§4.1 Exponential Functions

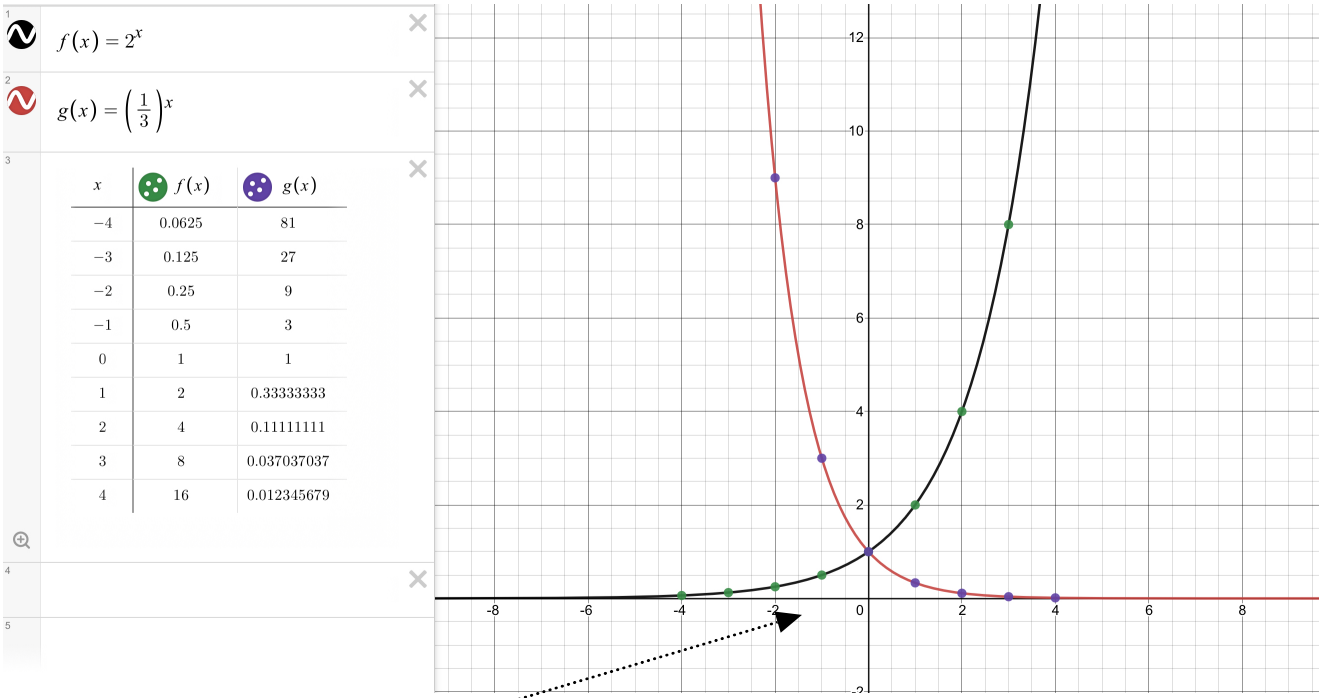
Def: Given $a > 0$ & $a \neq 1$,

the **exponential function with base a** is

$$f(x) = a^x$$

New: variable in exponent position!

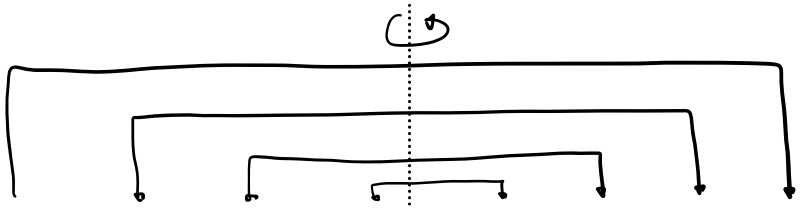
ex. Make a table of values & then sketch
(a) the exponential function with base 2
(b) the exponential function with base $\frac{1}{3}$



x	$f(x)$	$g(x)$
−4	0.0625	81
−3	0.125	27
−2	0.25	9
−1	0.5	3
0	1	1
1	2	0.33333333
2	4	0.11111111
3	8	0.037037037
4	16	0.012345679

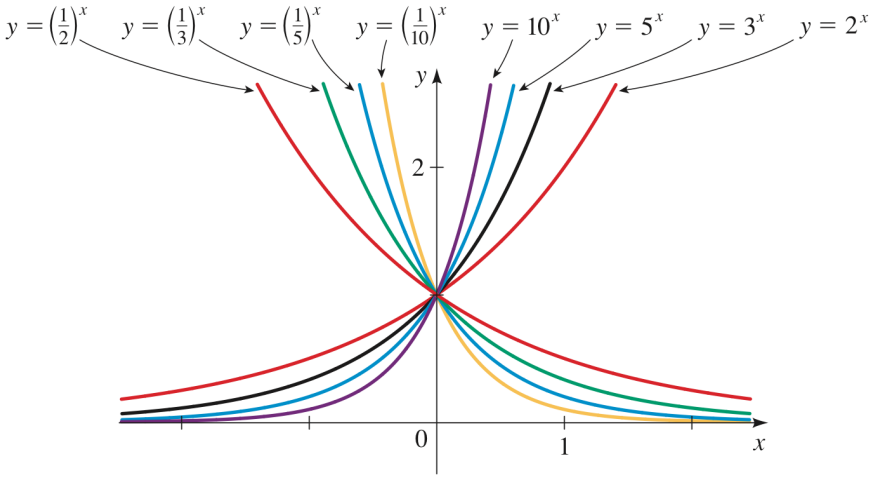
Note: Both graphs get closer & closer to x-axis $(y = 0)$ without ever intersecting it.

We say the x-axis $(y = 0)$ is a **horizontal asymptote** for these graphs.



IN GENERAL:

$\left(\frac{1}{a}\right)^x = a^{-x}$

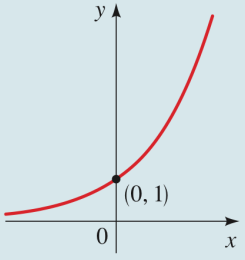


GRAPHS OF EXPONENTIAL FUNCTIONS

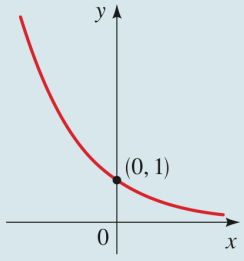
The exponential function

$$f(x) = a^x \qquad a > 0, a \neq 1$$

has domain $\mathbb{R}$ and range $(0, \infty)$. The line $y = 0$ (the x-axis) is a horizontal asymptote of f. The graph of f has one of the following shapes.



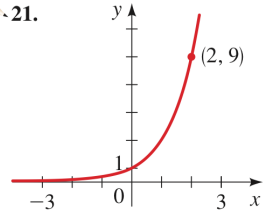
$f(x) = a^x$ for $a > 1$



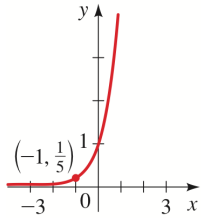
$f(x) = a^x$ for $0 < a < 1$

21–24 ■ Exponential Functions from a Graph Find the exponential function $f(x) = a^x$ whose graph is given.

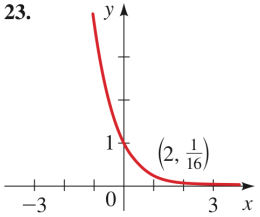
21.



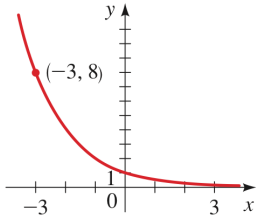
22.



23.



24.



27–40 ■ Graphing Exponential Functions Graph the function, not by plotting points, but by starting from the graphs in Figure 2. State the domain, range, and asymptote.

27. $g(x) = 2^x - 3$

28. $h(x) = 4 + \left(\frac{1}{2}\right)^x$

29. $f(x) = -3^x$

30. $f(x) = 10^{-x}$

31. $f(x) = 10^{x+3}$

32. $g(x) = 2^{x-3}$

33. $y = 5^{-x} + 1$

34. $h(x) = 6 - 3^x$

35. $y = 2 - \left(\frac{1}{3}\right)^x$

36. $y = 5^{-x} - 3$

37. $h(x) = 2^{x-4} + 1$

38. $y = 3 - 10^{x-1}$

39. $g(x) = 1 - 3^{-x}$

40. $y = 3 - \left(\frac{1}{5}\right)^x$

§4.2 THE NATURAL EXPONENTIAL FUNCTION

Def: THE NATURAL EXPONENTIAL FUNCTION IS THE EXPONENTIAL FUNCTION WITH BASE $e \approx 2.71828182\ldots$,

$$f(x) = e^x.$$

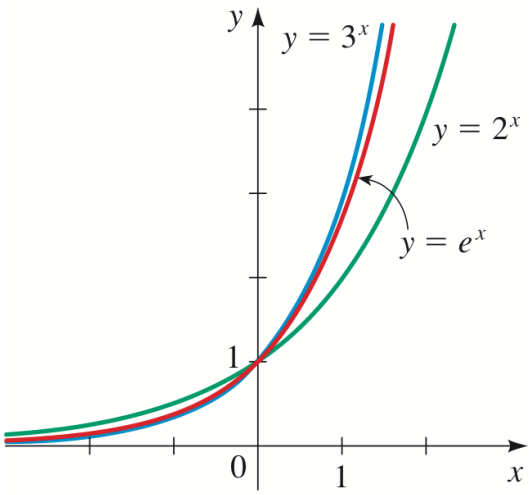


FIGURE 1 Graph of the natural exponential function

ex. SKETCH

(a) $y = e^{-x} + 3$

(b) $y = e^{x+2} - 1$

(c) $y = 5e^{x-1}$