§1.1 REAL NUMBERS

NATURAL NUMBERS	INTEGERS	RATIONAL NUMBERS	IRRATIONAL NUMBERS
$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	
$1, 2, 3, \ldots$	$0, -1, -2,$	$\frac{2}{3}, -\frac{4}{7},$	$\sqrt{2}, \pi, \ldots$

REAL NUMBERS $\mathbb{R}$

LET a, b, c BE REAL NUMBERS

	ADDITION	MULTIPLICATION
COMMUTATIVE PROPERTY	$a + b = b + a$	$ab = ba$
ASSOCIATIVE PROPERTY	$(a+b)+c = a + (b+c)$	$(ab)c = a(bc)$
DISTRIBUTIVE PROPERTY	$a(b+c) = ab + ac$ $(a+b)c = ac + bc$	

ex. $3(2x+5)$

ex. $(x+2)^2$

ex. $5(3+a)(4+b)$

ex. $(x+2)(x^2+5x+3)$

SUBTRACTION : $a - b = a + (-b)$ ADD NEGATIVE

DIVISION : $a \div b = \frac{a}{b} = a \cdot \frac{1}{b}$ MULTIPLY BY RECIPROCAL

ex. $(a-b)^2$

ex. HOW MANY WAY CAN YOU WRITE $\frac{ab}{c}$?

Properties of Fractions

1. $\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$

"Multiply across"

2. $\dfrac{a}{b} \div \dfrac{c}{d} = \dfrac{a}{b} \cdot \dfrac{d}{c}$

"Keep change flip"

3. $\dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a+b}{c}$

Addition requires common denominators

4. Simplify

$\dfrac{ac}{bc} = \dfrac{a}{b}$

Change the denominator / numerator

5. If $\dfrac{a}{b} = \dfrac{c}{d}$

Then $ad = bc$

"Cross multiplication"

ex $\dfrac{2}{3} \div \dfrac{5}{9}$

ex $\dfrac{5}{18} + \dfrac{7}{60}$

ex $\dfrac{1}{\frac{1}{a}}$

ex $\dfrac{\frac{3}{2} - \frac{2}{3}}{\frac{1}{9} + \frac{7}{12}}$

Intervals, Unions, & Intersections

Notation	Set description	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid a < x\}$	
$[a, \infty)$	$\{x \mid a \le x\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	

Def: Suppose A & B are two intervals.

$A \cup B$ ("A union B")

is the set of all numbers in A or B or both.

$A \cap B$ ("A intersect B")

is the set of all numbers in A and B (i.e. overlap).

ex. $(1,3) \cup [2,4]$

ex. $(1,3) \cap [2,4]$

ex. $\{x \mid x \geq 0,\ x \neq 2,\ x \neq 3\}$

Absolute Value

Def. $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Properties

1. $|a| \geq 0$
2. $|a| = |-a|$
3. $|ab| = |a||b|$
4. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
5. $|a+b| \leq |a| + |b|$

Def. The distance between 2 numbers a & b is $|b-a|$,

i.e. $|a-b|$.

Alt. Def. $|x| = |x-0|$ = distance between x & 0.

ex $|-4|$

ex $-|4|$

ex. $\{x \mid |x| < 2\}$

ex. $\{x \mid |x| \geq 1\}$

ex. $\{x \mid |x-5| \leq 3\}$

ex. $\{x \mid |x-1| > 4\}$

§1.2 Exponents & Radicals

Notation

Given any real number a & any positive integer n,

$$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}}$$

ex. $\left(\frac{2}{3}\right)^4$

ex. -3^2 vs. $(-3)^2$

Def.

$\forall a \in \mathbb{R}, \quad a^0 = 1$

$\forall a \neq 0, \quad a^{-n} = \frac{1}{a^n}$

Laws of Exponents

1. $a^m a^n = a^{m+n}$
2. $\frac{a^m}{a^n} = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
7. $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m}$

ex. Simplify : $(2a^4b^5)^2(4a^2b^3)^3$

ex. Simplify : $\frac{3x^{-2}\cancel{y^4}}{4x^3\cancel{y^{-5}}}$

ex. Simplify : $\left(6a^2(bc^3)^3\right)^2$

ex. Simplify : $\left(\frac{5a^2b}{c^3}\right)^{-2}$

Scientific Notation

Def: Given any real # $x \neq 0$, x is written in scientific notation when $x = a \times 10^n$, where

(1) $1 \le a < 10$ (single digit #)

(2) n is an integer (pos/neg. whole number)

e.g. Speed of light $299,792,458$ m/s $= 2.99792458 \times 10^8$ m/s

e.g. Mass of H atom $.00000000000000000000000166$ g (23 zeros)

$= 1.66 \times 10^{-24}$ g

ex. Expand 4.886×10^7

ex. Expand 4.886×10^{-4}

ex. Write in sci. not.

(a) 2874.13

(b) 0.0000585

ex. $(6.1 \times 10^4) \times (8 \times 10^6)$

Radicals

Def: Given $a \ge 0$,

$\sqrt{a}$ is the non-neg # x s.t. $x^2 = a$

ex. $\sqrt{64}$

ex. $\sqrt{\frac{49}{36}}$

ex. Solve: $x^2 = 16$

Def: n^{th} root: $\sqrt[n]{a}$ is # x s.t. $x^n = a$

- If n is even then we must have $a \ge 0$.

Properties of n^{th} roots:

1. $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$
2. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
4. $\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ is odd} \\ |a| & \text{if } n \text{ is even} \end{cases}$

Simplifying Radical Expressions

ex. Simplify $\sqrt{8}$, $\sqrt{72}$

ex. Simplify $\sqrt{32x^8y^{11}}$

ex. Simplify $\sqrt[3]{54}$, $\sqrt[3]{16x^5y^7}$, $\sqrt[4]{a^5b^{10}c^{15}}$

ex. $\sqrt{32} + \sqrt{200}$ ex. $\sqrt{25b} - \sqrt{b^3}$ ex. $\sqrt{49x^2 + 49}$

Alt. Notation:

$(\sqrt{a})^2 = a$

$(a^{\frac{1}{2}})^2 = a$

$\sqrt{a} = a^{1/2}$

$(\sqrt[n]{a})^n = a$

$(a^{\frac{1}{n}})^n = a$

$\sqrt[n]{a} = a^{1/n}$

ex. $8^{2/3}$ ex. $125^{-1/3}$ ex. $(2a^3b^4)^{3/2}$ ex. $\left(\dfrac{2x^{3/4}}{y^{1/3}}\right)^3\left(\dfrac{y^4}{x^{-1/2}}\right)$

ex. Rewrite with exponents: (a) $(2\sqrt{x})(3\sqrt[3]{x})$

(b) $\sqrt{x\sqrt{x}}$

RATIONALIZING DENOMINATORS ("STANDARD FORM")

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}, \quad a > 0$$

ex. $\frac{1}{\sqrt{2}}$

ex. $\frac{3}{2\sqrt{5}}$

MORE GENERALLY,

$$\frac{1}{\sqrt[n]{a^m}} = \frac{1}{\sqrt[n]{a^m}} \cdot \frac{\sqrt[n]{a^{n-m}}}{\sqrt[n]{a^{n-m}}} = \frac{\sqrt[n]{a^{n-m}}}{a}$$

ex. $\frac{1}{\sqrt[3]{5}}$

ex. $\sqrt[7]{\frac{1}{a^2}}$