§5.2 Trigonometry of Right Triangles

Consider a right triangle (one angle is $90° = \frac{\pi}{2}$ rad).

Let one of the acute angles be θ (the other must be $(90 - \theta)° = \frac{\pi}{2} - \theta$ rad.)

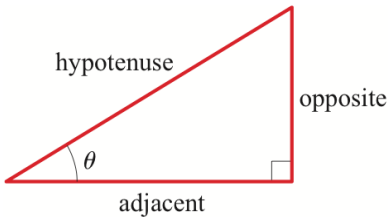


FIGURE 1

THE TRIGONOMETRIC RATIOS

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

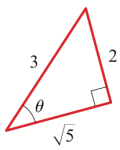


FIGURE 3

EXAMPLE 1 ■ Finding Trigonometric Ratios

Find the six trigonometric ratios of the angle θ in Figure 3.

SOLUTION By the definition of trigonometric ratios, we get

$$\sin\theta = \frac{2}{3} \qquad \cos\theta = \frac{\sqrt{5}}{3} \qquad \tan\theta = \frac{2}{\sqrt{5}}$$

$$\csc\theta = \frac{3}{2} \qquad \sec\theta = \frac{3}{\sqrt{5}} \qquad \cot\theta = \frac{\sqrt{5}}{2}$$

Now Try Exercise 3 ■

EXAMPLE 2 ■ Finding Trigonometric Ratios

If $\cos\alpha = \frac{3}{4}$, sketch a right triangle with acute angle α, and find the other five trigonometric ratios of α.

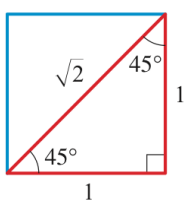
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

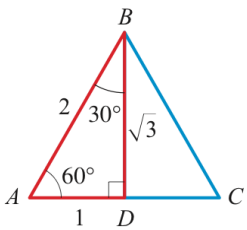
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

Special Triangles & Trig Table



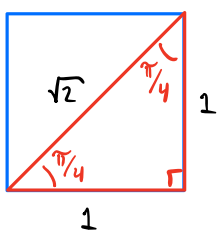
Square

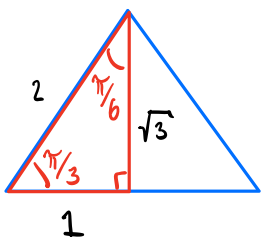


Equilateral Triangle

Both have shortest side length 1

(Pyth Thm: $a^2 + b^2 = c^2 \Rightarrow \sin^2\theta + \cos^2\theta = 1$)

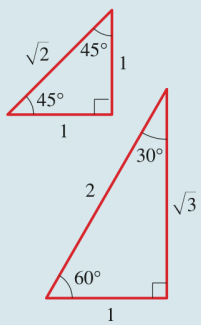




SPECIAL VALUES OF THE TRIGONOMETRIC FUNCTIONS

The following values of the trigonometric functions are obtained from the special triangles.

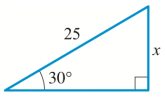
θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0	0	0	1	0	—	1	—
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	—	1	—	0



redundant

15–20 ■ Finding an Unknown Side Find the side labeled x. In Exercises 17 and 18 state your answer rounded to five decimal places.

15.



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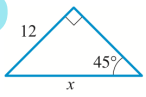

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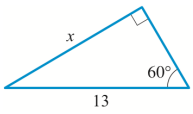

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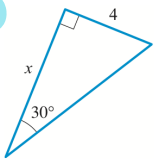

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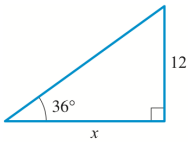

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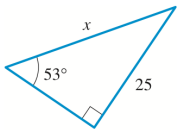
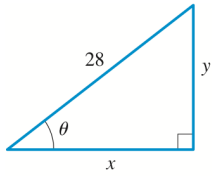


21–22 ■ Trigonometric Ratios Express x and y in terms of trigonometric ratios of θ.

21.



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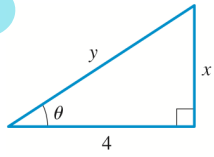

23–28 ■ Trigonometric Ratios Sketch a triangle that has acute angle θ, and find the other five trigonometric ratios of θ.

23. $\tan\theta = \frac{5}{6}$ **24.** $\cos\theta = \frac{12}{13}$ **25.** $\cot\theta = 1$

26. $\tan\theta = \sqrt{3}$ **27.** $\csc\theta = \frac{11}{6}$ **28.** $\cot\theta = \frac{5}{3}$

EXAMPLE 5 ■ Finding the Height of a Tree

A giant redwood tree casts a shadow 532 ft long. Find the height of the tree if the angle of elevation of the sun is 25.7°.

56. Distance at Sea From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocean is 23°. How far is the ship from the base of the lighthouse?