Solving Linear Equations

$ax = b \Rightarrow x = \frac{b}{a}$

ex. $\frac{3x}{2} = \frac{9}{5}$

Solving Quadratic Eq's

$ax^2 + bx + c = 0$

Factoring

Zero-Product Property: If $AB = 0$ then $A = 0$ or $B = 0$ (or both)

ex. solve $x^2 + 7x = x + 16$

ex. solve $3x^2 + 8 = 11x + 2$

ex. solve $2x(x-5) = 7(x-5)$

(1) Set equal to 0

(2) Factor

(3) Set factors equal to 0 & solve

Completing the Square

Suppose $x^2 + bx = c$

$x^2 + 2\left(\frac{b}{2}\right)x = c$

Add $\left(\frac{b}{2}\right)^2$ to both sides of eq to "complete the square"

$x^2 + 2\left(\frac{b}{2}\right)x + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$

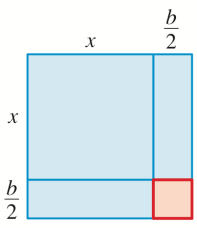
$\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$

Completing the Square

The area of the blue region is

$x^2 + 2\left(\frac{b}{2}\right)x = x^2 + bx$

Add a small square of area $(b/2)^2$ to "complete" the square.



ex. $x^2 + 12x = 3$ (cannot factor!)

ex. $x^2 - 5x - 1 = 0$ (cannot factor!)

The Quadratic Formula

$ax^2 + bx + c = 0 \quad \Rightarrow \quad x^2 + \frac{b}{a}x = -\frac{c}{a}$

Backup Plan

$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$

$\Rightarrow x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$

$\Rightarrow x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex. $10x^2 + 13x - 2 = 0$

$b^2 - 4ac$ is called the discriminant

- If $b^2 - 4ac > 0$ then 2 solutions
- If $b^2 - 4ac = 0$ then 1 solution
- If $b^2 - 4ac < 0$ then No solutions

ex. Solve $x^2 + x + 1 = 0$ (neg. discr. $\Rightarrow$ No solution)

§ 1.6 Solving Other Types of Equations

(1) Set Equal To 0

(2) Factor

(3) Set Factors Equal To 0 & Solve

ex. $8x^4 = 18x^2$

EXAMPLE 3 ■ An Equation Involving Fractional Expressions

Solve the equation $\frac{3}{x} - \frac{2}{x-3} = \frac{-12}{x^2-9}$.

MULTIPLY EACH TERM BY LCD TO CANCEL ALL DENOMINATORS

EXAMPLE 4 ■ An Equation Involving a Radical

Solve the equation $2x = 1 - \sqrt{2-x}$.

ISOLATE RADICAL EXPRESSION, SQUARE BOTH SIDES.

* SOLUTIONS TO EQUATIONS WITH DENOMINATORS OR RADICALS NEED TO BE CHECKED! *