§ P.4 Rational Exponents & Radicals

- Radicals
- Rational Exponents
- Rationalizing Denominators

Radicals

DEF: Given $a \geq 0$,

$\sqrt{a}$ is the non-neg # x s.t. $x^2 = a$

ex. $\sqrt{64} = 8$ ex. $\sqrt{\frac{49}{36}} = \frac{7}{6}$

ex. Solve: $x^2 = 16$

DEF: n^{th} root: $\sqrt[n]{a}$ is # x s.t. $x^n = a$

- If n is even then we must have $a \geq 0$, $b \geq 0$.

Properties of n^{th} roots:

1. $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$
2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
3. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$
4. $\sqrt[n]{a^n} = \begin{cases} a & \text{if } n \text{ is odd} \\ |a| & \text{if } n \text{ is even} \end{cases}$

Simplifying Radical Expressions

ex. Simplify $\sqrt{8}$, $\sqrt{72}$

ex. Simplify $\sqrt{32x^8y^{11}}$

ex. Simplify $\sqrt[3]{54}$, $\sqrt[3]{16x^5y^7}$, $\sqrt[4]{a^5b^{10}c^{15}}$

ex. $\sqrt{32} + \sqrt{200}$ ex. $\sqrt{25b} - \sqrt{b^3}$ ex. $\sqrt{49x^2 + 49}$

ALT. NOTATION:

$(\sqrt{a})^2 = a$

$(a^{\frac{1}{2}})^2 = a$

$\sqrt{a} = a^{1/2}$

$(\sqrt[n]{a})^n = a$

$(a^{\frac{1}{n}})^n = a$

$\sqrt[n]{a} = a^{1/n}$

ex. $8^{2/3}$ ex. $125^{-1/3}$ ex. $(2a^3b^4)^{3/2}$ ex. $\left(\frac{2x^{3/4}}{y^{1/3}}\right)^3 \left(\frac{y^4}{x^{-1/2}}\right)$

ex. REWRITE WITH EXPONENTS: $(2\sqrt{x})(3\sqrt[3]{x})$

$\sqrt{x\sqrt{x}}$

RATIONALIZING DENOMINATORS ("STANDARD FORM")

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}, \quad a > 0$$

ex. $\frac{1}{\sqrt{2}}$ ex. $\frac{3}{2\sqrt{5}}$

MORE GENERALLY, $\frac{1}{\sqrt[n]{a^m}} = \frac{1}{\sqrt[n]{a^m}} \cdot \frac{\sqrt[n]{a^{n-m}}}{\sqrt[n]{a^{n-m}}} = \frac{\sqrt[n]{a^{n-m}}}{a}$

ex. $\frac{1}{\sqrt[3]{5}}$ ex. $\sqrt[7]{\frac{1}{a^2}}$