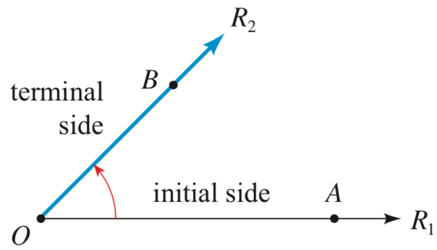
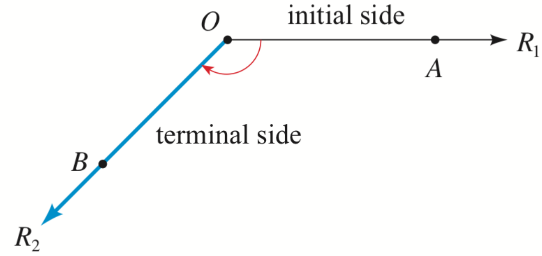


Positive angle



Negative angle

THROUGHOUT MATH & SCIENCE :

COUNTERCLOCKWISE	=	POSITIVE ROTATION
CLOCKWISE	=	NEGATIVE ROTATION

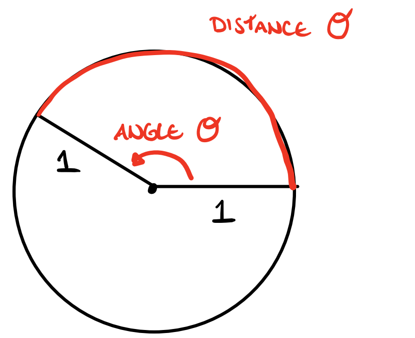
THE MEASURE OF AN ANGLE (POS OR NEG) IS THE AMOUNT OF ROTATION REQUIRED TO MOVE THE INITIAL SIDE ONTO THE TERMINAL SIDE.

DEGREES

$360° = 1$ REVOLUTION
$180° = \frac{1}{2}$ REVOLUTION
$90° = \frac{1}{4}$ REVOLUTION
$60° = \frac{1}{6}$ REVOLUTION
$45° = \frac{1}{8}$ REVOLUTION
$30° = \frac{1}{12}$ REVOLUTION

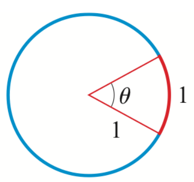
RADIANS

2π RAD $= 1$ REVOLUTION
π RAD $= \frac{1}{2}$ REVOLUTION
$\pi/2$ RAD $= \frac{1}{4}$ REVOLUTION
$\pi/3$ RAD $= \frac{1}{6}$ REVOLUTION
$\pi/4$ RAD $= \frac{1}{8}$ REVOLUTION
$\pi/6$ RAD $= \frac{1}{12}$ REVOLUTION



THE CIRCUMFERENCE OF UNIT CIRCLE IS 2π UNITS (WHATEVER THE UNIT IS).

THE MEASURE OF AN ANGLE IN RADIANS IS EQUAL TO THE LENGTH OF THE CORRESPONDING ARC ALONG THE UNIT CIRCLE.



Measure of $\theta = 1$ rad
Measure of $\theta \approx 57.296°$

RELATIONSHIP BETWEEN DEGREES AND RADIANS

$$180° = \pi \text{ rad} \qquad 1 \text{ rad} = \left(\frac{180}{\pi}\right)^{\circ} \qquad 1° = \frac{\pi}{180} \text{ rad}$$

1. To convert degrees to radians, multiply by $\frac{\pi}{180}$.
2. To convert radians to degrees, multiply by $\frac{180}{\pi}$.

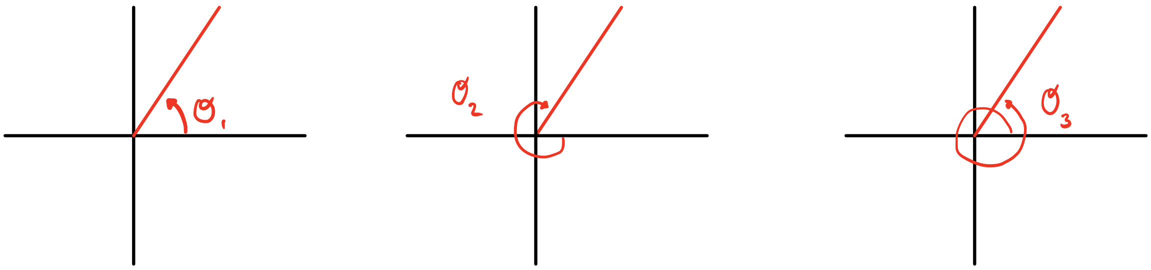
EXAMPLE 1 ■ Converting Between Radians and Degrees

(a) Express 60° in radians. **(b)** Express $\frac{\pi}{6}$ rad in degrees.

5. 15° **6.** 36° **7.** 54° **8.** 75°
9. −45° **10.** −30° **11.** 100° **12.** 200°
13. 1000° **14.** 3600° **15.** −70° **16.** −150°

17. $\frac{5\pi}{3}$ **18.** $\frac{3\pi}{4}$ **19.** $\frac{5\pi}{6}$
20. $-\frac{3\pi}{2}$ **21.** 3 **22.** −2
23. −1.2 **24.** 3.4 **25.** $\frac{\pi}{10}$
26. $\frac{5\pi}{18}$ **27.** $-\frac{2\pi}{15}$ **28.** $-\frac{13\pi}{12}$

STANDARD POSITION : VERTEX AT ORIGIN, INITIAL SIDE IS ON POSITIVE X-AXIS.



TWO ANGLES IN STAND. POS. ARE COTERMINAL IF THEIR SIDES COINCIDE.

IN THIS CASE THEY DIFFER BY SOME WHOLE NUMBER OF CLOCKWISE / C-CLOCK. REVOLUTIONS.

(i.e. A POS/NEG MULT. OF 2π RAD (OR 360°).)

EXAMPLE 2 ■ Coterminal Angles

(a) Find angles that are coterminal with the angle $\theta = 30^\circ$ in standard position.

(b) Find angles that are coterminal with the angle $\theta = \dfrac{\pi}{3}$ in standard position.

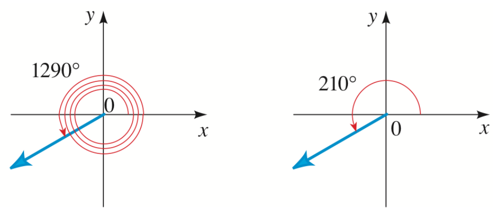
47–52 ■ Finding a Coterminal Angle Find an angle between 0 and 2π that is coterminal with the given angle.

47. $\dfrac{19\pi}{6}$ **48.** $-\dfrac{5\pi}{3}$ **49.** 25π

50. 10 **51.** $\dfrac{17\pi}{4}$ **52.** $\dfrac{51\pi}{2}$

EXAMPLE 3 ■ Coterminal Angles

Find an angle with measure between 0° and 360° that is coterminal with the angle of measure 1290° in standard position.



ARC LENGTH

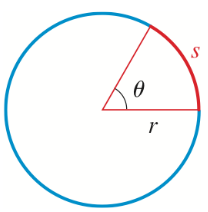


FIGURE 9 $s = \theta r$

$$s = \frac{\theta}{2\pi} \times \text{CIRCUMFERENCE} = \frac{\theta}{2\pi} \cdot 2\pi r = r\theta$$

LENGTH OF A CIRCULAR ARC

In a circle of radius r the length s of an arc that subtends a central angle of θ radians is

$$s = r\theta$$

$$\theta = \frac{s}{r}$$

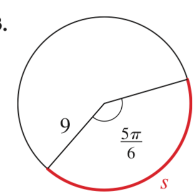
EXAMPLE 4 ■ Arc Length and Angle Measure

(a) Find the length of an arc of a circle with radius 10 m that subtends a central angle of 30°.

(b) A central angle θ in a circle of radius 4 m is subtended by an arc of length 6 m. Find the measure of θ in radians.

53–62 ■ Circular Arcs Find the length s of the circular arc, the radius r of the circle, or the central angle θ, as indicated.

53.



54.