

§1.3 ALGEBRAIC EXPRESSIONS

Def: **VARIABLE** (e.g. x, y, z) REPRESENTS A NUMBER.

- MAYBE UNKNOWN (SOLVE)

- MAYBE SPECIFIC #'S WILL BE PLUGGED IN LATER (PLACE HOLDER)

ALGEBRAIC EXPRESSION IS ANY COMBINATION OF VARIABLES AND REAL NUMBERS USING ADD, SUB, MULT, DIV, ANY EXPONENTS (INCLUDING ANY RADICALS).

$$8x^2 + \frac{7}{3}\sqrt{x} - \frac{1}{x^{2/3}} + 4 \quad : 4 \text{ terms (+/-)}$$

POLYNOMIAL OF DEGREE n :

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

WHERE $n \geq 0$ IS AN INTEGER & $a_n \neq 0$.

e.g. $3x^4 - \frac{2}{5}x^3$ (BINOMIAL)
DEGREE 4

e.g. $\sqrt{7}x^2$ (MONOMIAL)
DEGREE 2

e.g. $x^2 + 5x - \frac{1}{2}$ (TRINOMIAL)
DEGREE 2

e.g. $3x + \sqrt{x} + 1$
(NOT A POLYNOMIAL)

ADDING/SUBTRACTING POLYNOMIALS

$$2x^2 + 3x + 1 + x^2 - 2x + 3 = 3x^2 + x + 4$$

LIKE TERMS: SAME VARIABLES RAISED TO SAME POWERS (ONLY COEFF. MAY BE DIFFERENT)

(b) Find the difference $(x^3 - 6x^2 + 2x + 4) - (x^3 + 5x^2 - 7x)$.

MULTIPLYING ALGEBRAIC EXPRESSIONS

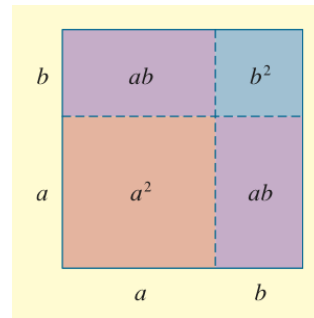
ex. $(3x + 2)(x^2 - 1)$

ex. $(x^3 - \sqrt{x})(2x + \sqrt{x})$

ex. $(3x^2 + x)(x^2 + 6x - 2)$

ex. $x^{3/2} \left(2\sqrt{x} - \frac{3}{\sqrt{x}} \right)$

ex. $-8t(t^2 + 3t - 1)$



SPECIAL PRODUCT FORMULAS

If A and B are any real numbers or algebraic expressions, then

1. $(A + B)(A - B) = A^2 - B^2$ Sum and difference of same terms

2. $(A + B)^2 = A^2 + 2AB + B^2$ Square of a sum

3. $(A - B)^2 = A^2 - 2AB + B^2$ Square of a difference

4. $(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$ Cube of a sum

5. $(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$ Cube of a difference

ex. $(x + 1)^2$

ex. $(x - 1)^2$

ex. $(x + 1)(x - 1)$

ex. $(3x^2 + 4x)^2$

ex. $(3x^2 - 4x)^2$

ex. $(3x^2 + 4x)(3x^2 - 4x)$

ex. $(2x + 3)^3$

ex. $(2x - 3)^3$

ex. $(3 + \sqrt{x})(3 - \sqrt{x})$

ex. $(x + y + 1)(x + y - 1)$

To Factor an expression means to rewrite it as a product

e.g.

$$(x+2)(x+3) = x^2 + 5x + 6$$

MULTIPLY

FACTOR

Common Factors

ex. $15x^6 - 20x^4 = 5x^4(3x^2 - 4)$

GCF OF COEFFICIENTS

COMMON VARIABLE(S)

RAISED TO LOWEST POWER

ex. $8x^6y^2 + 10x^5y^3 - 6x^4y^6$

(You can check your answers!)

ex. $12x^2(y-1) - 8x(y-1)^2$

ex. $3x(x+1) - (x+1)$

Factoring Trinomials (Quadratics)

Note: $(x+a)(x+b) = x^2 + ax + bx + ab = x^2 + (a+b)x + ab$

$$\Rightarrow x^2 + (a+b)x + ab = (x+a)(x+b)$$

e.g. $x^2 + 7x + 10$ Trial & Error: Find a, b such that

$$a+b=7, \quad ab=10$$

$$= x^2 + (5+2)x + 5 \cdot 2 = (x+5)(x+2)$$

(check!)

ex. $x^2 + x - 6$

ex. $x^2 - 8x - 15$

IF THE COEFFICIENT OF x^2 IS NOT 1:

NOTE: $(px + a)(qx + b) = pqx^2 + agx + bpx + ab = pqx^2 + (ag + bp)x + ab$

$$\Rightarrow pqx^2 + (ag + bp)x + ab = (px + a)(qx + b)$$

e.g. $6x^2 + 7x - 5$

$p = \pm 1$ $q = \pm 6$

$p = \pm 2$ $q = \pm 3$

(OR VICE VERSA)

$a = \pm 1$ $b = \mp 5$

(OR VICE VERSA)

$$(x + 1)(6x - 5)$$

$$(x - 1)(6x + 5)$$

$$(2x + 1)(3x - 5)$$

$$(2x - 1)(3x + 5)$$

ex. $12x^2 - x - 6$

$\swarrow \quad \searrow$
 -72
 $\swarrow \quad \searrow$
 $8 \quad -9 = -1$

$$= 12x^2 + 8x - 9x - 6$$

$$4x(3x + 2) - 3(3x + 2)$$

$$(4x - 3)(3x + 2)$$

SPECIAL FACTORING FORMULAS

FACTORING FORMULAS

Formula

1. $A^2 - B^2 = (A - B)(A + B)$

2. $A^2 + 2AB + B^2 = (A + B)^2$

3. $A^2 - 2AB + B^2 = (A - B)^2$

~~4.~~ $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$

~~5.~~ $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

Name

Difference of squares

Perfect square

Perfect square

Difference of cubes

Sum of cubes

ex. $81x^4 - 16x^8$

ex. $16x^2 + 24x + 9$

More Practice with Special Product/Factoring Formulas

ex. $(2x + x^3)^2$

ex. $x^4 - 81$

ex. $16x^2 - 48x + 36$

ex. $[x^3(x+5)]^2$

Correct multiplication property	Common error with addition
$(a \cdot b)^2 = a^2 \cdot b^2$	$(a + b)^2 \neq a^2 + b^2$
$\sqrt{a \cdot b} = \sqrt{a} \sqrt{b} \quad (a, b \geq 0)$	$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$
$\sqrt{a^2 \cdot b^2} = a \cdot b \quad (a, b \geq 0)$	$\sqrt{a^2 + b^2} \neq a + b$
$\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{a \cdot b}$	$\frac{1}{a} + \frac{1}{b} \neq \frac{1}{a + b}$
$\frac{ab}{a} = b$	$\frac{a + b}{a} \neq b$
$a^{-1} \cdot b^{-1} = (a \cdot b)^{-1}$	$a^{-1} + b^{-1} \neq (a + b)^{-1}$

§ 1.4 RATIONAL EXPRESSIONS

DOMAIN

ex. $\frac{x^2 + 2x + x}{x^2 - 2x - 8}$

ex. $\frac{\sqrt{x+4}}{x^3 - x}$

- DENOMINATORS CANNOT EQUAL 0
- CANNOT TAKE $\sqrt{\quad}$ OF NEGATIVE #'S

SIMPLIFYING RATIONAL EXPRESSIONS

$$\frac{AC}{BC} = \frac{A}{B}$$

ex. $\frac{x^2 + 10x + 25}{3x^4 + 15x^3}$

ex. What is WRONG WITH THIS?

$$\frac{\cancel{x^2} - 1}{\cancel{x^2} + x - 2} = \frac{-1}{x - 2} \quad (\otimes)$$

$$\hookrightarrow \frac{(x+1)\cancel{(x-1)}}{(x+2)(x-1)} = \frac{x+1}{x+2} \quad (\checkmark)$$

ex. $\frac{4x^3 + 4x^2}{2x^3 + 4x^2 + 2x}$

MULTIPLYING & DIVIDING RATIONAL EXPRESSIONS

Note: $\frac{24}{25} \cdot \frac{5}{12} = \frac{(2)(12)}{(5)(5)} \cdot \frac{\cancel{5}}{\cancel{12}} = \frac{2}{5} \quad (\checkmark)$

ex. $\frac{x^2 - 4}{x^2 + 6x + 9} \cdot \frac{x^2 + 4x + 3}{x^2 + 3x + 2}$

ex. $\frac{x-4}{x^2-4} \div \frac{x^2-3x-4}{x^2+5x+6}$

ADDING & SUBTRACTING RATIONAL EXPRESSIONS

Note: $\frac{5}{18} + \frac{11}{75} = \frac{5}{2 \cdot 3^2} + \frac{11}{3 \cdot 5^2}$

$$\text{LCD} = 2 \cdot 3^2 \cdot 5^2 = 450$$
$$= \frac{5}{2 \cdot 3^2} \cdot \frac{5^2}{5^2} + \frac{11}{3 \cdot 5^2} \cdot \frac{2 \cdot 3}{2 \cdot 3} = \frac{125 + 66}{450}$$

ex. $\frac{2}{2x+3} + \frac{5}{3x-2}$

① FACTOR ALL NUM. & DENOM. (SIMPLIFY FRACTIONS BEFORE ADD/SUB IF POSSIBLE)

② LCD IS PRODUCT OF ALL FACTORS IN DENOM. RAISED TO THE HIGHEST EXPONENT.

(COMMON MULTIPLE, NOT COMMON FACTOR)

③ MULTIPLY TOPS & BOTTOMS BY MISSING FACTORS TO MAKE ALL DENOM. EQUAL TO LCD.

ex. $\frac{1}{x^2-1} - \frac{1}{x^2+2x+1}$

ex. 55. $\frac{2}{x} + \frac{3}{x-1} - \frac{4}{x^2-x}$

ex 58. $\frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{x^2-1}$

SIMPLIFYING A COMPOUND FRACTION

60. $\frac{1 - \frac{2}{y}}{\frac{3}{y} - 1}$

64. $\frac{\frac{x-3}{x-4} - \frac{x+2}{x+1}}{x+3}$

Simplify: $\frac{(1+x^2)^{1/2} - x^2(1+x^2)^{-1/2}}{1+x^2}$