

§4.3 LOGARITHMIC FUNCTIONS

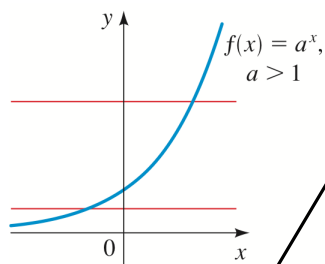


FIGURE 1 $f(x) = a^x$ is one-to-one.

$f(x) = a^x = \text{EXP}_a x = \text{EXPONENTIAL FUNCTION BASE } a$

HAS INVERSE $f^{-1}(x) = \log_a x = \text{LOGARITHMIC FUNCTION BASE } a$

Def: $\log_a x = y \iff a^y = x$

THE POWER WE RAISE a TO TO MAKE x .

ex. USE THE DEFINITION TO EVALUATE

(a) $\log_3 9 \longrightarrow \text{WRITE } \log_3 9 = p \iff 3^p = 9$

(b) $\log_{10} 10,000$

(c) $\log_2 \left(\frac{1}{8} \right)$

(d) $\log_{13} (13^5)$

PROPERTIES OF LOGARITHMS

Property

Reason

1. $\log_a 1 = 0$

We must raise a to the power 0 to get 1.

2. $\log_a a = 1$

We must raise a to the power 1 to get a .

3. $\log_a a^x = x$

We must raise a to the power x to get a^x .

4. $a^{\log_a x} = x$

$\log_a x$ is the power to which a must be raised to get x .

CANCELLATION
PROPERTIES

OF INVERSE
FUNCTIONS

$f(x) = a^x, f^{-1}(x) = \log_a x$

3. $f^{-1}(f(x)) = x$

4. $f(f^{-1}(x)) = x$

KNOW THESE GRAPHS!!

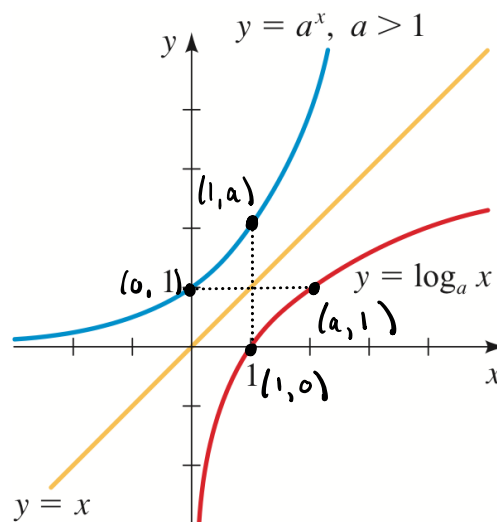
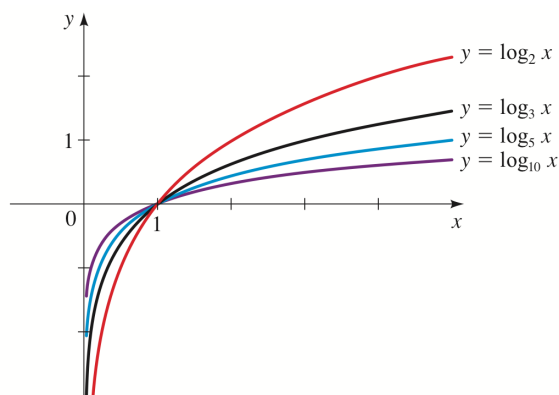


FIGURE 2 Graph of the logarithmic function $f(x) = \log_a x$



EXAMPLE 5 ■ Reflecting Graphs of Logarithmic Functions

Sketch the graph of each function. State the domain, range, and asymptote.

(a) $g(x) = -\log_2 x$ (b) $h(x) = \log_2(-x)$

EXAMPLE 6 ■ Shifting Graphs of Logarithmic Functions

Sketch the graph of each function. State the domain, range, and asymptote.

(a) $g(x) = 2 + \log_5 x$ (b) $h(x) = \log_{10}(x - 3)$

THE NATURAL LOGARITHM

Def: $\ln x = \log_e x$, $e = 2.718281828...$

THE INVERSE OF THE NATURAL EXPONENTIAL e^x

ex.

EVALUATE

(a) $\ln(e^x)$

(d) $e^{\ln x}$

(b) $\ln(e^3)$

(e) $e^{\ln 5}$

(c) $\ln\left(\frac{1}{\sqrt{e}}\right)$

PROPERTIES OF NATURAL LOGARITHMS

Property

Reason

1. $\ln 1 = 0$

We must raise e to the power 0 to get 1.

2. $\ln e = 1$

We must raise e to the power 1 to get e .

3. $\ln e^x = x$

We must raise e to the power x to get e^x .

4. $e^{\ln x} = x$

$\ln x$ is the power to which e must be raised to get x .

EXAMPLE 10 ■ Finding the Domain of a Logarithmic Function

Find the domain of the function $f(x) = \ln(4 - x^2)$.

FIND THE DOMAIN:

77. $h(x) = \ln x + \ln(2 - x)$

78. $h(x) = \sqrt{x - 2} - \log_5(10 - x)$