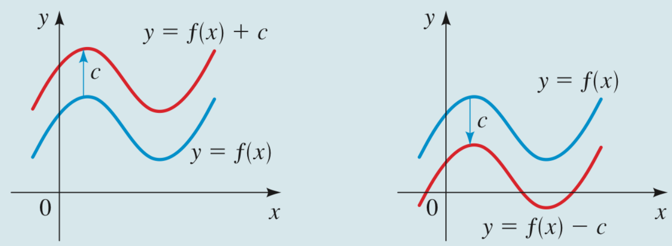
§2.6 Transformations of Functions

VERTICAL SHIFTS OF GRAPHS

Suppose $c > 0$.

To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upward c units.

To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downward c units.



ex. Sketch $y = x^2$ (without plotting - it doesn't have to be perfect).

Then sketch $y = x^2 + 2$ & $y = x^2 - 4$.

(If we let $f(x) = x^2$

Then these graphs are $y = f(x) + 2$ & $y = f(x) - 4$)

Vertical shifts!

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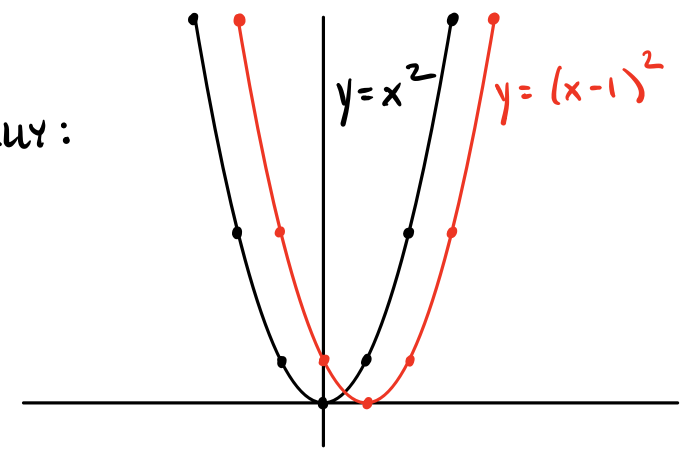
ex. Complete the table of values:

	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$(x-1)^2$	16	9	4	1	0	1	4

Horizontal shift 1 unit to the right!

Graphically:



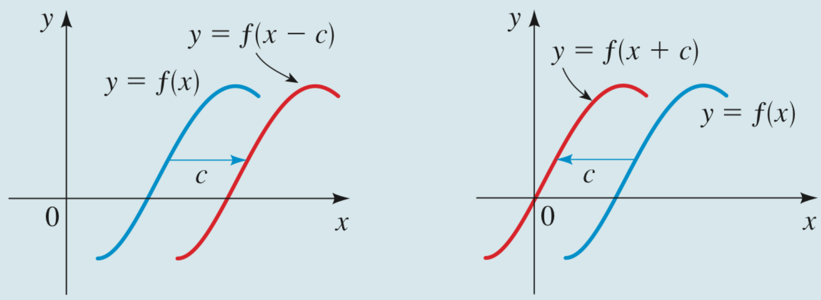
IN GENERAL:

HORIZONTAL SHIFTS OF GRAPHS

Suppose $c > 0$.

To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the right c units.

To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the left c units.



ex. SKETCH $y = \sqrt{x}$ (WITHOUT PLOTTING - IT DOESN'T HAVE TO BE PERFECT).

THEN SKETCH $y = \sqrt{x-4}$ & $y = \sqrt{x+3}$

(IF WE LET $f(x) = \sqrt{x}$

THEN THESE GRAPHS ARE $y = f(x-4)$ $y = f(x+3)$)

HORIZONTAL SHIFTS!

ex. SKETCH $y = |x|$ (WITHOUT PLOTTING - IT DOESN'T HAVE TO BE PERFECT).

THEN SKETCH (a) $y = |x-5| + 2$

(b) $y = |x+2| - 5$

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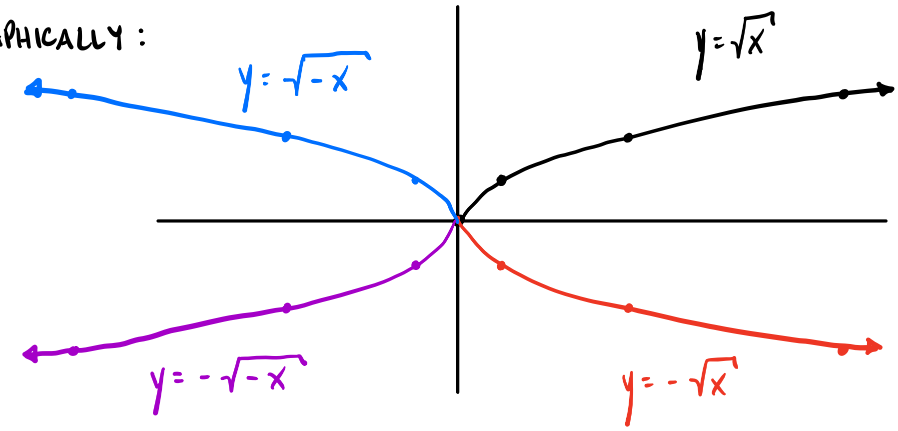
ex. COMPLETE THE TABLE OF VALUES

	-4	-3	-2	-1	0	1	2	3	4
$\sqrt{x}$	UND	UND	UND	UND	0	1	$\sqrt{2}$	$\sqrt{3}$	2
$-\sqrt{x}$	UND	UND	UND	UND	0	-1	$-\sqrt{2}$	$-\sqrt{3}$	-2
$\sqrt{-x}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0	UND	UND	UND	UND
$-\sqrt{-x}$	-2	$-\sqrt{3}$	$-\sqrt{2}$	-1	0	UND	UND	UND	UND

"SYMMETRY"

"REFLECTIONS"

GRAPHICALLY:



"SYMMETRY"

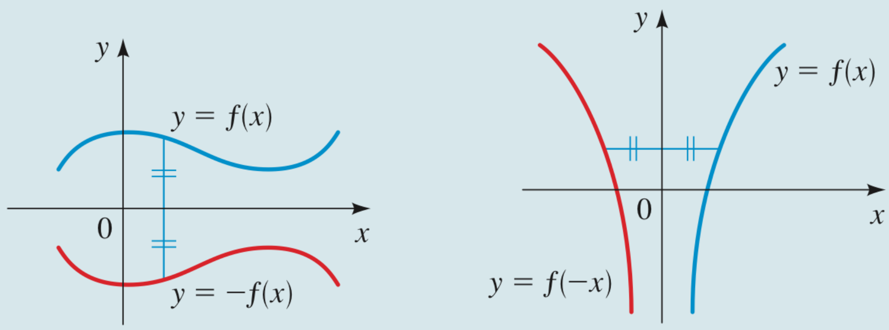
"REFLECTIONS"

IN GENERAL:

REFLECTING GRAPHS

To graph $y = -f(x)$, reflect the graph of $y = f(x)$ in the x-axis.

To graph $y = f(-x)$, reflect the graph of $y = f(x)$ in the y-axis.

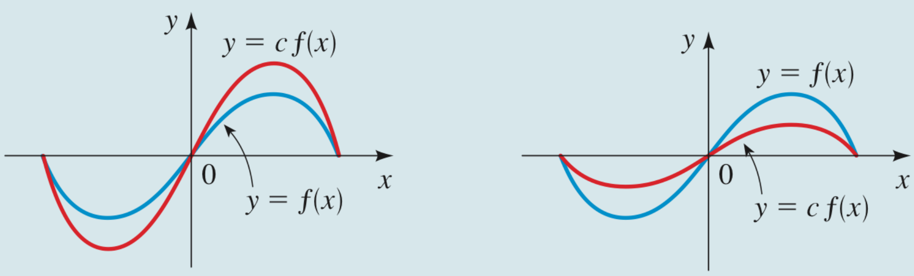


VERTICAL STRETCHING AND SHRINKING OF GRAPHS

To graph $y = cf(x)$:

If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of c.

If $0 < c < 1$, shrink the graph of $y = f(x)$ vertically by a factor of c.



$c > 1$ $\qquad$ $0 < c < 1$

ex. USE THE GRAPH OF $y = x^2$ TO SKETCH

(a) $y = \frac{1}{2}x^2$

(b) $y = 2(x-3)^2 + 1$

ex. USE THE GIVEN VALUES OF $f(x)$ TO COMPLETE THE TABLE OF VALUES AS MUCH AS POSSIBLE

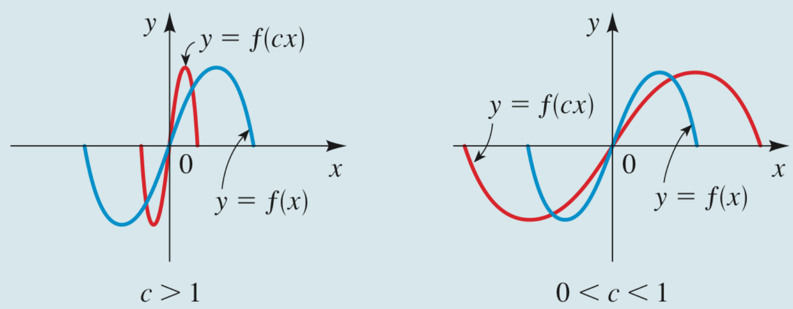
x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	
$f(x)$	1	4	1	5	9	2	6	5	3	5	8	9	7	
$f(2x)$				1	1	9	0	3	8	7				"SHRINK"
$f(\frac{1}{2}x)$	5		9		2		6		5		3		5	"STRETCH"

HORIZONTAL SHRINKING AND STRETCHING OF GRAPHS

To graph $y = f(cx)$:

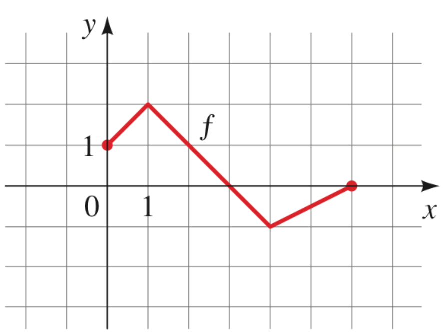
If $c > 1$, shrink the graph of $y = f(x)$ horizontally by a factor of $1/c$.

If $0 < c < 1$, stretch the graph of $y = f(x)$ horizontally by a factor of $1/c$.



ex. USE THE GIVEN GRAPH $y = f(x)$ TO SKETCH THE INDICATED GRAPH (TRANSFORMATION).

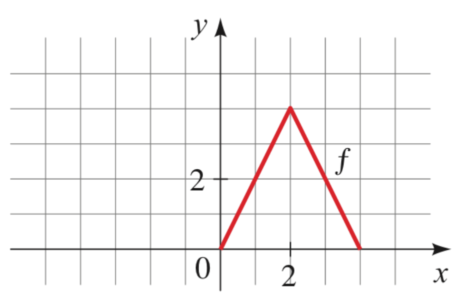
73. (a) $y = f(2x)$ **(b)** $y = f(\frac{1}{2}x)$



71. (a) $y = f(x - 2)$ **(b)** $y = f(x) - 2$

(c) $y = 2f(x)$ **(d)** $y = -f(x) + 3$

(e) $y = f(-x)$ **(f)** $y = \frac{1}{2}f(x - 1)$



ADDITIONAL EXERCISES :

45. $y = (x-3)^2 + 5$

46. $y = \sqrt{x+4} - 3$

47. $y = 3 - \frac{1}{2}(x-1)^2$

48. $y = 2 - \sqrt{x+1}$

49. $y = |x+2| + 2$

50. $y = 2 - |x|$

51. $y = \frac{1}{2}\sqrt{x+4} - 3$

52. $y = 3 - 2(x-1)^2$

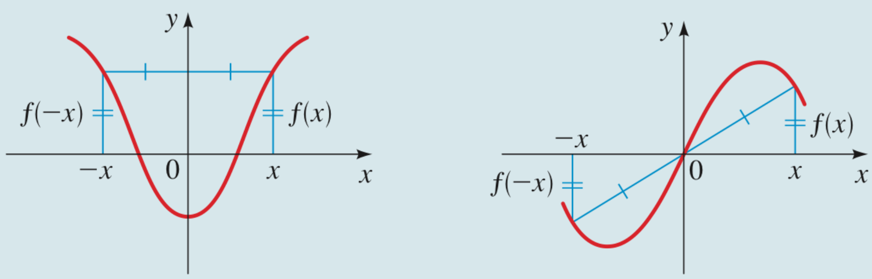
EVEN & ODD FUNCTIONS

EVEN AND ODD FUNCTIONS

Let f be a function.

f is **even** if $f(-x) = f(x)$ for all x in the domain of f.

f is **odd** if $f(-x) = -f(x)$ for all x in the domain of f.



The graph of an even function is symmetric with respect to the y-axis.

The graph of an odd function is symmetric with respect to the origin.

EXAMPLE 8 ■ Even and Odd Functions

Determine whether the functions are even, odd, or neither even nor odd.

(a) $f(x) = x^5 + x$

(b) $g(x) = 1 - x^4$

(c) $h(x) = 2x - x^2$