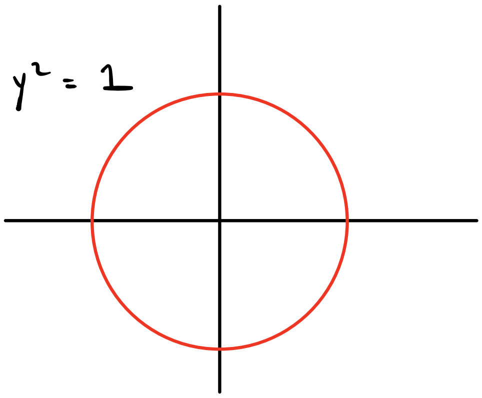
§5.1 The Unit Circle

$x^2 + y^2 = 1$



EXAMPLE 1 ■ **A Point on the Unit Circle**

Show that the point $P\left(\frac{\sqrt{3}}{3}, \frac{\sqrt{6}}{3}\right)$ is on the unit circle.

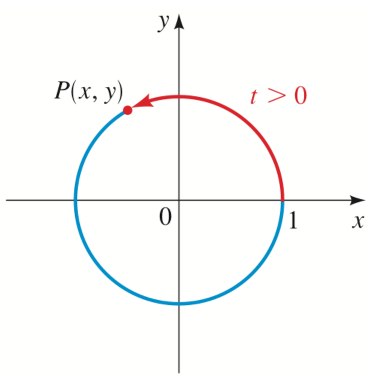
EXAMPLE 2 ■ **Locating a Point on the Unit Circle**

The point $P(\sqrt{3}/2, y)$ is on the unit circle in Quadrant IV. Find its y-coordinate.

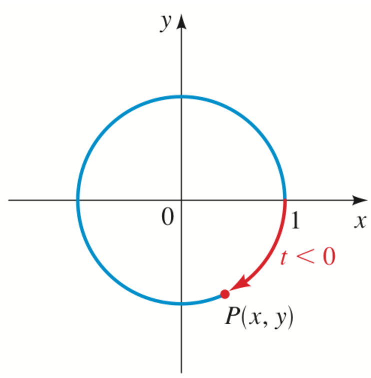
Def: Wrap a number line around the unit circle.

- Origin $\longrightarrow (1, 0)$
- Numbers increase in counter-clockwise direction
- Numbers decrease in clockwise direction

Given any real # t, just as it determines a point on a number line, it determines a point on the unit circle called the terminal point.

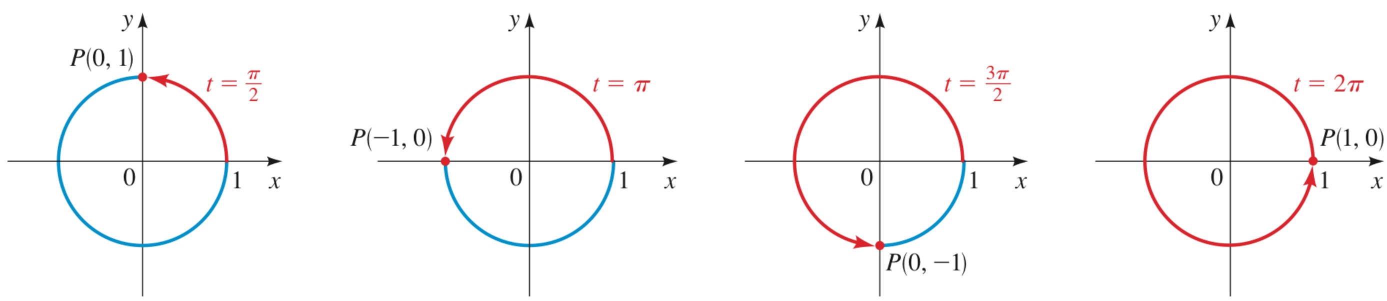


(a) Terminal point $P(x, y)$ determined by $t > 0$



(b) Terminal point $P(x, y)$ determined by $t < 0$

Note: The circumference of the unit circle is 2π.



EXAMPLE 3 ■ **Finding Terminal Points**

Find the terminal point on the unit circle determined by each real number t.

(a) $t = 3\pi$ **(b)** $t = -\pi$ **(c)** $t = -\frac{\pi}{2}$

TABLE 1

t	Terminal point determined by t
0	$(1, 0)$
$\frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ *
$\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ *
$\frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ *
$\frac{\pi}{2}$	$(0, 1)$

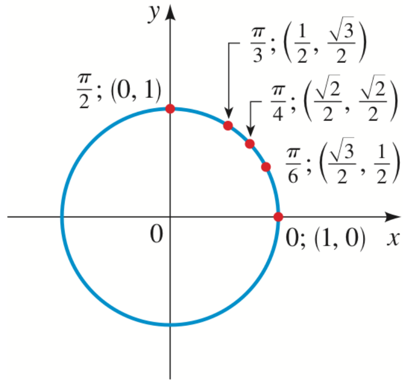


FIGURE 6

$x^2+y^2=1$, $x=y$

$\Rightarrow$ $2x^2=1$

$x=\pm\sqrt{\frac{1}{2}}$ ✓

$x^2+y^2=1$, $2y=\sqrt{x^2+(1-y)^2}$

$4y^2 = x^2+1-2y+y^2 = 2-2y$

$2y^2+y-1=0$

$y = \frac{-1\pm\sqrt{1+8}}{4} = \frac{-1\pm 3}{4} = \frac{1}{2}$ ✓ , $\cancel{-1}$

REFLECT $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

ACROSS $y=x$. ✓

EXAMPLE 4 ■ Finding Terminal Points

Find the terminal point determined by each given real number t.

(a) $t = -\frac{\pi}{4}$ **(b)** $t = \frac{3\pi}{4}$ **(c)** $t = -\frac{5\pi}{6}$

> **REFERENCE NUMBER**
>
> Let t be a real number. The **reference number** $\bar{t}$ associated with t is the shortest distance along the unit circle between the terminal point determined by t and the x-axis.

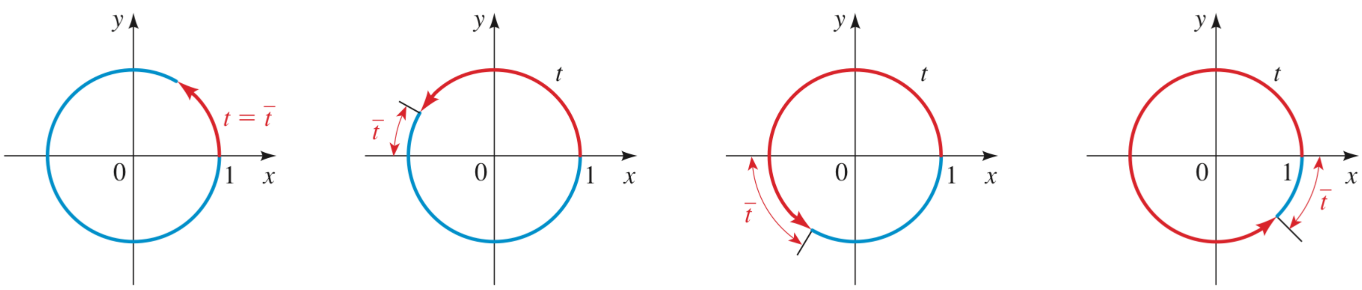


FIGURE 8 The reference number $\bar{t}$ for t

EXAMPLE 5 ■ Finding Reference Numbers

Find the reference number for each value of t.

(a) $t = \frac{5\pi}{6}$ **(b)** $t = \frac{7\pi}{4}$ **(c)** $t = -\frac{2\pi}{3}$ **(d)** $t = 5.80$

USING REFERENCE NUMBERS TO FIND TERMINAL POINTS

To find the terminal point P determined by any value of t, we use the following steps:

1. Find the reference number $\bar{t}$.
2. Find the terminal point $Q(a, b)$ determined by $\bar{t}$.
3. The terminal point determined by t is $P(\pm a, \pm b)$, where the signs are chosen according to the quadrant in which this terminal point lies.

EXAMPLE 6 ■ Using Reference Numbers to Find Terminal Points

Find the terminal point determined by each given real number t.

(a) $t = \dfrac{5\pi}{6}$ **(b)** $t = \dfrac{7\pi}{4}$ **(c)** $t = -\dfrac{2\pi}{3}$

EXAMPLE 7 ■ Finding the Terminal Point for Large t

Find the terminal point determined by $t = \dfrac{29\pi}{6}$.

41–54 ■ Terminal Points and Reference Numbers Find **(a)** the reference number for each value of t and **(b)** the terminal point determined by t.

41. $t = \dfrac{11\pi}{6}$ **42.** $t = \dfrac{2\pi}{3}$

43. $t = -\dfrac{4\pi}{3}$ **44.** $t = \dfrac{5\pi}{3}$

45. $t = -\dfrac{2\pi}{3}$ **46.** $t = -\dfrac{7\pi}{6}$

47. $t = \dfrac{13\pi}{4}$ **48.** $t = \dfrac{13\pi}{6}$

49. $t = \dfrac{41\pi}{6}$ **50.** $t = \dfrac{17\pi}{4}$

51. $t = -\dfrac{11\pi}{3}$ **52.** $t = \dfrac{31\pi}{6}$

53. $t = \dfrac{16\pi}{3}$ **54.** $t = -\dfrac{41\pi}{4}$