1.2 EXPONENTS AND RADICALS

■ Integer Exponents ■ Rules for Working with Exponents ■ Scientific Notation
■ Radicals ■ Rational Exponents ■ Rationalizing the Denominator; Standard Form

In this section we give meaning to expressions such as $a^{m/n}$ in which the exponent m/n is a rational number. To do this, we need to recall some facts about integer exponents, radicals, and nth roots.

■ Integer Exponents

A product of identical numbers is usually written in exponential notation. For example, $5 \cdot 5 \cdot 5$ is written as 5^3. In general, we have the following definition.

EXPONENTIAL NOTATION

If a is any real number and n is a positive integer, then the ***n*th power** of a is

$$a^n = \underbrace{a \cdot a \cdot \cdots \cdot a}_{n \text{ factors}}$$

The number a is called the **base**, and n is called the **exponent**.

EXAMPLE 1 ■ Exponential Notation

Note the distinction between $(-3)^4$ and -3^4. In $(-3)^4$ the exponent applies to -3, but in -3^4 the exponent applies only to 3.

(a) $\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{32}$

(b) $(-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$

(c) $-3^4 = -(3 \cdot 3 \cdot 3 \cdot 3) = -81$

Now Try Exercise 17 ■

We can state several useful rules for working with exponential notation. To discover the rule for multiplication, we multiply 5^4 by 5^2:

$$5^4 \cdot 5^2 = \underbrace{(5 \cdot 5 \cdot 5 \cdot 5)}_{4 \text{ factors}}\underbrace{(5 \cdot 5)}_{2 \text{ factors}} = \underbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}_{6 \text{ factors}} = 5^6 = 5^{4+2}$$

It appears that *to multiply two powers of the same base, we add their exponents*. In general, for any real number a and any positive integers m and n, we have

$$a^m a^n = \underbrace{(a \cdot a \cdot \cdots \cdot a)}_{m \text{ factors}}\underbrace{(a \cdot a \cdot \cdots \cdot a)}_{n \text{ factors}} = \underbrace{a \cdot a \cdot a \cdot \cdots \cdot a}_{m+n \text{ factors}} = a^{m+n}$$

Thus $a^m a^n = a^{m+n}$.

We would like this rule to be true even when m and n are 0 or negative integers. For instance, we must have

$$2^0 \cdot 2^3 = 2^{0+3} = 2^3$$

But this can happen only if $2^0 = 1$. Likewise, we want to have

$$5^4 \cdot 5^{-4} = 5^{4+(-4)} = 5^{4-4} = 5^0 = 1$$

and this will be true if $5^{-4} = 1/5^4$. These observations lead to the following definition.

ZERO AND NEGATIVE EXPONENTS

If $a \neq 0$ is a real number and n is a positive integer, then

$$a^0 = 1 \qquad \text{and} \qquad a^{-n} = \frac{1}{a^n}$$

EXAMPLE 2 ■ Zero and Negative Exponents

(a) $\left(\frac{4}{7}\right)^0 = 1$

(b) $x^{-1} = \dfrac{1}{x^1} = \dfrac{1}{x}$

(c) $(-2)^{-3} = \dfrac{1}{(-2)^3} = \dfrac{1}{-8} = -\dfrac{1}{8}$

Now Try Exercise 19 ■

■ Rules for Working with Exponents

Familiarity with the following rules is essential for our work with exponents and bases. In the table the bases a and b are real numbers, and the exponents m and n are integers.

LAWS OF EXPONENTS

Law	Example	Description
1. $a^m a^n = a^{m+n}$	$3^2 \cdot 3^5 = 3^{2+5} = 3^7$	To multiply two powers of the same number, add the exponents.
2. $\dfrac{a^m}{a^n} = a^{m-n}$	$\dfrac{3^5}{3^2} = 3^{5-2} = 3^3$	To divide two powers of the same number, subtract the exponents.
3. $(a^m)^n = a^{mn}$	$(3^2)^5 = 3^{2 \cdot 5} = 3^{10}$	To raise a power to a new power, multiply the exponents.
4. $(ab)^n = a^n b^n$	$(3 \cdot 4)^2 = 3^2 \cdot 4^2$	To raise a product to a power, raise each factor to the power.
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$	$\left(\dfrac{3}{4}\right)^2 = \dfrac{3^2}{4^2}$	To raise a quotient to a power, raise both numerator and denominator to the power.
6. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n$	$\left(\dfrac{3}{4}\right)^{-2} = \left(\dfrac{4}{3}\right)^2$	To raise a fraction to a negative power, invert the fraction and change the sign of the exponent.
7. $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}$	$\dfrac{3^{-2}}{4^{-5}} = \dfrac{4^5}{3^2}$	To move a number raised to a power from numerator to denominator or from denominator to numerator, change the sign of the exponent.

Proof of Law 3 If m and n are positive integers, we have

$$(a^m)^n = (\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}})^n$$

$$= \underbrace{(\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}})(\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}}) \cdots (\underbrace{a \cdot a \cdot \cdots \cdot a}_{m \text{ factors}})}_{n \text{ groups of factors}}$$

$$= \underbrace{a \cdot a \cdot \cdots \cdot a}_{mn \text{ factors}} = a^{mn}$$

The cases for which $m \le 0$ or $n \le 0$ can be proved by using the definition of negative exponents. ■

Proof of Law 4 If n is a positive integer, we have

$$(ab)^n = \underbrace{(ab)(ab)\cdots(ab)}_{n\text{ factors}} = \underbrace{(a\cdot a\cdot\cdots\cdot a)}_{n\text{ factors}}\cdot\underbrace{(b\cdot b\cdot\cdots\cdot b)}_{n\text{ factors}} = a^n b^n$$

Here we have used the Commutative and Associative Properties repeatedly. If $n \le 0$, Law 4 can be proved by using the definition of negative exponents. ■

You are asked to prove Laws 2, 5, 6, and 7 in Exercises 108 and 109.

EXAMPLE 3 ■ Using Laws of Exponents

(a) $x^4x^7 = x^{4+7} = x^{11}$ Law 1: $a^ma^n = a^{m+n}$

(b) $y^4y^{-7} = y^{4-7} = y^{-3} = \dfrac{1}{y^3}$ Law 1: $a^ma^n = a^{m+n}$

(c) $\dfrac{c^9}{c^5} = c^{9-5} = c^4$ Law 2: $\dfrac{a^m}{a^n} = a^{m-n}$

(d) $(b^4)^5 = b^{4\cdot5} = b^{20}$ Law 3: $(a^m)^n = a^{mn}$

(e) $(3x)^3 = 3^3x^3 = 27x^3$ Law 4: $(ab)^n = a^nb^n$

(f) $\left(\dfrac{x}{2}\right)^5 = \dfrac{x^5}{2^5} = \dfrac{x^5}{32}$ Law 5: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Now Try Exercises 29, 31, and 33 ■

EXAMPLE 4 ■ Simplifying Expressions with Exponents

Simplify:

(a) $(2a^3b^2)(3ab^4)^3$ **(b)** $\left(\dfrac{x}{y}\right)^3\left(\dfrac{y^2x}{z}\right)^4$

SOLUTION

(a)
$$\begin{aligned}(2a^3b^2)(3ab^4)^3 &= (2a^3b^2)[3^3a^3(b^4)^3] && \text{Law 4: } (ab)^n = a^nb^n\\ &= (2a^3b^2)(27a^3b^{12}) && \text{Law 3: } (a^m)^n = a^{mn}\\ &= (2)(27)a^3a^3b^2b^{12} && \text{Group factors with the same base}\\ &= 54a^6b^{14} && \text{Law 1: } a^ma^n = a^{m+n}\end{aligned}$$

(b)
$$\begin{aligned}\left(\frac{x}{y}\right)^3\left(\frac{y^2x}{z}\right)^4 &= \frac{x^3}{y^3}\,\frac{(y^2)^4x^4}{z^4} && \text{Laws 5 and 4}\\ &= \frac{x^3}{y^3}\,\frac{y^8x^4}{z^4} && \text{Law 3}\\ &= (x^3x^4)\left(\frac{y^8}{y^3}\right)\frac{1}{z^4} && \text{Group factors with the same base}\\ &= \frac{x^7y^5}{z^4} && \text{Laws 1 and 2}\end{aligned}$$

Now Try Exercises 35 and 39 ■

When simplifying an expression, you will find that many different methods will lead to the same result; you should feel free to use any of the rules of exponents to arrive at your own method. In the next example we see how to simplify expressions with negative exponents.

Mathematics in the Modern World

Although we are often unaware of its presence, mathematics permeates nearly every aspect of life in the modern world. With the advent of modern technology, mathematics plays an ever greater role in our lives. Today you were probably awakened by a digital alarm clock, sent a text, surfed the Internet, watched HDTV or a streaming video, listened to music on your cell phone, drove a car with digitally controlled fuel injection, then fell asleep in a room whose temperature is controlled by a digital thermostat. In each of these activities mathematics is crucially involved. In general, a property such as the intensity or frequency of sound, the oxygen level in the exhaust emission from a car, the colors in an image, or the temperature in your bedroom is transformed into sequences of numbers by sophisticated mathematical algorithms. These numerical data, which usually consist of many millions of bits (the digits 0 and 1), are then transmitted and reinterpreted. Dealing with such huge amounts of data was not feasible until the invention of computers, machines whose logical processes were invented by mathematicians.

The contributions of mathematics in the modern world are not limited to technological advances. The logical processes of mathematics are now used to analyze complex problems in the social, political, and life sciences in new and surprising ways. Advances in mathematics continue to be made, some of the most exciting of these just within the past decade.

In other *Mathematics in the Modern World,* we will describe in more detail how mathematics affects all of us in our everyday activities.

EXAMPLE 5 ■ Simplifying Expressions with Negative Exponents

Eliminate negative exponents, and simplify each expression.

(a) $\dfrac{6st^{-4}}{2s^{-2}t^2}$ **(b)** $\left(\dfrac{y}{3z^3}\right)^{-2}$

SOLUTION

(a) We use Law 7, which allows us to move a number raised to a power from the numerator to the denominator (or vice versa) by changing the sign of the exponent.

t^{-4} moves to denominator and becomes t^4

s^{-2} moves to numerator and becomes s^2

$$\frac{6st^{-4}}{2s^{-2}t^2} = \frac{6ss^2}{2t^2t^4} \qquad \text{Law 7}$$

$$= \frac{3s^3}{t^6} \qquad \text{Law 1}$$

(b) We use Law 6, which allows us to change the sign of the exponent of a fraction by inverting the fraction.

$$\left(\frac{y}{3z^3}\right)^{-2} = \left(\frac{3z^3}{y}\right)^2 \qquad \text{Law 6}$$

$$= \frac{9z^6}{y^2} \qquad \text{Laws 5 and 4}$$

Now Try Exercise 41 ■

■ Scientific Notation

Scientists use exponential notation as a compact way of writing very large numbers and very small numbers. For example, the nearest star beyond the sun, Proxima Centauri, is approximately 40,000,000,000,000 km away. The mass of a hydrogen atom is about 0.00000000000000000000000166 g. Such numbers are difficult to read and to write, so scientists usually express them in *scientific notation*.

SCIENTIFIC NOTATION

A positive number x is said to be written in **scientific notation** if it is expressed as follows:

$$x = a \times 10^n \qquad \text{where } 1 \le a < 10 \text{ and } n \text{ is an integer}$$

For instance, when we state that the distance to the star Proxima Centauri is 4×10^{13} km, the positive exponent 13 indicates that the decimal point should be moved 13 places to the *right*:

$$4 \times 10^{13} = 40{,}000{,}000{,}000{,}000$$

Move decimal point 13 places to the right

When we state that the mass of a hydrogen atom is 1.66×10^{-24} g, the exponent -24 indicates that the decimal point should be moved 24 places to the *left*:

$$1.66 \times 10^{-24} = 0.00000000000000000000000166$$

Move decimal point 24 places to the left

EXAMPLE 6 ■ Changing from Decimal to Scientific Notation

Write each number in scientific notation.

(a) 56,920 **(b)** 0.000093

SOLUTION

(a) $5\underbrace{6{,}920}_{\text{4 places}} = 5.692 \times 10^4$ **(b)** $0.\underbrace{00009}_{\text{5 places}}3 = 9.3 \times 10^{-5}$

Now Try Exercise 83 ■

EXAMPLE 7 ■ Changing from Scientific Notation to Decimal Notation

Write each number in decimal notation.

(a) 6.97×10^9 **(b)** 4.6271×10^{-6}

SOLUTION

(a) $6.97 \times 10^9 = 6\underbrace{{,}970{,}000{,}000}_{\text{9 places}}$ Move decimal 9 places to the right

(b) $4.6271 \times 10^{-6} = 0.\underbrace{000004}_{\text{6 places}}6271$ Move decimal 6 places to the left

Now Try Exercise 85 ■

To use scientific notation on a calculator, press the key labeled EE or EXP or EEX to enter the exponent. For example, to enter the number 3.629×10^{15} on a TI-83 or TI-84 calculator, we enter

3.629 2ND EE 15

and the display reads

3.629E15

Scientific notation is often used on a calculator to display a very large or very small number. For instance, if we use a calculator to square the number 1,111,111, the display panel may show (depending on the calculator model) the approximation

1.234568 12 or 1.234568 E12

Here the final digits indicate the power of 10, and we interpret the result as

$$1.234568 \times 10^{12}$$

EXAMPLE 8 ■ Calculating with Scientific Notation

If $a \approx 0.00046$, $b \approx 1.697 \times 10^{22}$, and $c \approx 2.91 \times 10^{-18}$, use a calculator to approximate the quotient ab/c.

SOLUTION We could enter the data using scientific notation, or we could use laws of exponents as follows:

$$\begin{aligned}\frac{ab}{c} &\approx \frac{(4.6 \times 10^{-4})(1.697 \times 10^{22})}{2.91 \times 10^{-18}}\\ &= \frac{(4.6)(1.697)}{2.91} \times 10^{-4+22+18}\\ &\approx 2.7 \times 10^{36}\end{aligned}$$

For guidelines on working with significant figures, see Appendix B, *Calculations and Significant Figures.* Go to **www.stewartmath.com**.

We state the answer rounded to two significant figures because the least accurate of the given numbers is stated to two significant figures.

Now Try Exercises 89 and 91 ■

■ Radicals

We know what 2^n means whenever n is an integer. To give meaning to a power, such as $2^{4/5}$, whose exponent is a rational number, we need to discuss radicals.

It is true that the number 9 has two square roots, 3 and -3, but the notation $\sqrt{9}$ is reserved for the *positive* square root of 9 (sometimes called the *principal square root* of 9). If we want the negative root, we must write $-\sqrt{9}$, which is -3.

The symbol $\sqrt{\ }$ means "the positive square root of." Thus

$$\sqrt{a} = b \qquad \text{means} \qquad b^2 = a \qquad \text{and} \qquad b \ge 0$$

Since $a = b^2 \ge 0$, the symbol $\sqrt{a}$ makes sense only when $a \ge 0$. For instance,

$$\sqrt{9} = 3 \qquad \text{because} \qquad 3^2 = 9 \qquad \text{and} \qquad 3 \ge 0$$

Square roots are special cases of nth roots. The nth root of x is the number that, when raised to the nth power, gives x.

DEFINITION OF *n*th ROOT

If n is any positive integer, then the **principal *n*th root** of a is defined as follows:

$$\sqrt[n]{a} = b \qquad \text{means} \qquad b^n = a$$

If n is even, we must have $a \ge 0$ and $b \ge 0$.

For example,

$$\sqrt[4]{81} = 3 \qquad \text{because} \qquad 3^4 = 81 \qquad \text{and} \qquad 3 \ge 0$$

$$\sqrt[3]{-8} = -2 \qquad \text{because} \qquad (-2)^3 = -8$$

But $\sqrt{-8}$, $\sqrt[4]{-8}$, and $\sqrt[6]{-8}$ are not defined. (For instance, $\sqrt{-8}$ is not defined because the square of every real number is nonnegative.)

Notice that

$$\sqrt{4^2} = \sqrt{16} = 4 \qquad \text{but} \qquad \sqrt{(-4)^2} = \sqrt{16} = 4 = |-4|$$

So the equation $\sqrt{a^2} = a$ is not always true; it is true only when $a \ge 0$. However, we can always write $\sqrt{a^2} = |a|$. This last equation is true not only for square roots, but for any even root. This and other rules used in working with nth roots are listed in the following box. In each property we assume that all the given roots exist.

PROPERTIES OF *n*th ROOTS

Property	Example
1. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$	$\sqrt[3]{-8 \cdot 27} = \sqrt[3]{-8}\sqrt[3]{27} = (-2)(3) = -6$
2. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt[4]{\dfrac{16}{81}} = \dfrac{\sqrt[4]{16}}{\sqrt[4]{81}} = \dfrac{2}{3}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$	$\sqrt{\sqrt[3]{729}} = \sqrt[6]{729} = 3$
4. $\sqrt[n]{a^n} = a$ if n is odd	$\sqrt[3]{(-5)^3} = -5,\quad \sqrt[5]{2^5} = 2$
5. $\sqrt[n]{a^n} = \|a\|$ if n is even	$\sqrt[4]{(-3)^4} = \|-3\| = 3$

EXAMPLE 9 ■ Simplifying Expressions Involving *n*th Roots

(a)
$$\begin{aligned} \sqrt[3]{x^4} &= \sqrt[3]{x^3 x} && \text{Factor out the largest cube} \\ &= \sqrt[3]{x^3}\sqrt[3]{x} && \text{Property 1: } \sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b} \\ &= x\sqrt[3]{x} && \text{Property 4: } \sqrt[3]{a^3} = a \end{aligned}$$

(b) $\sqrt[4]{81x^8y^4} = \sqrt[4]{81}\sqrt[4]{x^8}\sqrt[4]{y^4}$ — Property 1: $\sqrt[4]{abc} = \sqrt[4]{a}\sqrt[4]{b}\sqrt[4]{c}$

$= 3\sqrt[4]{(x^2)^4}|y|$ — Property 5: $\sqrt[4]{a^4} = |a|$

$= 3x^2|y|$ — Property 5: $\sqrt[4]{a^4} = |a|$, $|x^2| = x^2$

Now Try Exercises 45 and 47 ■

It is frequently useful to combine like radicals in an expression such as $2\sqrt{3} + 5\sqrt{3}$. This can be done by using the Distributive Property. For example,

$$2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3} = 7\sqrt{3}$$

The next example further illustrates this process.

Avoid making the following error:

$$\sqrt{a + b} \cancel{=} \sqrt{a} + \sqrt{b}$$

For instance, if we let $a = 9$ and $b = 16$, then we see the error:

$$\sqrt{9 + 16} \stackrel{?}{=} \sqrt{9} + \sqrt{16}$$

$$\sqrt{25} \stackrel{?}{=} 3 + 4$$

$$5 \stackrel{?}{=} 7 \quad \text{Wrong!}$$

EXAMPLE 10 ■ Combining Radicals

(a) $\sqrt{32} + \sqrt{200} = \sqrt{16 \cdot 2} + \sqrt{100 \cdot 2}$ — Factor out the largest squares

$= \sqrt{16}\sqrt{2} + \sqrt{100}\sqrt{2}$ — Property 1: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 4\sqrt{2} + 10\sqrt{2} = 14\sqrt{2}$ — Distributive Property

(b) If $b > 0$, then

$\sqrt{25b} - \sqrt{b^3} = \sqrt{25}\sqrt{b} - \sqrt{b^2}\sqrt{b}$ — Property 1: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 5\sqrt{b} - b\sqrt{b}$ — Property 5, $b > 0$

$= (5 - b)\sqrt{b}$ — Distributive Property

(c) $\sqrt{49x^2 + 49} = \sqrt{49(x^2 + 1)}$ — Factor out the perfect square

$= 7\sqrt{x^2 + 1}$ — Property 1: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

Now Try Exercises 49, 51, and 53 ■

■ Rational Exponents

To define what is meant by a *rational exponent* or, equivalently, a *fractional exponent* such as $a^{1/3}$, we need to use radicals. To give meaning to the symbol $a^{1/n}$ in a way that is consistent with the Laws of Exponents, we would have to have

$$(a^{1/n})^n = a^{(1/n)n} = a^1 = a$$

So by the definition of nth root,

$$\boxed{a^{1/n} = \sqrt[n]{a}}$$

In general, we define rational exponents as follows.

DEFINITION OF RATIONAL EXPONENTS

For any rational exponent m/n in lowest terms, where m and n are integers and $n > 0$, we define

$$a^{m/n} = (\sqrt[n]{a})^m \qquad \text{or equivalently} \qquad a^{m/n} = \sqrt[n]{a^m}$$

If n is even, then we require that $a \geq 0$.

With this definition it can be proved that *the Laws of Exponents also hold for rational exponents*.

DIOPHANTUS lived in Alexandria about 250 A.D. His book *Arithmetica* is considered the first book on algebra. In it he gives methods for finding integer solutions of algebraic equations. *Arithmetica* was read and studied for more than a thousand years. Fermat (see page 117) made some of his most important discoveries while studying this book. Diophantus' major contribution is the use of symbols to stand for the unknowns in a problem. Although his symbolism is not as simple as what we use today, it was a major advance over writing everything in words. In Diophantus' notation the equation

$$x^5 - 7x^2 + 8x - 5 = 24$$

is written

$$\Delta K^{\gamma} \alpha \varsigma \eta \pitchfork \Delta^{\gamma} \zeta \overset{\circ}{M} \varepsilon \iota^{\sigma} \kappa \delta$$

Our modern algebraic notation did not come into common use until the 17th century.

EXAMPLE 11 ■ Using the Definition of Rational Exponents

(a) $4^{1/2} = \sqrt{4} = 2$

(b) $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$ Alternative solution: $8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

(c) $125^{-1/3} = \dfrac{1}{125^{1/3}} = \dfrac{1}{\sqrt[3]{125}} = \dfrac{1}{5}$

Now Try Exercises 55 and 57 ■

EXAMPLE 12 ■ Using the Laws of Exponents with Rational Exponents

(a) $a^{1/3}a^{7/3} = a^{8/3}$ Law 1: $a^m a^n = a^{m+n}$

(b) $\dfrac{a^{2/5}a^{7/5}}{a^{3/5}} = a^{2/5+7/5-3/5} = a^{6/5}$ Law 1, Law 2: $\dfrac{a^m}{a^n} = a^{m-n}$

(c)
$$\begin{aligned}(2a^3b^4)^{3/2} &= 2^{3/2}(a^3)^{3/2}(b^4)^{3/2} && \text{Law 4: } (abc)^n = a^n b^n c^n\\ &= (\sqrt{2})^3 a^{3(3/2)} b^{4(3/2)} && \text{Law 3: } (a^m)^n = a^{mn}\\ &= 2\sqrt{2}a^{9/2}b^6\end{aligned}$$

(d)
$$\begin{aligned}\left(\frac{2x^{3/4}}{y^{1/3}}\right)^3\left(\frac{y^4}{x^{-1/2}}\right) &= \frac{2^3(x^{3/4})^3}{(y^{1/3})^3}\cdot(y^4x^{1/2}) && \text{Laws 5, 4, and 7}\\ &= \frac{8x^{9/4}}{y}\cdot y^4x^{1/2} && \text{Law 3}\\ &= 8x^{11/4}y^3 && \text{Laws 1 and 2}\end{aligned}$$

Now Try Exercises 61, 63, 67, and 69 ■

EXAMPLE 13 ■ Simplifying by Writing Radicals as Rational Exponents

(a) $\dfrac{1}{\sqrt[3]{x^4}} = \dfrac{1}{x^{4/3}} = x^{-4/3}$ Definition of rational and negative exponents

(b)
$$\begin{aligned}(2\sqrt{x})(3\sqrt[3]{x}) &= (2x^{1/2})(3x^{1/3}) && \text{Definition of rational exponents}\\ &= 6x^{1/2+1/3} = 6x^{5/6} && \text{Law 1}\end{aligned}$$

(c)
$$\begin{aligned}\sqrt{x\sqrt{x}} &= (xx^{1/2})^{1/2} && \text{Definition of rational exponents}\\ &= (x^{3/2})^{1/2} && \text{Law 1}\\ &= x^{3/4} && \text{Law 3}\end{aligned}$$

Now Try Exercises 73 and 77 ■

■ Rationalizing the Denominator; Standard Form

It is often useful to eliminate the radical in a denominator by multiplying both numerator and denominator by an appropriate expression. This procedure is called **rationalizing the denominator**. If the denominator is of the form $\sqrt{a}$, we multiply numerator and denominator by $\sqrt{a}$. In doing this we multiply the given quantity by 1, so we do not change its value. For instance,

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}\cdot 1 = \frac{1}{\sqrt{a}}\cdot\frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

Note that the denominator in the last fraction contains no radical. In general, if the denominator is of the form $\sqrt[n]{a^m}$ with $m < n$, then multiplying the numerator and denominator by $\sqrt[n]{a^{n-m}}$ will rationalize the denominator, because (for $a > 0$)

$$\sqrt[n]{a^m}\sqrt[n]{a^{n-m}} = \sqrt[n]{a^{m+n-m}} = \sqrt[n]{a^n} = a$$

A fractional expression whose denominator contains no radicals is said to be in **standard form**.

EXAMPLE 14 ■ Rationalizing Denominators

Put each fractional expression into standard form by rationalizing the denominator.

(a) $\dfrac{2}{\sqrt{3}}$ **(b)** $\dfrac{1}{\sqrt[3]{5}}$ **(c)** $\sqrt[7]{\dfrac{1}{a^2}}$

SOLUTION

This equals 1

(a) $\dfrac{2}{\sqrt{3}} = \dfrac{2}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$ Multiply by $\dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{2\sqrt{3}}{3}$ $\sqrt{3} \cdot \sqrt{3} = 3$

(b) $\dfrac{1}{\sqrt[3]{5}} = \dfrac{1}{\sqrt[3]{5}} \cdot \dfrac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$ Multiply by $\dfrac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$

$= \dfrac{\sqrt[3]{25}}{5}$ $\sqrt[3]{5} \cdot \sqrt[3]{5^2} = \sqrt[3]{5^3} = 5$

(c) $\sqrt[7]{\dfrac{1}{a^2}} = \dfrac{1}{\sqrt[7]{a^2}}$ Property 2: $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

$= \dfrac{1}{\sqrt[7]{a^2}} \cdot \dfrac{\sqrt[7]{a^5}}{\sqrt[7]{a^5}}$ Multiply by $\dfrac{\sqrt[7]{a^5}}{\sqrt[7]{a^5}}$

$= \dfrac{\sqrt[7]{a^5}}{a}$ $\sqrt[7]{a^2} \cdot \sqrt[7]{a^5} = a$

Now Try Exercises 79 and 81 ■

1.2 EXERCISES

CONCEPTS

1. (a) Using exponential notation, we can write the product $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ as ________.

(b) In the expression 3^4 the number 3 is called the ________, and the number 4 is called the ________.

2. (a) When we multiply two powers with the same base, we ________ the exponents. So $3^4 \cdot 3^5 =$ ________.

(b) When we divide two powers with the same base, we ________ the exponents. So $\dfrac{3^5}{3^2} =$ ________.

3. (a) Using exponential notation, we can write $\sqrt[3]{5}$ as ________.

(b) Using radicals, we can write $5^{1/2}$ as ________.

(c) Is there a difference between $\sqrt{5^2}$ and $(\sqrt{5})^2$? Explain.

4. Explain what $4^{3/2}$ means, then calculate $4^{3/2}$ in two different ways:

$(4^{1/2})^{\square} =$ ________ or $(4^3)^{\square} =$ ________

5. Explain how we rationalize a denominator, then complete the following steps to rationalize $\dfrac{1}{\sqrt{3}}$:

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\square}{\square} = \frac{\square}{\square}$$

6. Find the missing power in the following calculation: $5^{1/3} \cdot 5^{\square} = 5$.

7–8 ■ *Yes or No*? If *No*, give a reason.

7. (a) Is the expression $\left(\frac{2}{3}\right)^{-2}$ equal to $\frac{3}{4}$?

(b) Is there a difference between $(-5)^4$ and -5^4?

8. (a) Is the expression $(x^2)^3$ equal to x^5?

(b) Is the expression $(2x^4)^3$ equal to $2x^{12}$?

(c) Is the expression $\sqrt{4a^2}$ equal to $2a$?

(d) Is the expression $\sqrt{a^2 + 4}$ equal to $a + 2$?

SKILLS

9–16 ■ Radicals and Exponents Write each radical expression using exponents, and each exponential expression using radicals.

	Radical expression	Exponential expression
9.	$\dfrac{1}{\sqrt{3}}$	
10.	$\sqrt[3]{7^2}$	
11.		$4^{2/3}$
12.		$10^{-3/2}$
13.	$\sqrt[5]{5^3}$	
14.		$2^{-1.5}$
15.		$a^{2/5}$
16.	$\dfrac{1}{\sqrt{x^5}}$	

17–28 ■ Radicals and Exponents Evaluate each expression.

17. (a) -2^6 **(b)** $(-2)^6$ **(c)** $\left(\frac{1}{5}\right)^2 \cdot (-3)^3$

18. (a) $(-5)^3$ **(b)** -5^3 **(c)** $(-5)^2 \cdot \left(\frac{2}{5}\right)^2$

19. (a) $\left(\frac{5}{3}\right)^0 \cdot 2^{-1}$ **(b)** $\dfrac{2^{-3}}{3^0}$ **(c)** $\left(\frac{2}{3}\right)^{-2}$

20. (a) $-2^3 \cdot (-2)^0$ **(b)** $-2^{-3} \cdot (-2)^0$ **(c)** $\left(\frac{-3}{5}\right)^{-3}$

21. (a) $5^3 \cdot 5$ **(b)** $5^4 \cdot 5^{-2}$ **(c)** $(2^2)^3$

22. (a) $3^8 \cdot 3^5$ **(b)** $\dfrac{10^7}{10^4}$ **(c)** $(3^5)^4$

23. (a) $3\sqrt[3]{16}$ **(b)** $\dfrac{\sqrt{18}}{\sqrt{81}}$ **(c)** $\sqrt{\frac{27}{4}}$

24. (a) $2\sqrt[3]{81}$ **(b)** $\dfrac{\sqrt{18}}{\sqrt{25}}$ **(c)** $\sqrt{\frac{12}{49}}$

25. (a) $\sqrt{3}\sqrt{15}$ **(b)** $\dfrac{\sqrt{48}}{\sqrt{3}}$ **(c)** $\sqrt[3]{24}\sqrt[3]{18}$

26. (a) $\sqrt{10}\sqrt{32}$ **(b)** $\dfrac{\sqrt{54}}{\sqrt{6}}$ **(c)** $\sqrt[3]{15}\sqrt[3]{75}$

27. (a) $\dfrac{\sqrt{132}}{\sqrt{3}}$ **(b)** $\sqrt[3]{2}\sqrt[3]{32}$ **(c)** $\sqrt[4]{\frac{1}{4}}\sqrt[4]{\frac{1}{64}}$

28. (a) $\sqrt[5]{\frac{1}{8}}\sqrt[5]{\frac{1}{4}}$ **(b)** $\sqrt[6]{\frac{1}{2}}\sqrt[6]{128}$ **(c)** $\dfrac{\sqrt[3]{4}}{\sqrt[3]{108}}$

29–34 ■ Exponents Simplify each expression, and eliminate any negative exponents.

29. (a) $x^3 \cdot x^4$ **(b)** $(2y^2)^3$ **(c)** $y^{-2}y^7$

30. (a) $y^5 \cdot y^2$ **(b)** $(8x)^2$ **(c)** x^4x^{-3}

31. (a) $x^{-5} \cdot x^3$ **(b)** $w^{-2}w^{-4}w^5$ **(c)** $\dfrac{x^{16}}{x^{10}}$

32. (a) $y^2 \cdot y^{-5}$ **(b)** $z^5z^{-3}z^{-4}$ **(c)** $\dfrac{y^7y^0}{y^{10}}$

33. (a) $\dfrac{a^9a^{-2}}{a}$ **(b)** $(a^2a^4)^3$ **(c)** $\left(\dfrac{x}{2}\right)^3(5x^6)$

34. (a) $\dfrac{z^2z^4}{z^3z^{-1}}$ **(b)** $(2a^3a^2)^4$ **(c)** $(-3z^2)^3(2z^3)$

35–44 ■ Exponents Simplify each expression, and eliminate any negative exponents.

35. (a) $(3x^3y^2)(2y^3)$ **(b)** $(5w^2z^{-2})^2(z^3)$

36. (a) $(8m^{-2}n^4)\left(\frac{1}{2}n^{-2}\right)$ **(b)** $(3a^4b^{-2})^3(a^2b^{-1})$

37. (a) $\dfrac{x^2y^{-1}}{x^{-5}}$ **(b)** $\left(\dfrac{a^3}{2b^2}\right)^3$

38. (a) $\dfrac{y^{-2}z^{-3}}{y^{-1}}$ **(b)** $\left(\dfrac{x^3y^{-2}}{x^{-3}y^2}\right)^{-2}$

39. (a) $\left(\dfrac{a^2}{b}\right)^5\left(\dfrac{a^3b^2}{c^3}\right)^3$ **(b)** $\dfrac{(u^{-1}v^2)^2}{(u^3v^{-2})^3}$

40. (a) $\left(\dfrac{x^4z^2}{4y^5}\right)\left(\dfrac{2x^3y^2}{z^3}\right)^2$ **(b)** $\dfrac{(rs^2)^3}{(r^{-3}s^2)^2}$

41. (a) $\dfrac{8a^3b^{-4}}{2a^{-5}b^5}$ **(b)** $\left(\dfrac{y}{5x^{-2}}\right)^{-3}$

42. (a) $\dfrac{5xy^{-2}}{x^{-1}y^{-3}}$ **(b)** $\left(\dfrac{2a^{-1}b}{a^2b^{-3}}\right)^{-3}$

43. (a) $\left(\dfrac{3a}{b^3}\right)^{-1}$ **(b)** $\left(\dfrac{q^{-1}r^{-1}s^{-2}}{r^{-5}sq^{-8}}\right)^{-1}$

44. (a) $\left(\dfrac{s^2t^{-4}}{5s^{-1}t}\right)^{-2}$ **(b)** $\left(\dfrac{xy^{-2}z^{-3}}{x^2y^3z^{-4}}\right)^{-3}$

45–48 ■ Radicals Simplify the expression. Assume that the letters denote any positive real numbers.

45. (a) $\sqrt[4]{x^4}$ **(b)** $\sqrt[4]{16x^8}$

46. (a) $\sqrt[5]{x^{10}}$ **(b)** $\sqrt[3]{x^3y^6}$

47. (a) $\sqrt[6]{64a^6b^7}$ **(b)** $\sqrt[3]{a^2b}\sqrt[3]{64a^4b}$

48. (a) $\sqrt[4]{x^4y^2z^2}$ **(b)** $\sqrt[3]{\sqrt{64x^6}}$

49–54 ■ Radical Expressions Simplify the expression.

49. (a) $\sqrt{32} + \sqrt{18}$ **(b)** $\sqrt{75} + \sqrt{48}$

50. (a) $\sqrt{125} + \sqrt{45}$ **(b)** $\sqrt[3]{54} - \sqrt[3]{16}$

51. (a) $\sqrt{9a^3} + \sqrt{a}$ **(b)** $\sqrt{16x} + \sqrt{x^5}$

52. (a) $\sqrt[3]{x^4} + \sqrt[3]{8x}$ **(b)** $4\sqrt{18rt^3} + 5\sqrt{32r^3t^5}$

53. (a) $\sqrt{81x^2 + 81}$ **(b)** $\sqrt{36x^2 + 36y^2}$

54. (a) $\sqrt{27a^2 + 63a}$ **(b)** $\sqrt{75t + 100t^2}$

55–60 ■ Rational Exponents Evaluate each expression.

55. (a) $16^{1/4}$ **(b)** $-8^{1/3}$ **(c)** $9^{-1/2}$

56. (a) $27^{1/3}$ **(b)** $(-8)^{1/3}$ **(c)** $-\left(\frac{1}{8}\right)^{1/3}$

57. (a) $32^{2/5}$ **(b)** $\left(\frac{4}{9}\right)^{-1/2}$ **(c)** $\left(\frac{16}{81}\right)^{3/4}$

58. (a) $125^{2/3}$ **(b)** $\left(\frac{25}{64}\right)^{3/2}$ **(c)** $27^{-4/3}$

59. **(a)** $5^{2/3} \cdot 5^{1/3}$ **(b)** $\dfrac{3^{3/5}}{3^{2/5}}$ **(c)** $(\sqrt[3]{4})^3$

60. **(a)** $3^{2/7} \cdot 3^{12/7}$ **(b)** $\dfrac{7^{2/3}}{7^{5/3}}$ **(c)** $(\sqrt[5]{6})^{-10}$

61–70 ■ Rational Exponents Simplify the expression and eliminate any negative exponent(s). Assume that all letters denote positive numbers.

61. **(a)** $x^{3/4}x^{5/4}$ **(b)** $y^{2/3}y^{4/3}$

62. **(a)** $(4b)^{1/2}(8b^{1/4})$ **(b)** $(3a^{3/4})^2(5a^{1/2})$

63. **(a)** $\dfrac{w^{4/3}w^{2/3}}{w^{1/3}}$ **(b)** $\dfrac{a^{5/4}(2a^{3/4})^3}{a^{1/4}}$

64. **(a)** $(8y^3)^{-2/3}$ **(b)** $(u^4v^6)^{-1/3}$

65. **(a)** $(8a^6b^{3/2})^{2/3}$ **(b)** $(4a^6b^8)^{3/2}$

66. **(a)** $(x^{-5}y^{1/3})^{-3/5}$ **(b)** $(4r^8s^{-1/2})^{1/2}(32s^{-5/4})^{-1/5}$

67. **(a)** $\dfrac{(8s^3t^3)^{2/3}}{(s^4t^{-8})^{1/4}}$ **(b)** $\dfrac{(32x^5y^{-3/2})^{2/5}}{(x^{5/3}y^{2/3})^{3/5}}$

68. **(a)** $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4}$ **(b)** $\left(\dfrac{4s^3t^4}{s^2t^{9/2}}\right)^{-1/2}$

69. **(a)** $\left(\dfrac{x^{3/2}}{y^{-1/2}}\right)^4\left(\dfrac{x^{-2}}{y^3}\right)$ **(b)** $\left(\dfrac{4y^3z^{2/3}}{x^{1/2}}\right)^2\left(\dfrac{x^{-3}y^6}{8z^4}\right)^{1/3}$

70. **(a)** $\left(\dfrac{a^{1/6}b^{-3}}{x^{-1}y}\right)^3\left(\dfrac{x^{-2}b^{-1}}{a^{3/2}y^{1/3}}\right)$ **(b)** $\dfrac{(9st)^{3/2}}{(27s^3t^{-4})^{2/3}}\left(\dfrac{3s^{-2}}{4t^{1/3}}\right)^{-1}$

71–78 ■ Radicals Simplify the expression, and eliminate any negative exponents(s). Assume that all letters denote positive numbers.

71. **(a)** $\sqrt{x^3}$ **(b)** $\sqrt[5]{x^6}$

72. **(a)** $\sqrt{x^5}$ **(b)** $\sqrt[4]{x^6}$

73. **(a)** $\sqrt[6]{y^5}\sqrt[3]{y^2}$ **(b)** $(5\sqrt[3]{x})(2\sqrt[4]{x})$

74. **(a)** $\sqrt[4]{b^3}\sqrt{b}$ **(b)** $(2\sqrt{a})(\sqrt[3]{a^2})$

75. **(a)** $\sqrt{4st^3}\sqrt[6]{s^3t^2}$ **(b)** $\dfrac{\sqrt[4]{x^7}}{\sqrt[4]{x^3}}$

76. **(a)** $\sqrt[5]{x^3y^2}\sqrt[10]{x^4y^{16}}$ **(b)** $\dfrac{\sqrt[3]{8x^2}}{\sqrt{x}}$

77. **(a)** $\sqrt[3]{y\sqrt{y}}$ **(b)** $\sqrt{\dfrac{16u^3v}{uv^5}}$

78. **(a)** $\sqrt{s\sqrt{s^3}}$ **(b)** $\sqrt[3]{\dfrac{54x^2y^4}{2x^5y}}$

79–82 ■ Rationalize Put each fractional expression into standard form by rationalizing the denominator.

79. **(a)** $\dfrac{1}{\sqrt{6}}$ **(b)** $\sqrt{\frac{3}{2}}$ **(c)** $\dfrac{9}{\sqrt[4]{2}}$

80. **(a)** $\dfrac{12}{\sqrt{3}}$ **(b)** $\sqrt{\frac{12}{5}}$ **(c)** $\dfrac{8}{\sqrt[3]{5^2}}$

81. **(a)** $\dfrac{1}{\sqrt{5x}}$ **(b)** $\sqrt{\dfrac{x}{5}}$ **(c)** $\sqrt[5]{\dfrac{1}{x^3}}$

82. **(a)** $\sqrt{\dfrac{s}{3t}}$ **(b)** $\dfrac{a}{\sqrt[6]{b^2}}$ **(c)** $\dfrac{1}{c^{3/5}}$

83–84 ■ Scientific Notation Write each number in scientific notation.

83. **(a)** 69,300,000 **(b)** 7,200,000,000,000
(c) 0.000028536 **(d)** 0.0001213

84. **(a)** 129,540,000 **(b)** 7,259,000,000
(c) 0.0000000014 **(d)** 0.0007029

85–86 ■ Decimal Notation Write each number in decimal notation.

85. **(a)** 3.19×10^5 **(b)** 2.721×10^8
(c) 2.670×10^{-8} **(d)** 9.999×10^{-9}

86. **(a)** 7.1×10^{14} **(b)** 6×10^{12}
(c) 8.55×10^{-3} **(d)** 6.257×10^{-10}

87–88 ■ Scientific Notation Write the number indicated in each statement in scientific notation.

87. **(a)** A light-year, the distance that light travels in one year, is about 5,900,000,000,000 mi.
(b) The diameter of an electron is about 0.0000000000004 cm.
(c) A drop of water contains more than 33 billion billion molecules.

88. **(a)** The distance from the earth to the sun is about 93 million miles.
(b) The mass of an oxygen molecule is about 0.000000000000000000000053 g.
(c) The mass of the earth is about 5,970,000,000,000,000,000,000,000 kg.

89–94 ■ Scientific Notation Use scientific notation, the Laws of Exponents, and a calculator to perform the indicated operations. State your answer rounded to the number of significant digits indicated by the given data.

89. $(7.2 \times 10^{-9})(1.806 \times 10^{-12})$

90. $(1.062 \times 10^{24})(8.61 \times 10^{19})$

91. $\dfrac{1.295643 \times 10^9}{(3.610 \times 10^{-17})(2.511 \times 10^6)}$

92. $\dfrac{(73.1)(1.6341 \times 10^{28})}{0.0000000019}$

93. $\dfrac{(0.0000162)(0.01582)}{(594{,}621{,}000)(0.0058)}$ **94.** $\dfrac{(3.542 \times 10^{-6})^9}{(5.05 \times 10^4)^{12}}$

SKILLS Plus

95. Let a, b, and c be real numbers with $a > 0$, $b < 0$, and $c < 0$. Determine the sign of each expression.
(a) b^5 **(b)** b^{10} **(c)** ab^2c^3
(d) $(b - a)^3$ **(e)** $(b - a)^4$ **(f)** $\dfrac{a^3c^3}{b^6c^6}$

96. Comparing Roots Without using a calculator, determine which number is larger in each pair.
(a) $2^{1/2}$ or $2^{1/3}$ **(b)** $\left(\frac{1}{2}\right)^{1/2}$ or $\left(\frac{1}{2}\right)^{1/3}$
(c) $7^{1/4}$ or $4^{1/3}$ **(d)** $\sqrt[3]{5}$ or $\sqrt{3}$

APPLICATIONS

97. Distance to the Nearest Star Proxima Centauri, the star nearest to our solar system, is 4.3 light-years away. Use the information in Exercise 87(a) to express this distance in miles.

98. Speed of Light The speed of light is about 186,000 mi/s. Use the information in Exercise 88(a) to find how long it takes for a light ray from the sun to reach the earth.

99. Volume of the Oceans The average ocean depth is 3.7×10^3 m, and the area of the oceans is 3.6×10^{14} m^2. What is the total volume of the ocean in liters? (One cubic meter contains 1000 liters.)

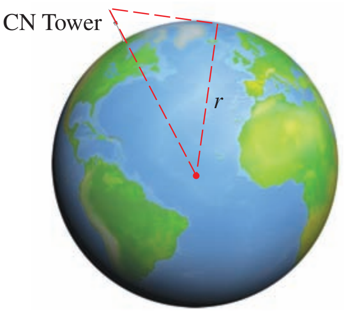
100. National Debt As of July 2013, the population of the United States was 3.164×10^8, and the national debt was 1.674×10^{13} dollars. How much was each person's share of the debt?
[*Source:* U.S. Census Bureau and U.S. Department of Treasury]

101. Number of Molecules A sealed room in a hospital, measuring 5 m wide, 10 m long, and 3 m high, is filled with pure oxygen. One cubic meter contains 1000 L, and 22.4 L of any gas contains 6.02×10^{23} molecules (Avogadro's number). How many molecules of oxygen are there in the room?

102. How Far Can You See? Because of the curvature of the earth, the maximum distance D that you can see from the top of a tall building of height h is estimated by the formula

$$D = \sqrt{2rh + h^2}$$

where $r = 3960$ mi is the radius of the earth and D and h are also measured in miles. How far can you see from the observation deck of the Toronto CN Tower, 1135 ft above the ground?



103. Speed of a Skidding Car Police use the formula $s = \sqrt{30fd}$ to estimate the speed s (in mi/h) at which a car is traveling if it skids d feet after the brakes are applied suddenly. The number f is the coefficient of friction of the road, which is a measure of the "slipperiness" of the road. The table gives some typical estimates for f.

	Tar	Concrete	Gravel
Dry	1.0	0.8	0.2
Wet	0.5	0.4	0.1

(a) If a car skids 65 ft on wet concrete, how fast was it moving when the brakes were applied?

(b) If a car is traveling at 50 mi/h, how far will it skid on wet tar?

104. Distance from the Earth to the Sun It follows from **Kepler's Third Law** of planetary motion that the average distance from a planet to the sun (in meters) is

$$d = \left(\frac{GM}{4\pi^2}\right)^{1/3} T^{2/3}$$

where $M = 1.99 \times 10^{30}$ kg is the mass of the sun, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2 is the gravitational constant, and T is the period of the planet's orbit (in seconds). Use the fact that the period of the earth's orbit is about 365.25 days to find the distance from the earth to the sun.

DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

105. DISCUSS: How Big is a Billion? If you had a million (10^6) dollars in a suitcase, and you spent a thousand (10^3) dollars each day, how many years would it take you to use all the money? Spending at the same rate, how many years would it take you to empty a suitcase filled with a *billion* (10^9) dollars?

106. DISCUSS: Easy Powers that Look Hard Calculate these expressions in your head. Use the Laws of Exponents to help you.

(a) $\dfrac{18^5}{9^5}$ **(b)** $20^6 \cdot (0.5)^6$