

1.1 REAL NUMBERS

■ Real Numbers ■ Properties of Real Numbers ■ Addition and Subtraction ■ Multiplication and Division ■ The Real Line ■ Sets and Intervals ■ Absolute Value and Distance

In the real world we use numbers to measure and compare different quantities. For example, we measure temperature, length, height, weight, blood pressure, distance, speed, acceleration, energy, force, angles, age, cost, and so on. Figure 1 illustrates some situations in which numbers are used. Numbers also allow us to express relationships between different quantities—for example, relationships between the radius and volume of a ball, between miles driven and gas used, or between education level and starting salary.



FIGURE 1 Measuring with real numbers

■ Real Numbers

Let's review the types of numbers that make up the real number system. We start with the **natural numbers**:

$$1, 2, 3, 4, \dots$$

The **integers** consist of the natural numbers together with their negatives and 0:

$$\dots, -3, -2, -1, 0, 1, 2, 3, 4, \dots$$

We construct the **rational numbers** by taking ratios of integers. Thus any rational number r can be expressed as

$$r = \frac{m}{n}$$

where m and n are integers and $n \neq 0$. Examples are

$$\frac{1}{2}, \quad -\frac{3}{7}, \quad 46 = \frac{46}{1}, \quad 0.17 = \frac{17}{100}$$

(Recall that division by 0 is always ruled out, so expressions like $\frac{3}{0}$ and $\frac{0}{0}$ are undefined.) There are also real numbers, such as $\sqrt{2}$, that cannot be expressed as a ratio of integers and are therefore called **irrational numbers**. It can be shown, with varying degrees of difficulty, that these numbers are also irrational:

$$\sqrt{3}, \quad \sqrt{5}, \quad \sqrt[3]{2}, \quad \pi, \quad \frac{3}{\pi^2}$$

The set of all real numbers is usually denoted by the symbol $\mathbb{R}$. When we use the word *number* without qualification, we will mean “real number.” Figure 2 is a diagram of the types of real numbers that we work with in this book.

Every real number has a decimal representation. If the number is rational, then its corresponding decimal is repeating. For example,

$$\frac{1}{2} = 0.5000\dots = 0.5\overline{0} \qquad \frac{2}{3} = 0.6666\dots = 0.\overline{6}$$

$$\frac{157}{495} = 0.3171717\dots = 0.31\overline{7} \qquad \frac{9}{7} = 1.285714285714\dots = 1.\overline{285714}$$

The different types of real numbers were invented to meet specific needs. For example, natural numbers are needed for counting, negative numbers for describing debt or below-zero temperatures, rational numbers for concepts like “half a gallon of milk,” and irrational numbers for measuring certain distances, like the diagonal of a square.

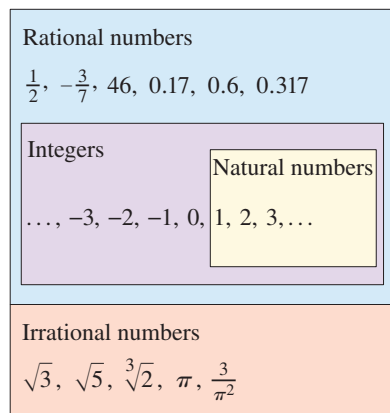


FIGURE 2 The real number system

A repeating decimal such as

$$x = 3.5474747\ldots$$

is a rational number. To convert it to a ratio of two integers, we write

$$\begin{array}{r} 1000x = 3547.47474747\ldots \\ 10x = 35.47474747\ldots \\ \hline 990x = 3512.0 \end{array}$$

Thus $x = \frac{3512}{990}$. (The idea is to multiply x by appropriate powers of 10 and then subtract to eliminate the repeating part.)

(The bar indicates that the sequence of digits repeats forever.) If the number is irrational, the decimal representation is nonrepeating:

$$\sqrt{2} = 1.414213562373095\ldots \quad \pi = 3.141592653589793\ldots$$

If we stop the decimal expansion of any number at a certain place, we get an approximation to the number. For instance, we can write

$$\pi \approx 3.14159265$$

where the symbol $\approx$ is read “is approximately equal to.” The more decimal places we retain, the better our approximation.

■ Properties of Real Numbers

We all know that $2 + 3 = 3 + 2$, and $5 + 7 = 7 + 5$, and $513 + 87 = 87 + 513$, and so on. In algebra we express all these (infinitely many) facts by writing

$$a + b = b + a$$

where a and b stand for any two numbers. In other words, “ $a + b = b + a$ ” is a concise way of saying that “when we add two numbers, the order of addition doesn’t matter.” This fact is called the *Commutative Property* of addition. From our experience with numbers we know that the properties in the following box are also valid.

PROPERTIES OF REAL NUMBERS

Property

Example

Description

Commutative Properties

$$a + b = b + a$$

$$7 + 3 = 3 + 7$$

When we add two numbers, order doesn’t matter.

$$ab = ba$$

$$3 \cdot 5 = 5 \cdot 3$$

When we multiply two numbers, order doesn’t matter.

Associative Properties

$$(a + b) + c = a + (b + c)$$

$$(2 + 4) + 7 = 2 + (4 + 7)$$

When we add three numbers, it doesn’t matter which two we add first.

$$(ab)c = a(bc)$$

$$(3 \cdot 7) \cdot 5 = 3 \cdot (7 \cdot 5)$$

When we multiply three numbers, it doesn’t matter which two we multiply first.

Distributive Property

$$a(b + c) = ab + ac$$

$$2 \cdot (3 + 5) = 2 \cdot 3 + 2 \cdot 5$$

When we multiply a number by a sum of two numbers, we get the same result as we get if we multiply the number by each of the terms and then add the results.

The Distributive Property is crucial because it describes the way addition and multiplication interact with each other.

The Distributive Property applies whenever we multiply a number by a sum. Figure 3 explains why this property works for the case in which all the numbers are positive integers, but the property is true for any real numbers a , b , and c .

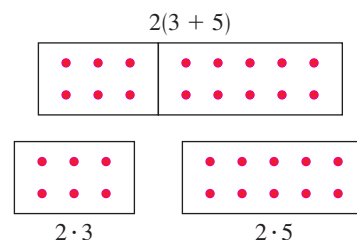


FIGURE 3 The Distributive Property

EXAMPLE 1 ■ Using the Distributive Property

$$\begin{aligned} \text{(a)} \quad 2(x + 3) &= 2 \cdot x + 2 \cdot 3 \\ &= 2x + 6 \end{aligned}$$

Distributive Property

Simplify

$$\begin{aligned} \text{(b)} \quad (a + b)(x + y) &= (a + b)x + (a + b)y \\ &= (ax + bx) + (ay + by) \\ &= ax + bx + ay + by \end{aligned}$$

Distributive Property

Distributive Property

Associative Property of Addition

In the last step we removed the parentheses because, according to the Associative Property, the order of addition doesn't matter.

 **Now Try Exercise 15**

 Don't assume that $-a$ is a negative number. Whether $-a$ is negative or positive depends on the value of a . For example, if $a = 5$, then $-a = -5$, a negative number, but if $a = -5$, then $-a = -(-5) = 5$ (Property 2), a positive number.

■ Addition and Subtraction

The number 0 is special for addition; it is called the **additive identity** because $a + 0 = a$ for any real number a . Every real number a has a **negative**, $-a$, that satisfies $a + (-a) = 0$. **Subtraction** is the operation that undoes addition; to subtract a number from another, we simply add the negative of that number. By definition

$$a - b = a + (-b)$$

To combine real numbers involving negatives, we use the following properties.

PROPERTIES OF NEGATIVES**Property****Example**

1. $(-1)a = -a$

$(-1)5 = -5$

2. $-(-a) = a$

$-(-5) = 5$

3. $(-a)b = a(-b) = -(ab)$

$(-5)7 = 5(-7) = -(5 \cdot 7)$

4. $(-a)(-b) = ab$

$(-4)(-3) = 4 \cdot 3$

5. $-(a + b) = -a - b$

$-(3 + 5) = -3 - 5$

6. $-(a - b) = b - a$

$-(5 - 8) = 8 - 5$

Property 6 states the intuitive fact that $a - b$ and $b - a$ are negatives of each other. Property 5 is often used with more than two terms:

$$-(a + b + c) = -a - b - c$$

EXAMPLE 2 ■ Using Properties of Negatives

Let x , y , and z be real numbers.

(a) $-(x + 2) = -x - 2$

Property 5: $-(a + b) = -a - b$

(b) $-(x + y - z) = -x - y - (-z)$

Property 5: $-(a + b) = -a - b$

$= -x - y + z$

Property 2: $-(-a) = a$

 **Now Try Exercise 23**

■ Multiplication and Division

The number 1 is special for multiplication; it is called the **multiplicative identity** because $a \cdot 1 = a$ for any real number a . Every nonzero real number a has an **inverse**, $1/a$, that satisfies $a \cdot (1/a) = 1$. **Division** is the operation that undoes multiplication; to divide by a number, we multiply by the inverse of that number. If $b \neq 0$, then, by definition,

$$a \div b = a \cdot \frac{1}{b}$$

We write $a \cdot (1/b)$ as simply a/b . We refer to a/b as the **quotient** of a and b or as the **fraction** a over b ; a is the **numerator** and b is the **denominator** (or **divisor**). To combine real numbers using the operation of division, we use the following properties.

PROPERTIES OF FRACTIONS

Property	Example	Description
1. $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$	$\frac{2}{3} \cdot \frac{5}{7} = \frac{2 \cdot 5}{3 \cdot 7} = \frac{10}{21}$	When multiplying fractions , multiply numerators and denominators.
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$	$\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \cdot \frac{7}{5} = \frac{14}{15}$	When dividing fractions , invert the divisor and multiply.
3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$	$\frac{2}{5} + \frac{7}{5} = \frac{2+7}{5} = \frac{9}{5}$	When adding fractions with the same denominator , add the numerators.
4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{2}{5} + \frac{3}{7} = \frac{2 \cdot 7 + 3 \cdot 5}{35} = \frac{29}{35}$	When adding fractions with different denominators , find a common denominator. Then add the numerators.
5. $\frac{ac}{bc} = \frac{a}{b}$	$\frac{2 \cdot 5}{3 \cdot 5} = \frac{2}{3}$	Cancel numbers that are common factors in numerator and denominator.
6. If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$	$\frac{2}{3} = \frac{6}{9}$, so $2 \cdot 9 = 3 \cdot 6$	Cross-multiply .

When adding fractions with different denominators, we don't usually use Property 4. Instead we rewrite the fractions so that they have the smallest possible common denominator (often smaller than the product of the denominators), and then we use Property 3. This denominator is the **Least Common Denominator (LCD)** described in the next example.

EXAMPLE 3 ■ Using the LCD to Add Fractions

Evaluate: $\frac{5}{36} + \frac{7}{120}$

SOLUTION Factoring each denominator into prime factors gives

$$36 = 2^2 \cdot 3^2 \quad \text{and} \quad 120 = 2^3 \cdot 3 \cdot 5$$

We find the least common denominator (LCD) by forming the product of all the prime factors that occur in these factorizations, using the highest power of each prime factor. Thus the LCD is $2^3 \cdot 3^2 \cdot 5 = 360$. So

$$\begin{aligned} \frac{5}{36} + \frac{7}{120} &= \frac{5 \cdot \cancel{10}}{36 \cdot \cancel{10}} + \frac{7 \cdot \cancel{3}}{120 \cdot \cancel{3}} && \text{Use common denominator} \\ &= \frac{50}{360} + \frac{21}{360} = \frac{71}{360} && \text{Property 3: Adding fractions with the same denominator} \end{aligned}$$

 **Now Try Exercise 29**

■ The Real Line

The real numbers can be represented by points on a line, as shown in Figure 4. The positive direction (toward the right) is indicated by an arrow. We choose an arbitrary reference point O , called the **origin**, which corresponds to the real number 0. Given any convenient unit of measurement, each positive number x is represented by the point on the line a distance of x units to the right of the origin, and each negative number $-x$ is represented by the point x units to the left of the origin. The number associated with the point P is called the **coordinate of P** , and the line is then called a **coordinate line**, or a **real number line**, or simply a **real line**. Often we identify the point with its coordinate and think of a number as being a point on the real line.

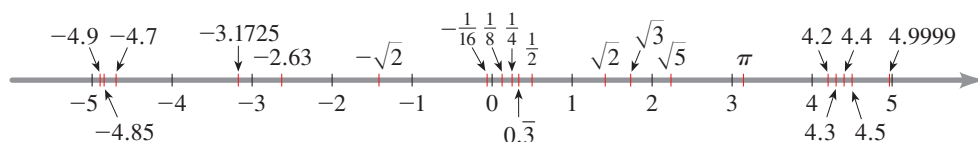


FIGURE 4 The real line

The real numbers are *ordered*. We say that **a is less than b** and write $a < b$ if $b - a$ is a positive number. Geometrically, this means that a lies to the left of b on the number line. Equivalently, we can say that **b is greater than a** and write $b > a$. The symbol $a \leq b$ (or $b \geq a$) means that either $a < b$ or $a = b$ and is read “ a is less than or equal to b .” For instance, the following are true inequalities (see Figure 5):

$$7 < 7.4 < 7.5 \quad -\pi < -3 \quad \sqrt{2} < 2 \quad 2 \leq 2$$

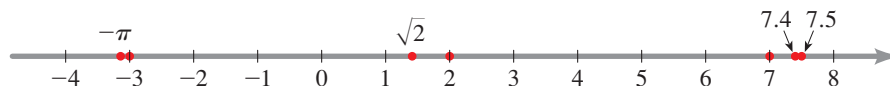


FIGURE 5

■ Sets and Intervals

A **set** is a collection of objects, and these objects are called the **elements** of the set. If S is a set, the notation $a \in S$ means that a is an element of S , and $b \notin S$ means that b is not an element of S . For example, if Z represents the set of integers, then $-3 \in Z$ but $\pi \notin Z$.

Some sets can be described by listing their elements within braces. For instance, the set A that consists of all positive integers less than 7 can be written as

$$A = \{1, 2, 3, 4, 5, 6\}$$

We could also write A in **set-builder notation** as

$$A = \{x \mid x \text{ is an integer and } 0 < x < 7\}$$

which is read “ A is the set of all x such that x is an integer and $0 < x < 7$.”



DISCOVERY PROJECT

Real Numbers in the Real World

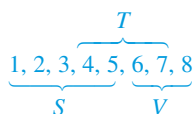
Real-world measurements always involve units. For example, we usually measure distance in feet, miles, centimeters, or kilometers. Some measurements involve different types of units. For example, speed is measured in miles per hour or meters per second. We often need to convert a measurement from one type of unit to another. In this project we explore different types of units used for different purposes and how to convert from one type of unit to another. You can find the project at www.stewartmath.com.

If S and T are sets, then their **union** $S \cup T$ is the set that consists of all elements that are in S or T (or in both). The **intersection** of S and T is the set $S \cap T$ consisting of all elements that are in both S and T . In other words, $S \cap T$ is the common part of S and T . The **empty set**, denoted by $\emptyset$, is the set that contains no element.

EXAMPLE 4 ■ Union and Intersection of Sets

If $S = \{1, 2, 3, 4, 5\}$, $T = \{4, 5, 6, 7\}$, and $V = \{6, 7, 8\}$, find the sets $S \cup T$, $S \cap T$, and $S \cap V$.

SOLUTION



$$S \cup T = \{1, 2, 3, 4, 5, 6, 7\}$$

All elements in S or T

$$S \cap T = \{4, 5\}$$

Elements common to both S and T

$$S \cap V = \emptyset$$

S and V have no element in common

Now Try Exercise 41



FIGURE 6 The open interval (a, b)



FIGURE 7 The closed interval $[a, b]$

Certain sets of real numbers, called **intervals**, occur frequently in calculus and correspond geometrically to line segments. If $a < b$, then the **open interval** from a to b consists of all numbers between a and b and is denoted (a, b) . The **closed interval** from a to b includes the endpoints and is denoted $[a, b]$. Using set-builder notation, we can write

$$(a, b) = \{x \mid a < x < b\} \quad [a, b] = \{x \mid a \leq x \leq b\}$$

Note that parentheses $()$ in the interval notation and open circles on the graph in Figure 6 indicate that endpoints are *excluded* from the interval, whereas square brackets $[]$ and solid circles in Figure 7 indicate that the endpoints are *included*. Intervals may also include one endpoint but not the other, or they may extend infinitely far in one direction or both. The following table lists the possible types of intervals.

Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \leq x \leq b\}$	
$[a, b)$	$\{x \mid a \leq x < b\}$	
$(a, b]$	$\{x \mid a < x \leq b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \leq x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \leq b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

The symbol ∞ (“infinity”) does not stand for a number. The notation (a, ∞) , for instance, simply indicates that the interval has no endpoint on the right but extends infinitely far in the positive direction.

EXAMPLE 5 ■ Graphing Intervals

Express each interval in terms of inequalities, and then graph the interval.

(a) $[-1, 2) = \{x \mid -1 \leq x < 2\}$

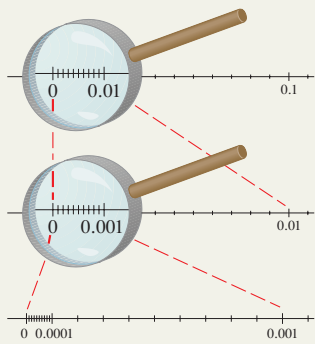
(b) $[1.5, 4] = \{x \mid 1.5 \leq x \leq 4\}$

(c) $(-3, \infty) = \{x \mid -3 < x\}$

Now Try Exercise 47

No Smallest or Largest Number in an Open Interval

Any interval contains infinitely many numbers—every point on the graph of an interval corresponds to a real number. In the closed interval $[0, 1]$, the smallest number is 0 and the largest is 1, but the open interval $(0, 1)$ contains no smallest or largest number. To see this, note that 0.01 is close to zero, but 0.001 is closer, 0.0001 is closer yet, and so on. We can always find a number in the interval $(0, 1)$ closer to zero than any given number. Since 0 itself is not in the interval, the interval contains no smallest number. Similarly, 0.99 is close to 1, but 0.999 is closer, 0.9999 closer yet, and so on. Since 1 itself is not in the interval, the interval has no largest number.

**EXAMPLE 6 ■ Finding Unions and Intersections of Intervals**

Graph each set.

- (a) $(1, 3) \cap [2, 7]$ (b) $(1, 3) \cup [2, 7]$

SOLUTION

- (a) The intersection of two intervals consists of the numbers that are in both intervals. Therefore

$$\begin{aligned}(1, 3) \cap [2, 7] &= \{x \mid 1 < x < 3 \text{ and } 2 \leq x \leq 7\} \\ &= \{x \mid 2 \leq x < 3\} = [2, 3)\end{aligned}$$

This set is illustrated in Figure 8.

- (b) The union of two intervals consists of the numbers that are in either one interval or the other (or both). Therefore

$$\begin{aligned}(1, 3) \cup [2, 7] &= \{x \mid 1 < x < 3 \text{ or } 2 \leq x \leq 7\} \\ &= \{x \mid 1 < x \leq 7\} = (1, 7]\end{aligned}$$

This set is illustrated in Figure 9.

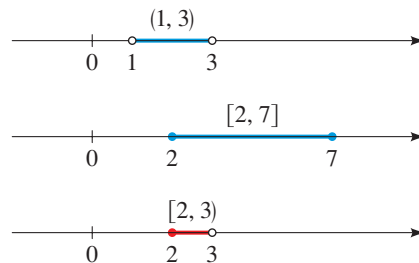


FIGURE 8 $(1, 3) \cap [2, 7] = [2, 3)$

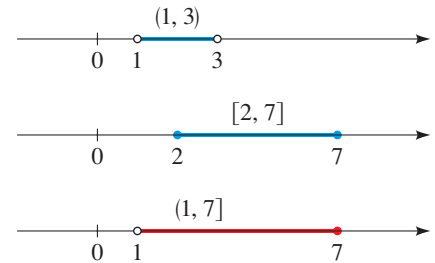


FIGURE 9 $(1, 3) \cup [2, 7] = (1, 7]$

Now Try Exercise 61

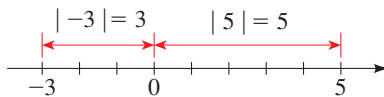


FIGURE 10

■ Absolute Value and Distance

The **absolute value** of a number a , denoted by $|a|$, is the distance from a to 0 on the real number line (see Figure 10). Distance is always positive or zero, so we have $|a| \geq 0$ for every number a . Remembering that $-a$ is positive when a is negative, we have the following definition.

DEFINITION OF ABSOLUTE VALUE

If a is a real number, then the **absolute value** of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

EXAMPLE 7 ■ Evaluating Absolute Values of Numbers

- (a) $|3| = 3$
 (b) $|-3| = -(-3) = 3$
 (c) $|0| = 0$
 (d) $|3 - \pi| = -(3 - \pi) = \pi - 3$ (since $3 < \pi \Rightarrow 3 - \pi < 0$)

Now Try Exercise 67

When working with absolute values, we use the following properties.

PROPERTIES OF ABSOLUTE VALUE

Property	Example	Description
1. $ a \geq 0$	$ -3 = 3 \geq 0$	The absolute value of a number is always positive or zero.
2. $ a = -a $	$ 5 = -5 $	A number and its negative have the same absolute value.
3. $ ab = a b $	$ -2 \cdot 5 = -2 5 $	The absolute value of a product is the product of the absolute values.
4. $\left \frac{a}{b}\right = \frac{ a }{ b }$	$\left \frac{12}{-3}\right = \frac{ 12 }{ -3 }$	The absolute value of a quotient is the quotient of the absolute values.
5. $ a + b \leq a + b $	$ -3 + 5 \leq -3 + 5 $	Triangle Inequality

What is the distance on the real line between the numbers -2 and 11 ? From Figure 11 we see that the distance is 13 . We arrive at this by finding either $|11 - (-2)| = 13$ or $|(-2) - 11| = 13$. From this observation we make the following definition (see Figure 12).

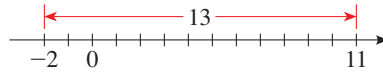


FIGURE 11

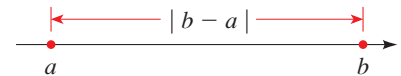


FIGURE 12 Length of a line segment is $|b - a|$

DISTANCE BETWEEN POINTS ON THE REAL LINE

If a and b are real numbers, then the **distance** between the points a and b on the real line is

$$d(a, b) = |b - a|$$

From Property 6 of negatives it follows that

$$|b - a| = |a - b|$$

This confirms that, as we would expect, the distance from a to b is the same as the distance from b to a .

EXAMPLE 8 ■ Distance Between Points on the Real Line

The distance between the numbers -8 and 2 is

$$d(a, b) = |2 - (-8)| = |-10| = 10$$

We can check this calculation geometrically, as shown in Figure 13.

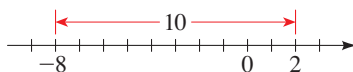


FIGURE 13

 **Now Try Exercise 75**

1.1 EXERCISES

CONCEPTS

- Give an example of each of the following:
 - A natural number
 - An integer that is not a natural number
 - A rational number that is not an integer
 - An irrational number
 - Complete each statement and name the property of real numbers you have used.
 - $ab = \underline{\hspace{2cm}}$; $\underline{\hspace{2cm}}$ Property
 - $a + (b + c) = \underline{\hspace{2cm}}$; $\underline{\hspace{2cm}}$ Property
 - $a(b + c) = \underline{\hspace{2cm}}$; $\underline{\hspace{2cm}}$ Property
 - Express the set of real numbers between but not including 2 and 7 as follows.
 - In set-builder notation: $\underline{\hspace{4cm}}$
 - In interval notation: $\underline{\hspace{4cm}}$
 - The symbol $|x|$ stands for the $\underline{\hspace{2cm}}$ of the number x .
If x is not 0, then the sign of $|x|$ is always $\underline{\hspace{2cm}}$.
 - The distance between a and b on the real line is $d(a, b) = \underline{\hspace{2cm}}$. So the distance between -5 and 2 is $\underline{\hspace{2cm}}$.
- 6–8 ■ Yes or No?** If *No*, give a reason. Assume that a and b are nonzero real numbers.
- Is the sum of two rational numbers always a rational number?
 - Is the sum of two irrational numbers always an irrational number?
 - Is $a - b$ equal to $b - a$?
 - Is $-2(a - 5)$ equal to $-2a - 10$?
 - Is the distance between any two different real numbers always positive?
 - Is the distance between a and b the same as the distance between b and a ?

SKILLS

- 9–10 ■ Real Numbers** List the elements of the given set that are
- natural numbers
 - integers
 - rational numbers
 - irrational numbers

9. $\{-1.5, 0, \frac{5}{2}, \sqrt{7}, 2.71, -\pi, 3.14, 100, -8\}$

10. $\{1.3, 1.3333\ldots, \sqrt{5}, 5.34, -500, 1\frac{2}{3}, \sqrt{16}, \frac{246}{579}, -\frac{20}{5}\}$

- 11–18 ■ Properties of Real Numbers** State the property of real numbers being used.

11. $3 + 7 = 7 + 3$

12. $4(2 + 3) = (2 + 3)4$

13. $(x + 2y) + 3z = x + (2y + 3z)$

14. $2(A + B) = 2A + 2B$

15. $(5x + 1)3 = 15x + 3$

16. $(x + a)(x + b) = (x + a)x + (x + a)b$

17. $2x(3 + y) = (3 + y)2x$

18. $7(a + b + c) = 7(a + b) + 7c$

- 19–22 ■ Properties of Real Numbers** Rewrite the expression using the given property of real numbers.

19. Commutative Property of Addition, $x + 3 = \underline{\hspace{2cm}}$

20. Associative Property of Multiplication, $7(3x) = \underline{\hspace{2cm}}$

21. Distributive Property, $4(A + B) = \underline{\hspace{2cm}}$

22. Distributive Property, $5x + 5y = \underline{\hspace{2cm}}$

- 23–28 ■ Properties of Real Numbers** Use properties of real numbers to write the expression without parentheses.

23. $3(x + y)$

24. $(a - b)8$

25. $4(2m)$

26. $\frac{4}{3}(-6y)$

27. $-\frac{5}{2}(2x - 4y)$

28. $(3a)(b + c - 2d)$

- 29–32 ■ Arithmetic Operations** Perform the indicated operations.

29. (a) $\frac{3}{10} + \frac{4}{15}$

(b) $\frac{1}{4} + \frac{1}{5}$

30. (a) $\frac{2}{3} - \frac{3}{5}$

(b) $1 + \frac{5}{8} - \frac{1}{6}$

31. (a) $\frac{2}{3}(6 - \frac{3}{2})$

(b) $(3 + \frac{1}{4})(1 - \frac{4}{5})$

32. (a) $\frac{2}{\frac{2}{3}} - \frac{2}{2}$

(b) $\frac{\frac{2}{5} + \frac{1}{2}}{\frac{1}{10} + \frac{3}{15}}$

- 33–34 ■ Inequalities** Place the correct symbol ($<$, $>$, or $=$) in the space.

33. (a) $3 \underline{\hspace{1cm}} \frac{7}{2}$ (b) $-3 \underline{\hspace{1cm}} -\frac{7}{2}$ (c) $3.5 \underline{\hspace{1cm}} \frac{7}{2}$

34. (a) $\frac{2}{3} \underline{\hspace{1cm}} 0.67$ (b) $\frac{2}{3} \underline{\hspace{1cm}} -0.67$

(c) $|0.67| \underline{\hspace{1cm}} |-0.67|$

- 35–38 ■ Inequalities** State whether each inequality is true or false.

35. (a) $-3 < -4$

(b) $3 < 4$

36. (a) $\sqrt{3} > 1.7325$

(b) $1.732 \geq \sqrt{3}$

37. (a) $\frac{10}{2} \geq 5$

(b) $\frac{6}{10} \geq \frac{5}{6}$

38. (a) $\frac{7}{11} \geq \frac{8}{13}$

(b) $-\frac{3}{5} > -\frac{3}{4}$

- 39–40 ■ Inequalities** Write each statement in terms of inequalities.

39. (a) x is positive.

(b) t is less than 4.

(c) a is greater than or equal to π .

(d) x is less than $\frac{1}{3}$ and is greater than -5 .

(e) The distance from p to 3 is at most 5.

40. (a) y is negative.
 (b) z is greater than 1.
 (c) b is at most 8.
 (d) w is positive and is less than or equal to 17.
 (e) y is at least 2 units from π .

41–44 ■ **Sets** Find the indicated set if

$$A = \{1, 2, 3, 4, 5, 6, 7\} \quad B = \{2, 4, 6, 8\} \\ C = \{7, 8, 9, 10\}$$

41. (a) $A \cup B$ (b) $A \cap B$
 42. (a) $B \cup C$ (b) $B \cap C$
 43. (a) $A \cup C$ (b) $A \cap C$
 44. (a) $A \cup B \cup C$ (b) $A \cap B \cap C$

45–46 ■ **Sets** Find the indicated set if

$$A = \{x \mid x \geq -2\} \quad B = \{x \mid x < 4\} \\ C = \{x \mid -1 < x \leq 5\}$$

45. (a) $B \cup C$ (b) $B \cap C$
 46. (a) $A \cap C$ (b) $A \cap B$

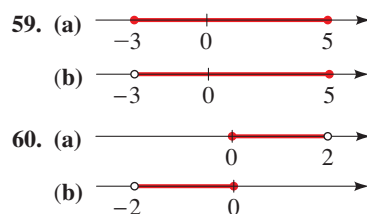
47–52 ■ **Intervals** Express the interval in terms of inequalities, and then graph the interval.

47. $(-3, 0)$ 48. $(2, 8]$
 49. $[2, 8)$ 50. $[-6, -\frac{1}{2}]$
 51. $[2, \infty)$ 52. $(-\infty, 1)$

53–58 ■ **Intervals** Express the inequality in interval notation, and then graph the corresponding interval.

53. $x \leq 1$ 54. $1 \leq x \leq 2$
 55. $-2 < x \leq 1$ 56. $x \geq -5$
 57. $x > -1$ 58. $-5 < x < 2$

59–60 ■ **Intervals** Express each set in interval notation.



61–66 ■ **Intervals** Graph the set.

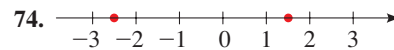
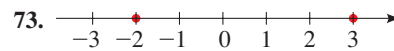
61. $(-2, 0) \cup (-1, 1)$ 62. $(-2, 0) \cap (-1, 1)$
 63. $[-4, 6] \cap [0, 8)$ 64. $[-4, 6) \cup [0, 8)$
 65. $(-\infty, -4) \cup (4, \infty)$ 66. $(-\infty, 6] \cap (2, 10)$

67–72 ■ **Absolute Value** Evaluate each expression.

67. (a) $|100|$ (b) $|-73|$
 68. (a) $|\sqrt{5} - 5|$ (b) $|10 - \pi|$

69. (a) $||-6| - |-4||$ (b) $\frac{-1}{|-1|}$
 70. (a) $|2 - |-12||$ (b) $-1 - |1 - |-1||$
 71. (a) $|(-2) \cdot 6|$ (b) $|(-\frac{1}{3})(-15)|$
 72. (a) $|\frac{-6}{24}|$ (b) $|\frac{7-12}{12-7}|$

73–76 ■ **Distance** Find the distance between the given numbers.



75. (a) 2 and 17 (b) -3 and 21 (c) $\frac{11}{8}$ and $-\frac{3}{10}$
 76. (a) $\frac{7}{15}$ and $-\frac{1}{21}$ (b) -38 and -57 (c) -2.6 and -1.8

SKILLS Plus

77–78 ■ **Repeating Decimal** Express each repeating decimal as a fraction. (See the margin note on page 3.)

77. (a) $0.\overline{7}$ (b) $0.2\overline{8}$ (c) $0.5\overline{7}$
 78. (a) $5.\overline{23}$ (b) $1.3\overline{7}$ (c) $2.1\overline{35}$

79–82 ■ **Simplifying Absolute Value** Express the quantity without using absolute value.

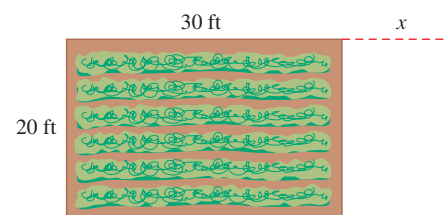
79. $|\pi - 3|$ 80. $|1 - \sqrt{2}|$
 81. $|a - b|$, where $a < b$
 82. $a + b + |a - b|$, where $a < b$

83–84 ■ **Signs of Numbers** Let a , b , and c be real numbers such that $a > 0$, $b < 0$, and $c < 0$. Find the sign of each expression.

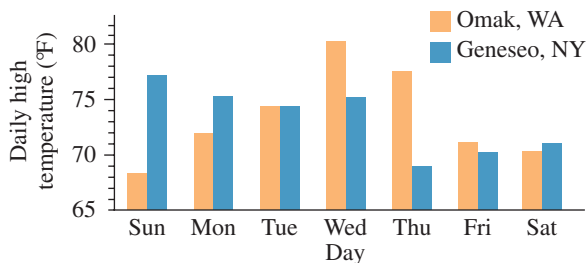
83. (a) $-a$ (b) bc (c) $a - b$ (d) $ab + ac$
 84. (a) $-b$ (b) $a + bc$ (c) $c - a$ (d) ab^2

APPLICATIONS

85. **Area of a Garden** Mary's backyard vegetable garden measures 20 ft by 30 ft, so its area is $20 \times 30 = 600 \text{ ft}^2$. She decides to make it longer, as shown in the figure, so that the area increases to $A = 20(30 + x)$. Which property of real numbers tells us that the new area can also be written $A = 600 + 20x$?



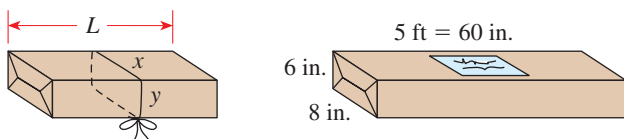
- 86. Temperature Variation** The bar graph shows the daily high temperatures for Omak, Washington, and Geneseo, New York, during a certain week in June. Let T_O represent the temperature in Omak and T_G the temperature in Geneseo. Calculate $T_O - T_G$ and $|T_O - T_G|$ for each day shown. Which of these two values gives more information?



- 87. Mailing a Package** The post office will accept only packages for which the length plus the “girth” (distance around) is no more than 108 in. Thus for the package in the figure, we must have

$$L + 2(x + y) \leq 108$$

- (a) Will the post office accept a package that is 6 in. wide, 8 in. deep, and 5 ft long? What about a package that measures 2 ft by 2 ft by 4 ft?
- (b) What is the greatest acceptable length for a package that has a square base measuring 9 in. by 9 in.?



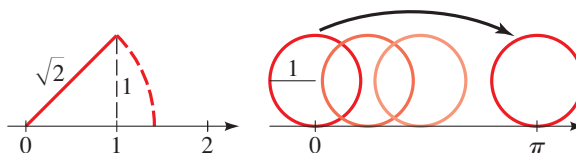
DISCUSS ■ DISCOVER ■ PROVE ■ WRITE

- 88. DISCUSS: Sums and Products of Rational and Irrational Numbers** Explain why the sum, the difference, and the product of two rational numbers are rational numbers. Is the product of two irrational numbers necessarily irrational? What about the sum?
- 89. DISCOVER ■ PROVE: Combining Rational and Irrational Numbers** Is $\frac{1}{2} + \sqrt{2}$ rational or irrational? Is $\frac{1}{2} \cdot \sqrt{2}$ rational or irrational? Experiment with sums and products of other rational and irrational numbers. Prove the following.
- (a) The sum of a rational number r and an irrational number t is irrational.
- (b) The product of a rational number r and an irrational number t is irrational.
- [Hint: For part (a), suppose that $r + t$ is a rational number q , that is, $r + t = q$. Show that this leads to a contradiction. Use similar reasoning for part (b).]
- 90. DISCOVER: Limiting Behavior of Reciprocals** Complete the tables. What happens to the size of the fraction $1/x$ as x gets large? As x gets small?

x	$1/x$
1	
2	
10	
100	
1000	

x	$1/x$
1.0	
0.5	
0.1	
0.01	
0.001	

- 91. DISCOVER: Locating Irrational Numbers on the Real Line** Using the figures below, explain how to locate the point $\sqrt{2}$ on a number line. Can you locate $\sqrt{5}$ by a similar method? How can the circle shown in the figure help us to locate π on a number line? List some other irrational numbers that you can locate on a number line.



- 92. PROVE: Maximum and Minimum Formulas** Let $\max(a, b)$ denote the maximum and $\min(a, b)$ denote the minimum of the real numbers a and b . For example, $\max(2, 5) = 5$ and $\min(-1, -2) = -2$.

(a) Prove that $\max(a, b) = \frac{a + b + |a - b|}{2}$.

(b) Prove that $\min(a, b) = \frac{a + b - |a - b|}{2}$.

[Hint: Take cases and write these expressions without absolute values. See Exercises 81 and 82.]

- 93. WRITE: Real Numbers in the Real World** Write a paragraph describing different real-world situations in which you would use natural numbers, integers, rational numbers, and irrational numbers. Give examples for each type of situation.
- 94. DISCUSS: Commutative and Noncommutative Operations** We have learned that addition and multiplication are both commutative operations.
- (a) Is subtraction commutative?
- (b) Is division of nonzero real numbers commutative?
- (c) Are the actions of putting on your socks and putting on your shoes commutative?
- (d) Are the actions of putting on your hat and putting on your coat commutative?
- (e) Are the actions of washing laundry and drying it commutative?
- 95. PROVE: Triangle Inequality** We prove Property 5 of absolute values, the Triangle Inequality:
- $$|x + y| \leq |x| + |y|$$
- (a) Verify that the Triangle Inequality holds for $x = 2$ and $y = 3$, for $x = -2$ and $y = -3$, and for $x = -2$ and $y = 3$.
- (b) Prove that the Triangle Inequality is true for all real numbers x and y . [Hint: Take cases.]