Answer each non-graph question CLEARLY on the line provided.

Name: *ANSWER KEY* ID: _____

| Page | Points | Score |
|---|---|---|
| 1 | 8 | |
| 2 | 12 | |
| 3 | 12 | |
| 4 | 12 | |
| 5 | 12 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 12 | |
| 9 | 8 | |
| Total: | 100 | |

1. (4 points) Perform the indicated operations $\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{5}-\frac{6}{15}}$ and simplify as much as possible.

$$\frac{\frac{2}{5}+\frac{1}{2}}{\frac{1}{5}-\frac{6}{15}} \cdot \frac{30}{30} = \frac{12+15}{6-12}$$

$$= \frac{27}{-6} = -\frac{9}{2}$$

ALTERNATIVELY,

$$\left(\frac{2}{5}+\frac{1}{2}\right) \div \left(\frac{1}{5}-\frac{6}{15}\right) = \left(\frac{4}{10}+\frac{5}{10}\right) \div \left(\frac{1}{5}-\frac{2}{5}\right)$$

$$= \frac{9}{10} \div \frac{-1}{5} = \frac{9}{10} \times \frac{-5}{1} = -\frac{9}{2}$$

1. $-\frac{9}{2}$

2. (4 points) Evaluate and simplify $\left(\frac{64}{81}\right)^{-\frac{1}{2}}$ completely. Eliminate any negative exponents.

$$\left(\frac{81}{64}\right)^{1/2} = \sqrt{\frac{81}{64}} = \frac{\sqrt{81}}{\sqrt{64}} = \frac{9}{8}$$

2. $\frac{9}{8}$

3. (4 points) Perform the multiplication $(3x-4)(3x+4)$ and simplify completely. Leave no parenthesis in final answer.

$(3x-4)(3x+4)$

$9x^2 + 12x - 12x - 16$

$0$

3. $9x^2 - 16$

4. (4 points) Factor $z^3 - 2z^2 - 3z$ completely.

GCF = z :

$z(z^2 - 2z - 3)$

$z(z-3)(z+1)$

FACTOR THE FACTORS AS MUCH AS POSSIBLE

4. $z(z-3)(z+1)$

5. (4 points) Perform the multiplication $\frac{x^2-36}{x^2-16} \cdot \frac{2x+8}{x-6}$ and simplify completely.

FACTOR FIRST!

$$\frac{(x+6)(x-6)}{(x+4)(x-4)} \cdot \frac{2(x+4)}{(x-6)} = \frac{2(x+6)}{x-4}$$

$$= \frac{2x+12}{x-4}$$

EITHER IS FINE

5. $\frac{2x+12}{x-4}$

6. (4 points) Perform the addition $1 + \frac{2}{x} + \frac{3}{x^2}$ and simplify completely as one rational expression.

6. $\frac{x^2 + 2x + 3}{x^2}$

LCD = $x^2$

$$\frac{x^2}{x^2} + \frac{2x}{x^2} + \frac{3}{x^2} = \frac{x^2 + 2x + 3}{x^2}$$

7. (4 points) Find an equation of the line through the point $(0, -2)$ and parallel to the line $2x + 3y = 4$.

7. $y = -\frac{2}{3}x - 2$

1. FIND SLOPE OF $2x + 3y = 4$

$$y = -\frac{2}{3}x + \frac{4}{3}$$

PARALLEL LINES HAVE THE SAME SLOPE

2. USE POINT SLOPE EQ: $y - y_1 = m(x - x_1)$

$$y + 2 = -\frac{2}{3}(x - 0) \quad \Rightarrow \quad y = -\frac{2}{3}x - 2$$

EITHER IS FINE

8. (4 points) Find all solutions $x$ to $\sqrt{6x + 4} + 2 = x$.

8. $x = 10$

1. ISOLATE THE RADICAL EXPRESSION

$$\sqrt{6x + 4} = x - 2$$

2. SQUARE BOTH SIDES (MAY INTRODUCE FALSE SOLUTIONS!)

$$6x + 4 = x^2 - 4x + 4$$

$$0 = x^2 - 10x = x(x - 10)$$

$$x = 0, 10$$

3. CHECK: $x = 0$: $\sqrt{0 + 4} + 2 = 0$ FALSE

$x = 10$: $\sqrt{60 + 4} + 2 = 10$ TRUE ✓

$g(x+5) - g(x)$ $g(x) = 2x^2$

9. (4 points) Evaluate and simplify the expression $g(a+5) - g(5)$ completely when $g(t) = 2t^2$.

$g(a+5) = 2(a+5)^2 = 2(a^2 + 10a + 25)$

$= 2a^2 + 20a + 50$

$g(5) = 2 \cdot 5^2 = 50$

$\therefore \; g(a+5) - g(5) = 2a^2 + 20a + \underbrace{50 - 50}_{0}$

9. $2a^2 + 20a$

10. (4 points) Find all solutions $x$ to $(x-4)^2 = 5$.

$x - 4 = \pm\sqrt{5}$

$x = 4 \pm \sqrt{5}$

10. $x = 4 \pm \sqrt{5}$

$a = -1, \; b = 2, \; c = 10$

$-x^2 + 2x + 10$

11. (4 points) Find the domain and range of $f(x) = 10 + 2x - x^2$. Write your answer using interval notation.

DOMAIN $(-\infty, \infty)$

SINCE $f$ IS POLYNOMIAL, DOMAIN IS $(-\infty, \infty)$.

11. RANGE $(-\infty, 11]$

SINCE $f$ IS QUADRATIC WITH $a = -1 < 0$,

GRAPH $y = f(x)$ OPENS DOWNWARDS  $\Rightarrow$ $f$ HAS MAXIMUM.

MAX VALUE IS $y$-COORD OF VERTEX $= f\left(-\frac{b}{2a}\right) = f(1) = 11$

$= f\left(\frac{-2}{2(-1)}\right) = f(1) = 10 + 2(1) - (1)^2$

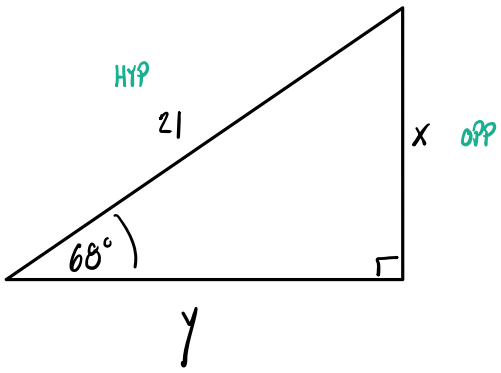
$= 10 + 2 - 1$

ALT: COMPLETE THE SQUARE

$f(x) = -(x^2 - 2x) + 10$

$f(x) = -(x^2 - 2x$

12. (4 points) A 21−ft ladder leans against a building so that the angle between the ground and the ladder is 68°. How high does the ladder reach on the building? (You may leave sin, cos, or tan in your answer).




12. $21 \sin 68°$

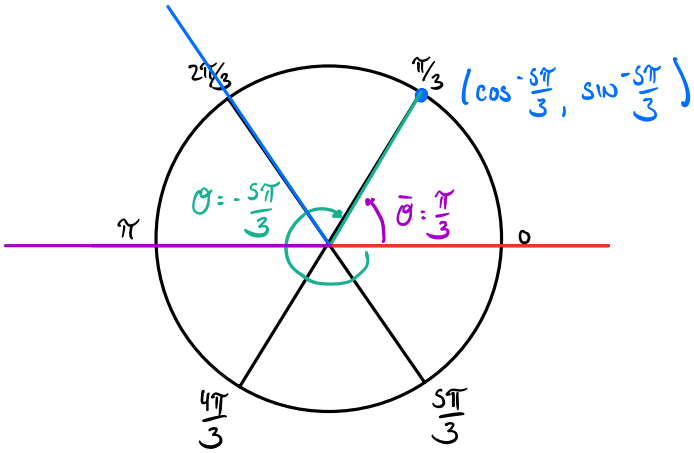
$\sin 68° = \frac{x}{21}$

$21 \sin 68° = x$

| S | A |
|---|---|
| T | C |

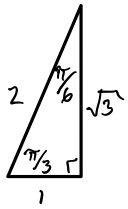
$\begin{matrix}\sin \\ \cos \\ \tan\end{matrix}(\theta) = \pm \begin{matrix}\sin \\ \cos \\ \tan\end{matrix}(\bar{\theta})$

13. (4 points) Evaluate $\sin\left(-\frac{5\pi}{3}\right)$.




13. $\frac{\sqrt{3}}{2}$

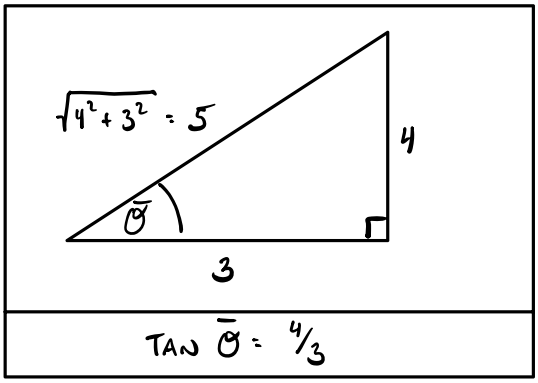
REFERENCE ANGLE FOR $\theta = -\frac{5\pi}{3}$ IS $\bar{\theta} = \frac{\pi}{3}$




$\Rightarrow \sin \frac{-5\pi}{3} = \pm \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

POSITIVE BECAUSE $\theta = \frac{-5\pi}{3}$ TERMINATES IN QI, WHERE BOTH $x = \cos\theta$ & $y = \sin\theta$ ARE POSITIVE.

14. (4 points) Evaluate $\cos\theta$ if $\tan\theta = \frac{4}{3}$ and $\theta$ is in Quadrant III.




(REFERENCE ANGLE)

14. $-\frac{3}{5}$

$\Rightarrow \cos\bar{\theta} = \frac{3}{5}$

SINCE $\theta$ TERMINATES IN QIII, WHERE $x = \cos\theta < 0$, $\cos\theta = -\frac{3}{5}$

15. (4 points) Find all solutions $x$ to $6^{3x-4} = \frac{1}{6}$.

$6^{3x-4} = 6^{-1}$

$3x - 4 = -1$

$3x = 3$

$x = 1$

15. $x = 1$

16. (4 points) Evaluate $\log_6(\frac{1}{36})$.

$\log_6 \frac{1}{36} = w$

$\Updownarrow$

$6^w = \frac{1}{36} = 6^{-2} \quad \Rightarrow \quad w = -2$

16. $-2$

17. (4 points) Find the degree measure of the angle with radian measure $-\frac{3\pi}{2}$.

$\text{DEG} = \text{RAD} \times \frac{180}{\pi}$ $\quad \left( \text{RAD} = \text{DEG} \times \frac{\pi}{180} \right)$

$= -\frac{3\pi}{2} \times \frac{180}{\pi} = -270^\circ$ (90)

17. $-270^\circ$

18. (4 points) Find $f^{-1}(x)$ when $f(x) = \frac{x}{x+2}$.

Let $y = \frac{x}{x+2}$. Solve for $x = f^{-1}(y)$.

$y(x+2) = x$

$xy + 2y = x$

$xy - x = -2y$

$x(y-1) = -2y$

$x = \frac{-2y}{y-1} \leftarrow f^{-1}(y)$

$\therefore f^{-1}(x) = \frac{-2x}{x-1}$

18. $\frac{-2x}{x-1}$

19. (4 points) Find $\sin^{-1}(-\frac{1}{2})$

SKIP THIS QUESTION

( §5.4 )

19. N.A.

20. (4 points) Find all solutions $x$ to $x^2 - 18x = 19$.

$x^2 - 18x + 81 = 19 + 81$ ( COMPLETING THE SQUARE )

$(x-9)^2 = 100$

$x - 9 = \pm 10$

$x = 9 \pm 10$

( OR USE QUAD. FORMULA )

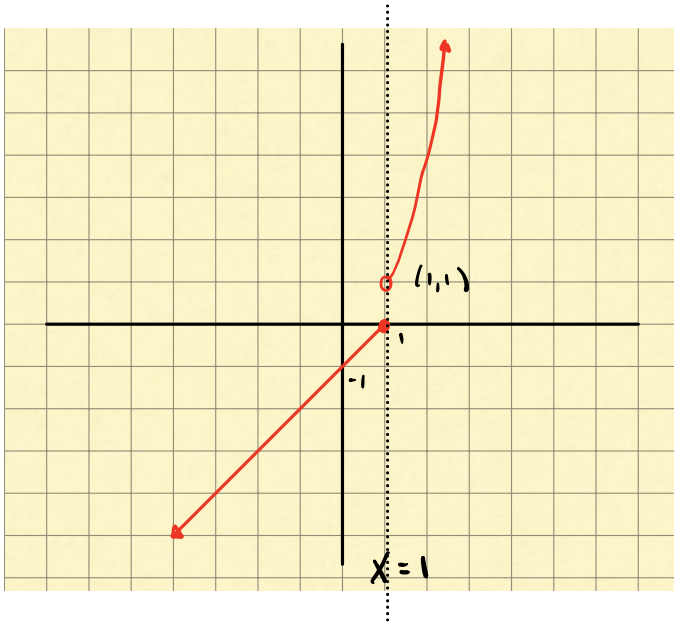
20. $x = -1, 19$

21. (4 points) Simplify $\frac{y^{-2}z^{-3}}{y^{-1}}$ as much as possible and eliminate any negative exponents.

$$\frac{y^1}{y^2 z^3} = \frac{1}{y z^3}$$

21. $\frac{1}{y z^3}$

22. (4 points) Sketch the graph of piecewise defined function $f(x) = \begin{cases} x-1 & \text{if } x \le 1 \\ x^2 & \text{if } x > 1 \end{cases}$




23. (4 points) Sketch the graph of $h(x) = x^4 - 4x^2$. Label all intercepts on your graph.

x-INT: $x^4 - 4x^2 = 0$

$x(x+2)(x-2) = 0$

$x=0$ BOUNCE

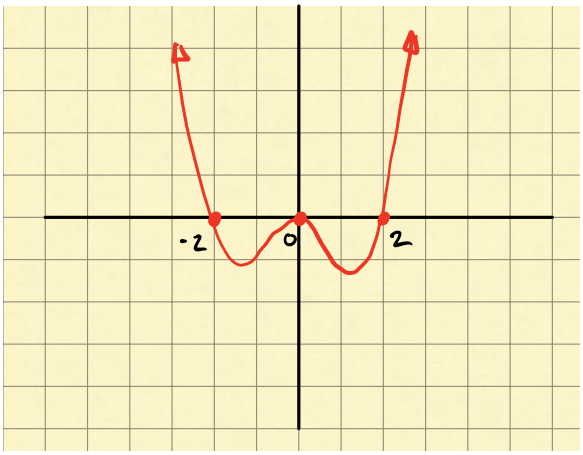
$x=-2$ CROSS

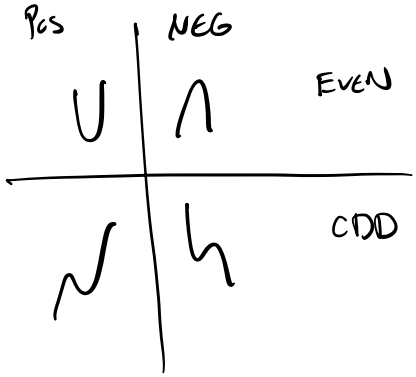
$x=2$ CROSS

y-INT: $h(0) = 0$.

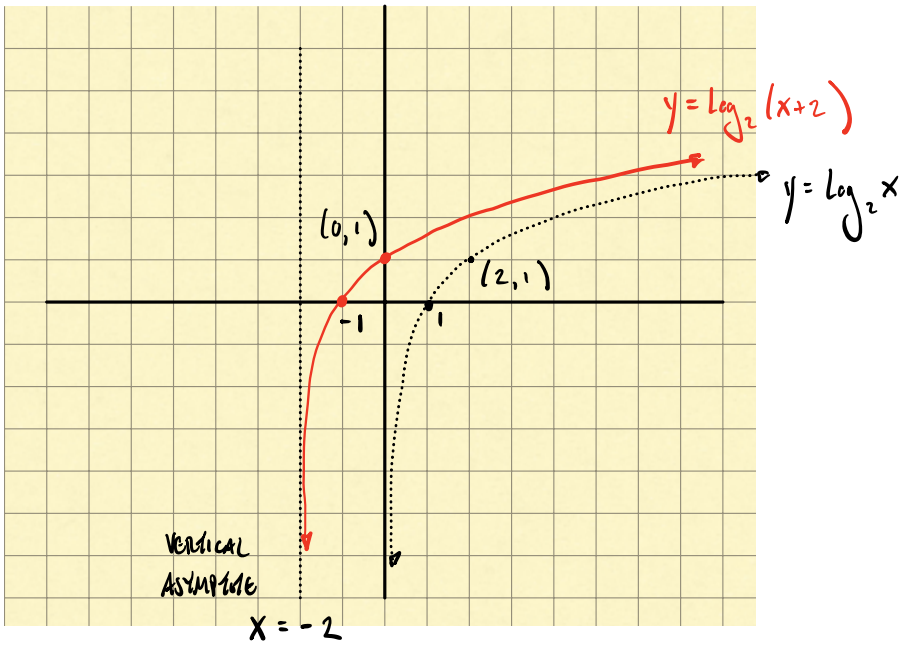
END BEHAVIOR ~ $y = x^4$ ∪

(As $x \to -\infty$, $y \to \infty$. As $x \to \infty$, $y \to \infty$)

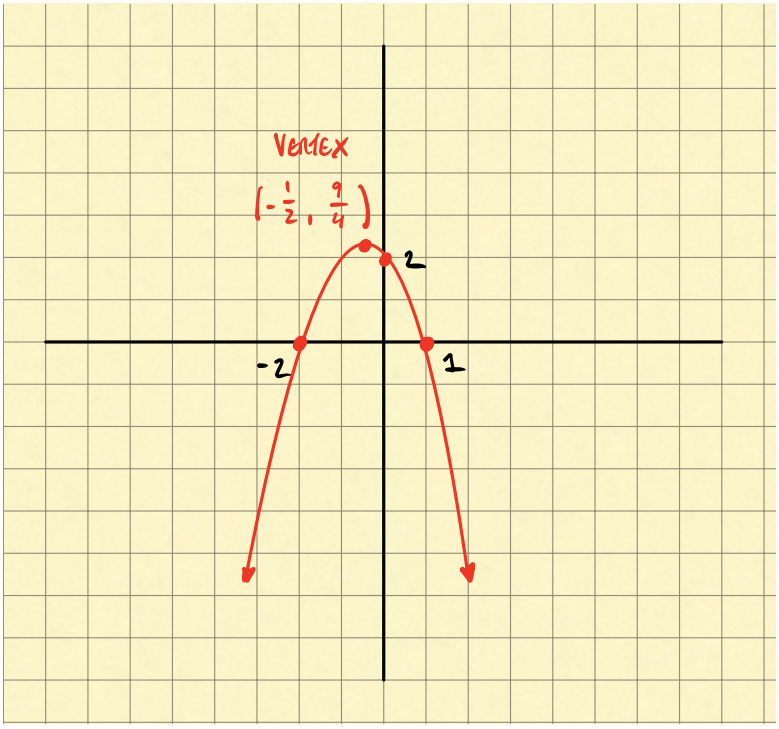


24. (4 points) Sketch the graph of $y = \log_2(x+2)$ not by plotting points but by starting with the graph of a standard function and applying transformations. Label all intercepts and asymptotes on the graph.




25. (4 points) Sketch the graph $f(x) = 2 - x - x^2$. Label the vertex and all intercepts on your graph.




$f(x) = -(x^2 + x) + 2$

$= -(x^2 + x + \frac{1}{4}) + 2 + \frac{1}{4}$

$= -(x + \frac{1}{2})^2 + \frac{9}{4}$

x-int: $0 = -(x + \frac{1}{2})^2 + \frac{9}{4}$

$(x + \frac{1}{2})^2 = \frac{9}{4}$

$x + \frac{1}{2} = \pm \frac{3}{2}$

$x = -\frac{1}{2} \pm \frac{3}{2} = -2, 1$

y-int: $f(0) = 2$