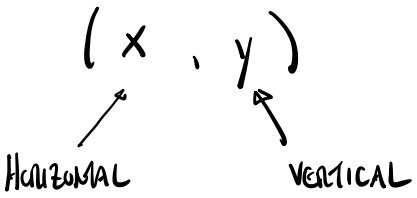
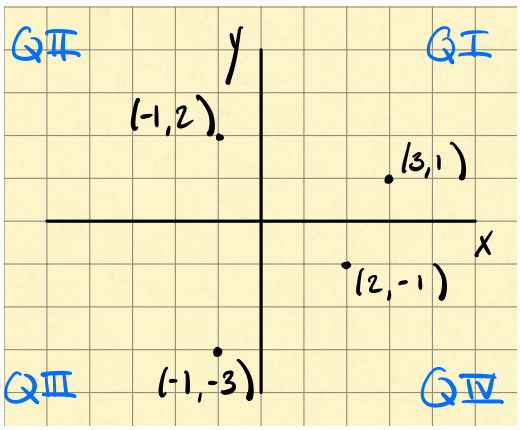
1.9 THE COORDINATE PLANE; GRAPHS OF EQUATIONS; CIRCLES

The Coordinate Plane ■ The Distance and Midpoint Formulas ■ Graphs of Equations in Two Variables ■ Intercepts ■ Circles ■ Symmetry

1 REAL # VISUALIZED AS POINT ON NUMBER LINE

2 REAL #'s (ORDERED PAIR) VISUALIZED AS POINT IN COORDINATE PLANE





Q. WHAT IS THE DISTANCE BETWEEN (2,-4) & (8,8) ?

DISTANCE FORMULA

The distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Q. WHAT POINT IS THE MIDPOINT OF THE LINE SEGMENT CONNECTING (-5, 2) & (7, 6) ?

MIDPOINT FORMULA

The midpoint of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

GRAPHS OF EQUATIONS WITH 2 VARIABLES

THE GRAPH OF AN EQUATION

The **graph** of an equation in x and y is the set of all points (x, y) in the coordinate plane that satisfy the equation.

e.g. WHICH POINTS ARE ON GRAPH OF EQUATION $x^2 + y^2 - 10x + 2y = -17$?

$(5,2)$ ✓ $(0,4)$ ✗

$(8,-1)$ ✓ $(1,-3)$ ✗

BUILDING TABLES & PLOTTING POINTS

MAY NEED TO DO THIS FIRST — STEP 0

WHEN AN EQUATION IS SOLVED FOR y, IT IS EASY TO SKETCH THE GRAPH OF THE EQUATION BY

PLUGGING IN VALUES FOR x, — STEP 1

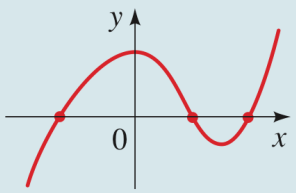
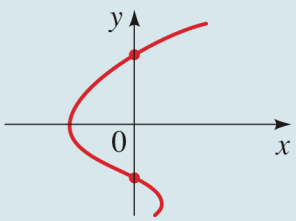
CALCULATING THE CORRESPONDING y, — STEP 2

PLOTTING THE POINT. — STEP 3

ex. PLOT 3 POINTS ON GRAPH OF $4x - 2y + 10 = 0$

ex. PLOT 4 POINTS ON GRAPH OF $x^2 - y + 5 = 3x$

DEFINITION OF INTERCEPTS

Intercepts	How to find them	Where they are on the graph
***x*-intercepts:** The x-coordinates of points where the graph of an equation intersects the x-axis	Set $y = 0$ and solve for x	
***y*-intercepts:** The y-coordinates of points where the graph of an equation intersects the y-axis	Set $x = 0$ and solve for y	

ex. FIND x & y INTERCEPT(S) OF

(a) $3x - 2y = 12$ (b) $x^2 + 2xy = y + 3$ (c) $\frac{x^2}{3} + \frac{y^2}{5} = 1$

EQUATIONS OF CIRCLES

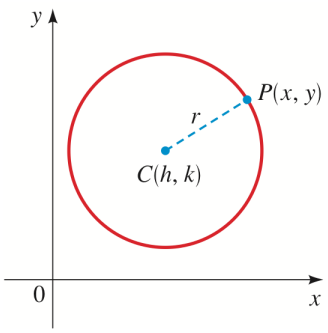
EQUATION OF A CIRCLE

An equation of the circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

This is called the **standard form** for the equation of the circle. If the center of the circle is the origin $(0, 0)$, then the equation is

$$x^2 + y^2 = r^2$$



ex. DESCRIBE THE GRAPH $(x-4)^2 + (y-1)^2 = 6$

ex. GIVE EQ OF CIRCLE WITH CENTER $(-2, 3)$ WITH RADIUS 4.

(b) Find an equation of the circle that has the points $P(1, 8)$ and $Q(5, -6)$ as the endpoints of a diameter.

1.10 LINES

The Slope of a Line ■ Point-Slope Form of the Equation of a Line ■ Slope-Intercept Form of the Equation of a Line ■ Vertical and Horizontal Lines ■ General Equation of a Line ■ Parallel and Perpendicular Lines

SLOPE OF A LINE

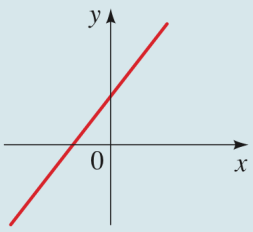
The **slope** m of a nonvertical line that passes through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

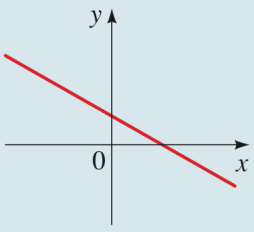
The slope of a vertical line is not defined.

SLOPE OF A LINE

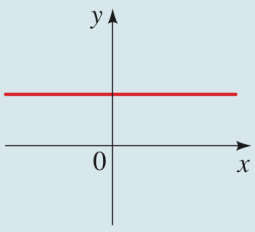
Positive Slope



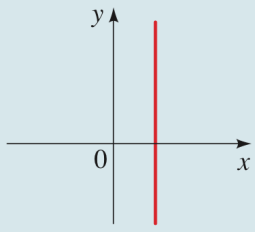
Negative Slope



Zero Slope



No Slope

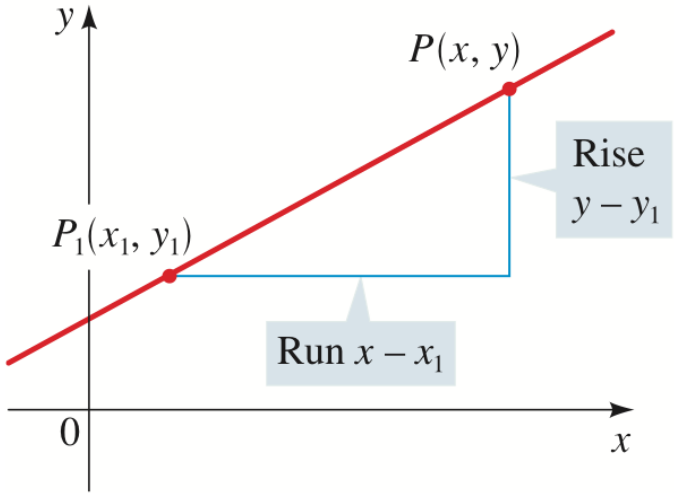


ex. Find the slope of the line through $(1, 3)$ & $(6, -2)$.

POINT-SLOPE FORM OF THE EQUATION OF A LINE

An equation of the line that passes through the point (x_1, y_1) and has slope m is

$$y - y_1 = m(x - x_1)$$



ex. Give an equation for the line through $(-2, 1)$ with slope $\frac{3}{4}$.

ex. Give an equation for the line through $(1, -2)$ that...

(a) also goes through $(3, 1)$.

(b) is horizontal.

(c) is vertical.

SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE

An equation of the line that has slope m and y-intercept b is

$$y = mx + b$$

ex. Find the slope & y-intercept of the line with equation

$$4x - 5y = 2.$$

ex. Let ℓ be the line with equation $7x + 2y = 1$.

Give an equation for the line through $(-3, -2)$ that is

(a) parallel to ℓ.

(b) perpendicular to ℓ.

PARALLEL LINES

Two nonvertical lines are parallel if and only if they have the same slope.

PERPENDICULAR LINES

Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$, that is, their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

Also, a horizontal line (slope 0) is perpendicular to a vertical line (no slope).