressions Perform the indicated operations and simplify.

47. $(x + 2)(x^2 + 2x + 3)$ **48.** $(x + 1)(2x^2 - x + 1)$

49. $(2x - 5)(x^2 - x + 1)$ **50.** $(1 + 2x)(x^2 - 3x + 1)$

51. $\sqrt{x}(x - \sqrt{x})$ **52.** $x^{3/2}(\sqrt{x} - 1/\sqrt{x})$

53. $y^{1/3}(y^{2/3} + y^{5/3})$ **54.** $x^{1/4}(2x^{3/4} - x^{1/4})$

55. $(x^2 - a^2)(x^2 + a^2)$

56. $(x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})$

57. $(\sqrt{a} - b)(\sqrt{a} + b)$

58. $(\sqrt{h^2 + 1} + 1)(\sqrt{h^2 + 1} - 1)$

59. $((x - 1) + x^2)((x - 1) - x^2)$

60. $(x + (2 + x^2))(x - (2 + x^2))$

61. $(2x + y - 3)(2x + y + 3)$

62. $(x + y + z)(x - y - z)$

63–68 ■ Factoring Common Factor Factor out the common factor.

63. $-2x^3 + x$ **64.** $3x^4 - 6x^3 - x^2$

65. $y(y - 6) + 9(y - 6)$ **66.** $(z + 2)^2 - 5(z + 2)$

67. $2x^2y - 6xy^2 + 3xy$ **68.** $-7x^4y^2 + 14xy^3 + 21xy^4$

69–76 ■ Factoring Trinomials Factor the trinomial.

69. $x^2 + 8x + 7$ **70.** $x^2 + 4x - 5$

71. $8x^2 - 14x - 15$ **72.** $6y^2 + 11y - 21$

73. $3x^2 - 16x + 5$ **74.** $5x^2 - 7x - 6$

75. $(3x + 2)^2 + 8(3x + 2) + 12$

76. $2(a + b)^2 + 5(a + b) - 3$

77–84 ■ Using Special Factoring Formulas Use a Special Factoring Formula to factor the expression.

77. $9a^2 - 16$ **78.** $(x + 3)^2 - 4$

79. $27x^3 + y^3$ **80.** $a^3 - b^6$

81. $8s^3 - 125t^3$ **82.** $1 + 1000y^3$

83. $x^2 + 12x + 36$ **84.** $16z^2 - 24z + 9$

85–90 ■ Factoring by Grouping Factor the expression by grouping terms.

85. $x^3 + 4x^2 + x + 4$ **86.** $3x^3 - x^2 + 6x - 2$

87. $5x^3 + x^2 + 5x + 1$ **88.** $18x^3 + 9x^2 + 2x + 1$

89. $x^3 + x^2 + x + 1$ **90.** $x^5 + x^4 + x + 1$

91–96 ■ Fractional Exponents Factor the expression completely. Begin by factoring out the lowest power of each common factor.

91. $x^{5/2} - x^{1/2}$ **92.** $3x^{-1/2} + 4x^{1/2} + x^{3/2}$

93. $x^{-3/2} + 2x^{-1/2} + x^{1/2}$ **94.** $(x - 1)^{7/2} - (x - 1)^{3/2}$

95. $(x^2 + 1)^{1/2} + 2(x^2 + 1)^{-1/2}$

96. $x^{-1/2}(x + 1)^{1/2} + x^{1/2}(x + 1)^{-1/2}$

97–126 ■ Factoring Completely Factor the expression completely.

97. $12x^3 + 18x$ **98.** $30x^3 + 15x^4$

99. $x^2 - 2x - 8$ **100.** $x^2 - 14x + 48$

101. $2x^2 + 5x + 3$ **102.** $2x^2 + 7x - 4$

103. $9x^2 - 36x - 45$ **104.** $8x^2 + 10x + 3$

105. $49 - 4y^2$ **106.** $4t^2 - 9s^2$

107. $t^2 - 6t + 9$ **108.** $x^2 + 10x + 25$

109. $4x^2 + 4xy + y^2$ **110.** $r^2 - 6rs + 9s^2$

111. $(a + b)^2 - (a - b)^2$ **112.** $\left(1 + \frac{1}{x}\right)^2 - \left(1 - \frac{1}{x}\right)^2$

113. $x^2(x^2 - 1) - 9(x^2 - 1)$ **114.** $(a^2 - 1)b^2 - 4(a^2 - 1)$

115. $8x^3 - 125$ **116.** $x^6 + 64$

117. $x^3 + 2x^2 + x$ **118.** $3x^3 - 27x$

119. $x^4y^3 - x^2y^5$ **120.** $18y^3x^2 - 2xy^4$

121. $3x^3 - x^2 - 12x + 4$ **122.** $9x^3 + 18x^2 - x - 2$

123. $(x - 1)(x + 2)^2 - (x - 1)^2(x + 2)$

124. $y^4(y + 2)^3 + y^5(y + 2)^4$

125. $(a^2 + 1)^2 - 7(a^2 + 1) + 10$

126. $(a^2 + 2a)^2 - 2(a^2 + 2a) - 3$

127–130 ■ Factoring Completely Factor the expression completely. (This type of expression arises in calculus when using the "Product Rule.")

127. $5(x^2 + 4)^4(2x)(x - 2)^4 + (x^2 + 4)^5(4)(x - 2)^3$

128. $3(2x - 1)^2(2)(x + 3)^{1/2} + (2x - 1)^3(\frac{1}{2})(x + 3)^{-1/2}$

129. $(x^2 + 3)^{-1/3} - \frac{2}{3}x^2(x^2 + 3)^{-4/3}$

130. $\frac{1}{2}x^{-1/2}(3x + 4)^{1/2} - \frac{3}{2}x^{1/2}(3x + 4)^{-1/2}$

SKILLS Plus

131–132 ■ Verifying Identities Show that the following identities hold.

131. (a) $ab = \frac{1}{2}[(a + b)^2 - (a^2 + b^2)]$

(b) $(a^2 + b^2)^2 - (a^2 - b^2)^2 = 4a^2b^2$

132. $(a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2$

133. Factoring Completely Factor the following expression completely: $4a^2c^2 - (a^2 - b^2 + c^2)^2$.

134. Factoring $x^4 + ax^2 + b$ A trinomial of the form $x^4 + ax^2 + b$ can sometimes be factored easily. For example,

$$x^4 + 3x^2 - 4 = (x^2 + 4)(x^2 - 1)$$

But $x^4 + 3x^2 + 4$ cannot be factored in this way. Instead, we can use the following method.

$$x^4 + 3x^2 + 4 = (x^4 + 4x^2 + 4) - x^2 \quad \text{Add and subtract } x^2$$
$$= (x^2 + 2)^2 - x^2 \quad \text{Factor perfect square}$$
$$= [(x^2 + 2) - x][(x^2 + 2) + x] \quad \text{Difference of squares}$$
$$= (x^2 - x + 2)(x^2 + x + 2)$$

Factor the following, using whichever method is appropriate.

(a) $x^4 + x^2 - 2$ **(b)** $x^4 + 2x^2 + 9$

(c) $x^4 + 4x^2 + 16$ **(d)** $x^4 + 2x^2 + 1$

APPLICATIONS

135. Volume of Concrete A culvert is constructed out of large cylindrical shells cast in concrete, as shown in the figure. Using the formula for the volume of a cylinder given on the inside front cover of this book, explain why the volume of the cylindrical shell is

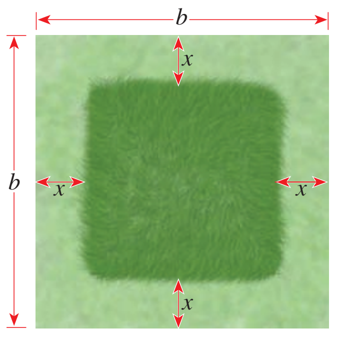
$$V = \pi R^2 h - \pi r^2 h$$

Factor to show that

$$V = 2\pi \cdot \text{average radius} \cdot \text{height} \cdot \text{thickness}$$

Use the "unrolled" diagram to explain why this makes sense geometrically.

136. Mowing a Field A square field in a certain state park is mowed around the edges every week. The rest of the field is kept unmowed to serve as a habitat for birds and small animals (see the figure). The field measures b feet by b feet, and the mowed strip is x feet wide.



(a) Explain why the area of the mowed portion is $b^2 - (b - 2x)^2$.

(b) Factor the expression in part (a) to show that the area of the mowed portion is also $4x(b - x)$.

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

137. DISCOVER: Degree of a Sum or Product of Polynomials Make up several pairs of polynomials, then calculate the sum and product of each pair. On the basis of your experiments and observations, answer the following questions.

(a) How is the degree of the product related to the degrees of the original polynomials?

(b) How is the degree of the sum related to the degrees of the original polynomials?

138. DISCUSS: The Power of Algebraic Formulas Use the Difference of Squares Formula $A^2 - B^2 = (A + B)(A - B)$ to evaluate the following differences of squares in your head. Make up more such expressions that you can do in your head.

(a) $528^2 - 527^2$

(b) $122^2 - 120^2$

(c) $1020^2 - 1010^2$

139. DISCUSS: The Power of Algebraic Formulas Use the Special Product Formula $(A + B)(A - B) = A^2 - B^2$ to evaluate the following products of numbers in your head. Make up more such products that you can do in your head.

(a) $501 \cdot 499$

(b) $79 \cdot 61$

(c) $2007 \cdot 1993$

140. DISCOVER: Differences of Even Powers

(a) Factor the expressions completely: $A^4 - B^4$ and $A^6 - B^6$.

(b) Verify that $18{,}335 = 12^4 - 7^4$ and that $2{,}868{,}335 = 12^6 - 7^6$.

(c) Use the results of parts (a) and (b) to factor the integers 18,335 and 2,868,335. Then show that in both of these factorizations, all the factors are prime numbers.

141. DISCOVER: Factoring $A^n - 1$

(a) Verify the following formulas by expanding and simplifying the right-hand side.

$$A^2 - 1 = (A - 1)(A + 1)$$
$$A^3 - 1 = (A - 1)(A^2 + A + 1)$$
$$A^4 - 1 = (A - 1)(A^3 + A^2 + A + 1)$$

(b) On the basis of the pattern displayed in this list, how do you think $A^5 - 1$ would factor? Verify your conjecture. Now generalize the pattern you have observed to obtain a factoring formula for $A^n - 1$, where n is a positive integer.

142. PROVE: Special Factoring Formulas Prove the following formulas by expanding the right-hand side.

(a) Difference of Cubes:
$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

(b) Sum of Cubes:
$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$