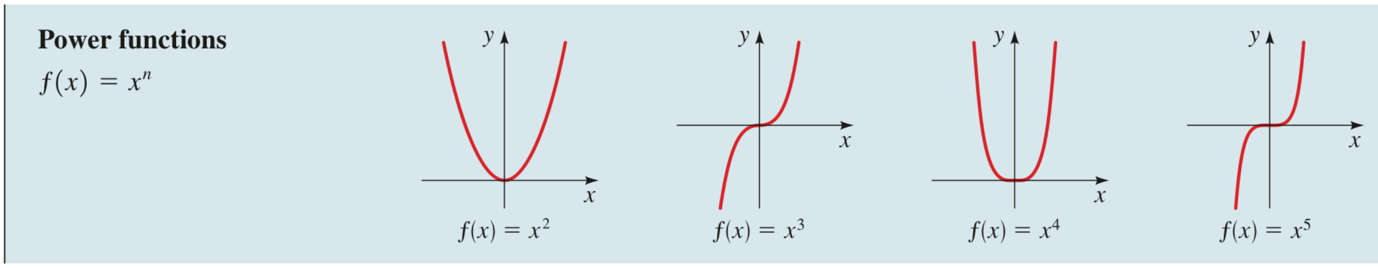
# §3.2 Polynomial Functions & Their Graphs

## Know the Graphs of Power Functions (aka Monomials)




ex. Sketch the Graphs of

(a) $y = (x+2)^4$

(b) $y = -\frac{1}{4}x^3$

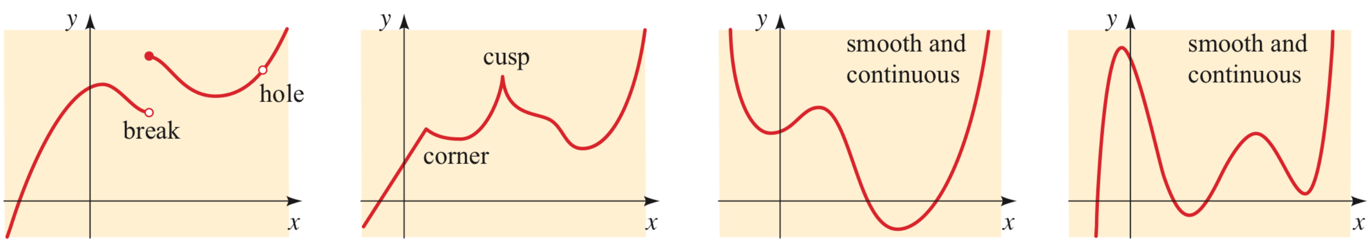
(c) $y = -(x+5)^5 + 1$

Big Idea: Graphs of Polynomials $(y = P(x))$ are

- Continuous - No gaps, holes, jumps, can be drawn without lifting your pencil

and

- Smooth - No corners or cusps (in calculus you will use the word "differentiable")




Not the graph of a polynomial function | Not the graph of a polynomial function | Graph of a polynomial function | Graph of a polynomial function

**BIG IDEA:** IF $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ AND $|x|$ IS LARGE,

THEN $f(x) \approx a_n x^n$

**LEAD TERM DOMINATES!**

e.g.

| $x$ | $x^5 + 2x^3 + 2$ | $x^5$ |
|---|---|---|
| 3 | 299 | 243 |
| 1,000,000 | 1,000,000,000,000,200,000,000,000,000,002 | 1,000,000,000,000,000,000,000,000,000,000 |

DIFFERENCE IS NEGLIGIBLE WHEN $x$ IS LARGE

IMPLICATION: GRAPH OF $y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

LOOKS LIKE THE GRAPH OF $y = a_n x^n$ WHEN $|x|$ IS LARGE.

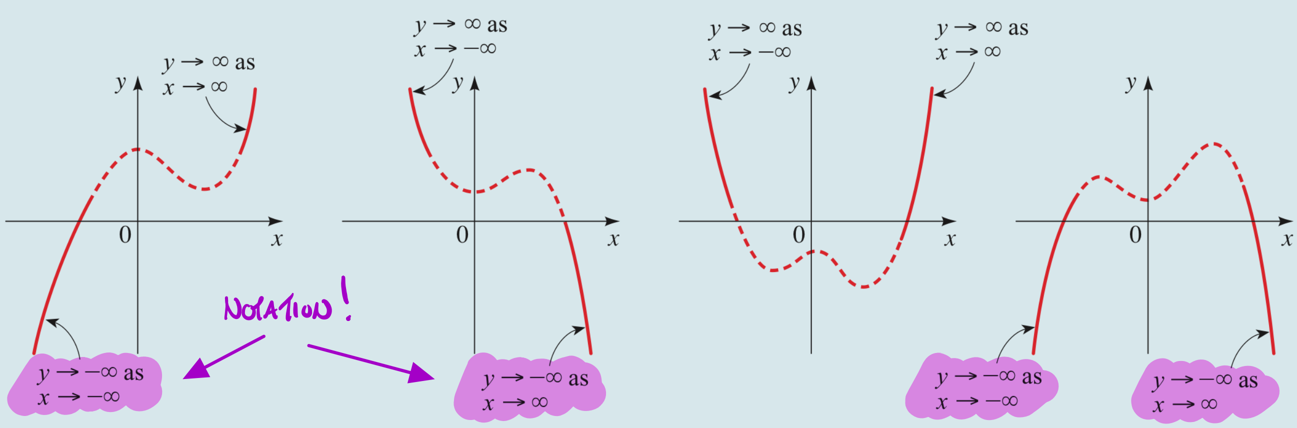
i.e. IDENTICAL **TAILS**

i.e. IDENTICAL **END BEHAVIOR**.

## END BEHAVIOR OF POLYNOMIALS

The end behavior of the polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is determined by the degree $n$ and the sign of the leading coefficient $a_n$, as indicated in the following graphs.

***P* has odd degree** | ***P* has even degree**




Leading coefficient positive | Leading coefficient negative | Leading coefficient positive | Leading coefficient negative

**ex.** DETERMINE THE END BEHAVIOR OF

(a) $y = -\frac{2}{3}x^3 + 5x^2 - \frac{1}{2}x + 7$

(b) $y = -9x^4 + 2x^3 + 7x - 128$

(c) $y = 4(x-1)(x-2)(x-3)(x-4)(x-5)$

## REAL ZEROS OF POLYNOMIALS

If $P$ is a polynomial and $c$ is a real number, then the following are equivalent:

1. $c$ is a zero of $P$.
2. $x = c$ is a solution of the equation $P(x) = 0$.
3. $x - c$ is a factor of $P(x)$.
4. $c$ is an $x$-intercept of the graph of $P$.

5. C IS A ROOT OF THE POLYNOMIAL $P(x)$

## INTERMEDIATE VALUE THEOREM FOR POLYNOMIALS

If $P$ is a polynomial function and $P(a)$ and $P(b)$ have opposite signs, then there exists at least one value $c$ between $a$ and $b$ for which $P(c) = 0$.

## EXAMPLE 5 ■ Finding Zeros and Graphing a Polynomial Function

Let $P(x) = x^3 - 2x^2 - 3x$.

**(a)** Find the zeros of $P$. **(b)** Sketch a graph of $P$.

USE A SIGN CHART TO DETERMINE WHETHER GRAPH IS ABOVE/BELOW X-AXIS IN BETWEEN THE ZEROS (X-INTERCEPTS)

## EXAMPLE 6 ■ Finding Zeros and Graphing a Polynomial Function

Let $P(x) = -2x^4 - x^3 + 3x^2$.

**(a)** Find the zeros of $P$. **(b)** Sketch a graph of $P$.

## EXAMPLE 7 ■ Finding Zeros and Graphing a Polynomial Function

Let $P(x) = x^3 - 2x^2 - 4x + 8$.

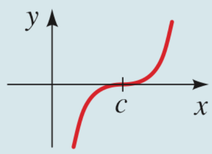
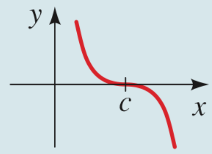
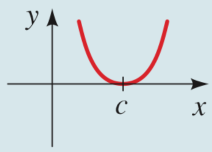
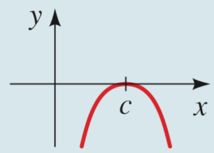
**(a)** Find the zeros of $P$. **(b)** Sketch a graph of $P$.

Def: EACH FACTOR OF A POLYNOMIAL CORRESPONDS TO A ZERO/ROOT OF THE POLYNOMIAL. THE EXPONENT ATTACHED TO THAT FACTOR IS CALLED THE MULTIPLICITY OF THE CORRESPONDING ZERO/ROOT.

IT DETERMINES THE SHAPE OF $y = P(x)$ AROUND THE ZERO/ROOT/X-INTERCEPT.

## SHAPE OF THE GRAPH NEAR A ZERO OF MULTIPLICITY $m$

If $c$ is a zero of $P$ of multiplicity $m$, then the shape of the graph of $P$ near $c$ is as follows.

| **Multiplicity of $c$** | **Shape of the graph of $P$ near the $x$-intercept $c$** | | |
|---|---|---|---|
| $m$ odd, $m > 1$ |  | OR |  |
| $m$ even, $m > 1$ |  | OR |  |

ex. GRAPH THE POLYNOMIAL:

**26.** $P(x) = -(x+1)^2(x-1)^3(x-2)$

**27.** $P(x) = \frac{1}{12}(x+2)^2(x-3)^2$

**28.** $P(x) = (x-1)^2(x+2)^3$

**29.** $P(x) = x^3(x+2)(x-3)^2$

**30.** $P(x) = (x-3)^2(x+1)^2$

**40.** $P(x) = \frac{1}{8}(2x^4 + 3x^3 - 16x - 24)^2$

**41.** $P(x) = x^4 - 2x^3 - 8x + 16$

**42.** $P(x) = x^4 - 2x^3 + 8x - 16$

**43.** $P(x) = x^4 - 3x^2 - 4$

**44.** $P(x) = x^6 - 2x^3 + 1$