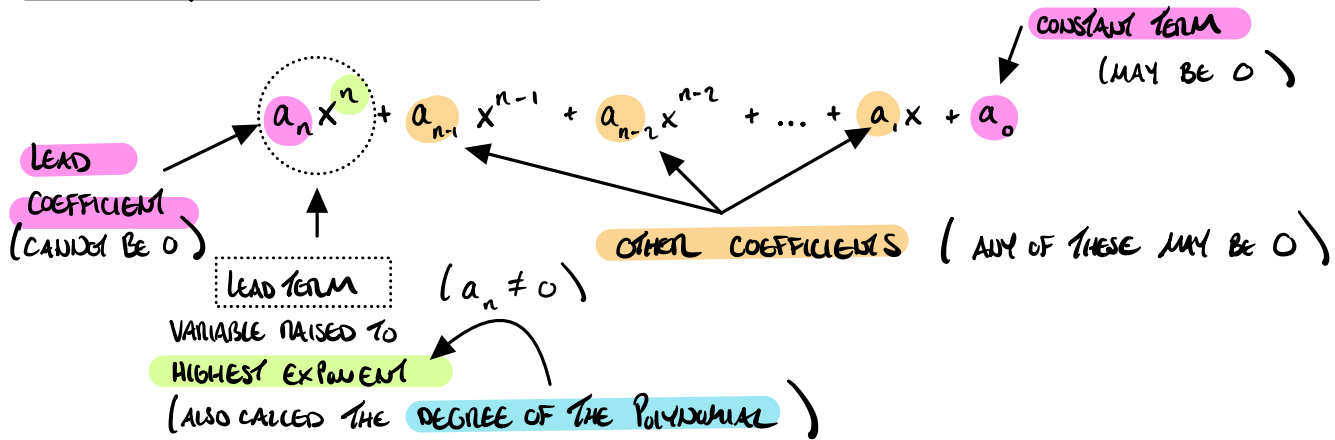


§3.1 QUADRATICS

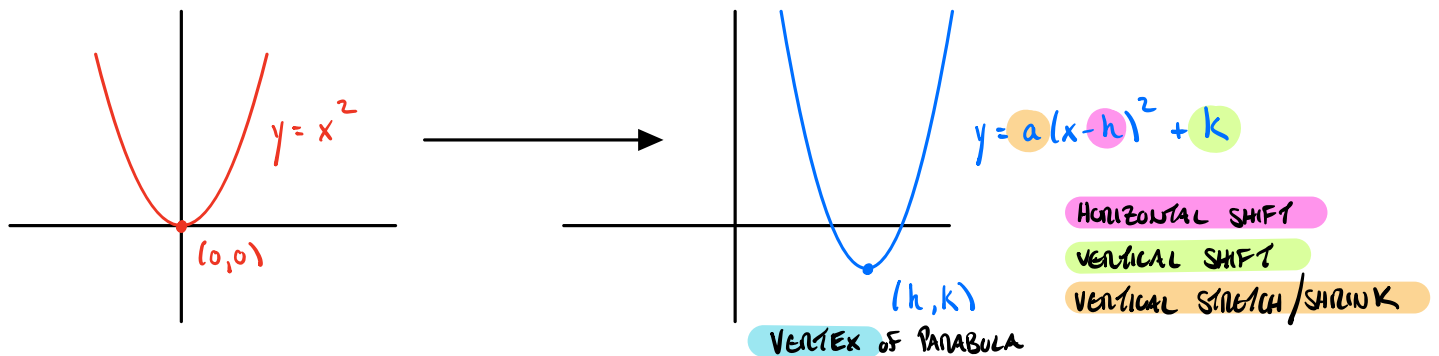
RECALL DEF. OF POLYNOMIAL:



QUADRATIC FUNCTIONS ARE DEGREE 2 POLYNOMIAL FUNCTIONS

$$a_2 x^2 + a_1 x + a_0 \xrightarrow[\text{WE RENAME THE COEFFICIENTS}]{\text{BY CONVENTION}} f(x) = ax^2 + bx + c$$

STANDARD FORM OF QUADRATIC FUNCTIONS: $f(x) = a(x-h)^2 + k$



$$f(x) = ax^2 + bx + c$$

CONVERT BY COMPLETING THE SQUARE

$$f(x) = a(x-h)^2 + k$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - \frac{b^2}{4a}$$

SUBTRACTING SAME THING!

ADDING $a\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a}$

$$= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

$$= a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

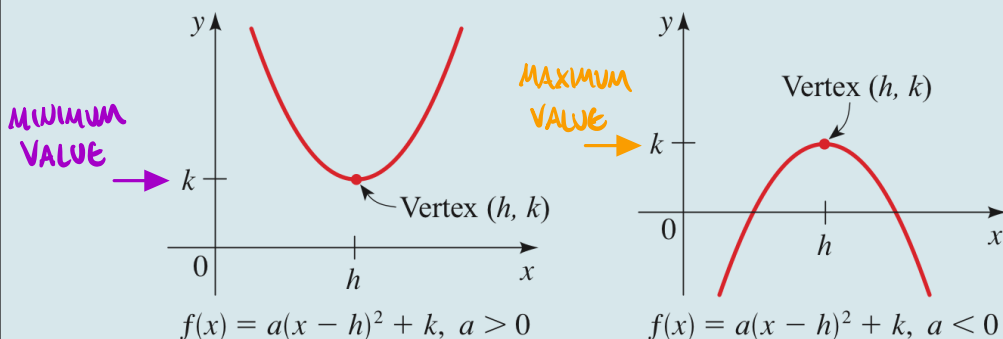
Annotations: $h = -\frac{b}{2a}$ and $k = c - \frac{b^2}{4a}$ are indicated by arrows from the final equation.

STANDARD FORM OF A QUADRATIC FUNCTION

A quadratic function $f(x) = ax^2 + bx + c$ can be expressed in the **standard form**

$$f(x) = a(x - h)^2 + k$$

by completing the square. The graph of f is a parabola with **vertex** (h, k) ; the parabola opens **upward** if $a > 0$ or **downward** if $a < 0$.



ex. Let $f(x) = x^2 + 8x + 13$

($a = 1$: EASIER)

- EXPRESS f IN STANDARD FORM $(x+4)^2 - 3$
- FIND VERTEX OF GRAPH $y = f(x)$
- FIND x - & y -INTERCEPTS OF $y = f(x)$
- SKETCH $y = f(x)$
- MINIMUM / MAXIMUM VALUE?

ex. Let $f(x) = -3x^2 - 30x - 73$

($a \neq 1$: TRICKY)

- EXPRESS f IN STANDARD FORM $-3(x+5)^2 + 2$
- FIND VERTEX OF GRAPH $y = f(x)$
- FIND x - & y -INTERCEPTS OF $y = f(x)$
- SKETCH $y = f(x)$
- MINIMUM / MAXIMUM VALUE?

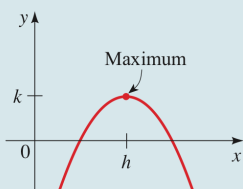
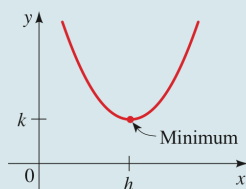
SHORTCUTS TO MAX/MIN VALUES:

MAXIMUM OR MINIMUM VALUE OF A QUADRATIC FUNCTION

Let f be a quadratic function with standard form $f(x) = a(x - h)^2 + k$. The maximum or minimum value of f occurs at $x = h$.

If $a > 0$, then the **minimum** value of f is $f(h) = k$.

If $a < 0$, then the **maximum** value of f is $f(h) = k$.



MAXIMUM OR MINIMUM VALUE OF A QUADRATIC FUNCTION

The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at

$$x = -\frac{b}{2a}$$

← INPUT THAT PRODUCES MAX/MIN OUTPUT

If $a > 0$, then the **minimum** value is $f\left(-\frac{b}{2a}\right)$.

If $a < 0$, then the **maximum** value is $f\left(-\frac{b}{2a}\right)$.

ADDITIONAL PRACTICE:

25–34 ■ Maximum and Minimum Values A quadratic function f is given. (a) Express f in standard form. (b) Sketch a graph of f . (c) Find the maximum or minimum value of f .

25. $f(x) = x^2 + 2x - 1$

26. $f(x) = x^2 - 8x + 8$

 27. $f(x) = 3x^2 - 6x + 1$

28. $f(x) = 5x^2 + 30x + 4$

 29. $f(x) = -x^2 - 3x + 3$

30. $f(x) = 1 - 6x - x^2$

31. $g(x) = 3x^2 - 12x + 13$

32. $g(x) = 2x^2 + 8x + 11$

33. $h(x) = 1 - x - x^2$

34. $h(x) = 3 - 4x - 4x^2$

35–44 ■ Formula for Maximum and Minimum Values

Find the maximum or minimum value of the function.

 35. $f(x) = 2x^2 + 4x - 1$

36. $f(x) = 3 - 4x - x^2$

 37. $f(t) = -3 + 80t - 20t^2$

38. $f(x) = 6x^2 - 24x - 100$

39. $f(s) = s^2 - 1.2s + 16$

40. $g(x) = 100x^2 - 1500x$

41. $h(x) = \frac{1}{2}x^2 + 2x - 6$

42. $f(x) = -\frac{x^2}{3} + 2x + 7$

43. $f(x) = 3 - x - \frac{1}{2}x^2$

44. $g(x) = 2x(x - 4) + 7$