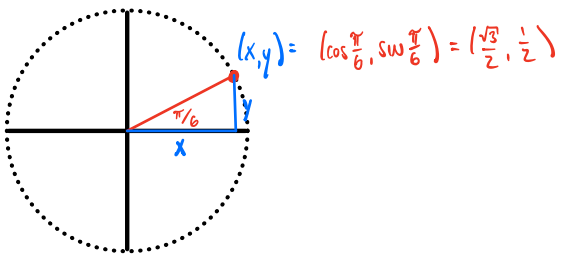
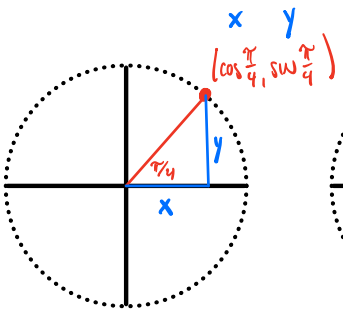
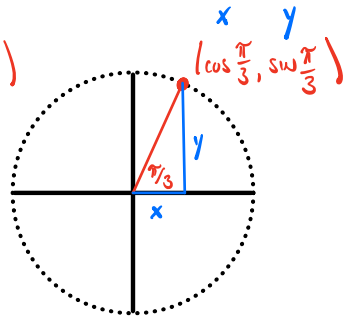
§6.3 Trigonometric Functions of Angles

So far, two ideas: Angles in standard position (xy-plane) + Trig ratios of sides of right triangles







$\cos\frac{\pi}{6} = \frac{x}{1} \Rightarrow x = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

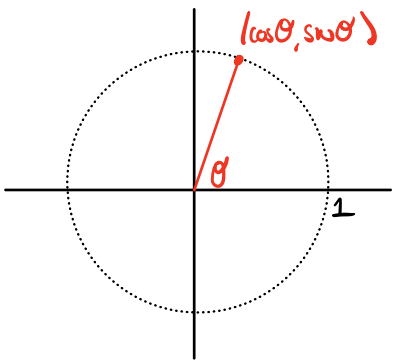
$\sin\frac{\pi}{6} = \frac{y}{1} \Rightarrow y = \sin\frac{\pi}{6} = \frac{1}{2}$

For an angle in standard position.

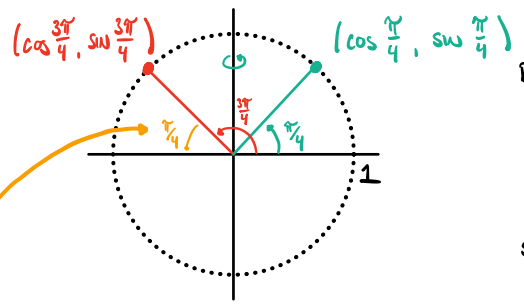
Consider the point where terminal side meets unit circle.

This point has coordinates $x = \cos\theta$, $y = \sin\theta$.

This generalizes to ALL angles!



Ex. Find $\cos\frac{3\pi}{4}$, $\sin\frac{3\pi}{4}$, ...



By symmetry,

$\cos\frac{3\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$ negative

$\sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ positive

REFERENCE ANGLE

Let θ be an angle in standard position. The **reference angle** $\bar{\theta}$ associated with θ is the acute angle formed by the terminal side of θ and the x-axis.

EVALUATING TRIGONOMETRIC FUNCTIONS FOR ANY ANGLE

To find the values of the trigonometric functions for any angle θ, we carry out the following steps.

1. Find the reference angle $\bar{\theta}$ associated with the angle θ.
2. Determine the sign of the trigonometric function of θ by noting the quadrant in which θ lies.
3. The value of the trigonometric function of θ is the same, except possibly for sign, as the value of the trigonometric function of $\bar{\theta}$.

ex. FIND $\tan 390^\circ$.

EXAMPLE 3 ■ Using the Reference Angle to Evaluate Trigonometric Functions

Find **(a)** $\sin 240^\circ$ and **(b)** $\cot 495^\circ$.

EXAMPLE 4 ■ Using the Reference Angle to Evaluate Trigonometric Functions

Find **(a)** $\sin \frac{16\pi}{3}$ and **(b)** $\sec\left(-\frac{\pi}{4}\right)$.

FUNDAMENTAL IDENTITIES

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

EXAMPLE 5 ■ Expressing One Trigonometric Function in Terms of Another

(a) Express $\sin\theta$ in terms of $\cos\theta$.

(b) Express $\tan\theta$ in terms of $\sin\theta$, where θ is in Quadrant II.

EXAMPLE 6 ■ Evaluating a Trigonometric Function

If $\tan\theta = \frac{2}{3}$ and θ is in Quadrant III, find $\cos\theta$.

EXAMPLE 7 ■ Evaluating Trigonometric Functions

If $\sec\theta = 2$ and θ is in Quadrant IV, find the other five trigonometric functions of θ.

13–36 ■ Values of Trigonometric Functions Find the exact value of the trigonometric function.

13. $\cos 150^\circ$
14. $\sin 240^\circ$
15. $\tan 330^\circ$
16. $\sin(-30^\circ)$
17. $\cot(-120^\circ)$
18. $\csc 300^\circ$
19. $\csc(-630^\circ)$
20. $\cot 210^\circ$
21. $\cos 570^\circ$
22. $\sec 120^\circ$
23. $\tan 750^\circ$
24. $\cos 660^\circ$
25. $\sin \frac{3\pi}{2}$
26. $\cos \frac{4\pi}{3}$
27. $\tan\left(-\frac{4\pi}{3}\right)$
28. $\cos\left(-\frac{11\pi}{6}\right)$
29. $\csc\left(-\frac{5\pi}{6}\right)$
30. $\sec \frac{7\pi}{6}$
31. $\sec \frac{17\pi}{3}$
32. $\csc \frac{5\pi}{4}$
33. $\cot\left(-\frac{\pi}{4}\right)$
34. $\cos \frac{7\pi}{4}$
35. $\tan \frac{5\pi}{2}$
36. $\sin \frac{11\pi}{6}$

37–40 ■ Quadrant in Which an Angle Lies Find the quadrant in which θ lies from the information given.

37. $\sin\theta < 0$ and $\cos\theta < 0$
38. $\tan\theta < 0$ and $\sin\theta < 0$
39. $\sec\theta > 0$ and $\tan\theta < 0$
40. $\csc\theta > 0$ and $\cos\theta < 0$

41–46 ■ Expressing One Trigonometric Function in Terms of Another Write the first trigonometric function in terms of the second for θ in the given quadrant.

41. $\tan\theta$, $\cos\theta$; θ in Quadrant III
42. $\cot\theta$, $\sin\theta$; θ in Quadrant II
43. $\cos\theta$, $\sin\theta$; θ in Quadrant IV
44. $\sec\theta$, $\sin\theta$; θ in Quadrant I
45. $\sec\theta$, $\tan\theta$; θ in Quadrant II
46. $\csc\theta$, $\cot\theta$; θ in Quadrant III

47–54 ■ Values of Trigonometric Functions Find the values of the trigonometric functions of θ from the information given.

47. $\sin\theta = -\frac{4}{5}$, θ in Quadrant IV
48. $\tan\theta = \frac{4}{3}$, θ in Quadrant III
49. $\cos\theta = \frac{7}{12}$, $\sin\theta < 0$
50. $\cot\theta = -\frac{8}{9}$, $\cos\theta > 0$
51. $\csc\theta = 2$, θ in Quadrant I
52. $\cot\theta = \frac{1}{4}$, $\sin\theta < 0$
53. $\cos\theta = -\frac{2}{7}$, $\tan\theta < 0$
54. $\tan\theta = -4$, $\sin\theta > 0$