

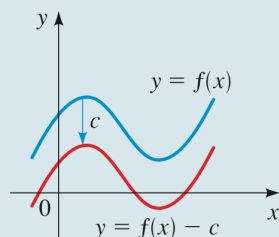
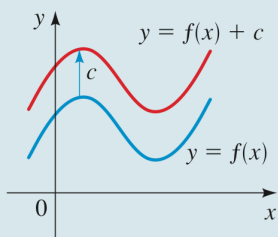
§2.6 TRANSFORMATIONS OF FUNCTIONS

VERTICAL SHIFTS OF GRAPHS

Suppose $c > 0$.

To graph $y = f(x) + c$, shift the graph of $y = f(x)$ upward c units.

To graph $y = f(x) - c$, shift the graph of $y = f(x)$ downward c units.



ex. Sketch $y = x^2$ (WITHOUT PLOTTING - IT DOESN'T HAVE TO BE PERFECT).

THEN SKETCH $y = x^2 + 2$ & $y = x^2 - 4$.

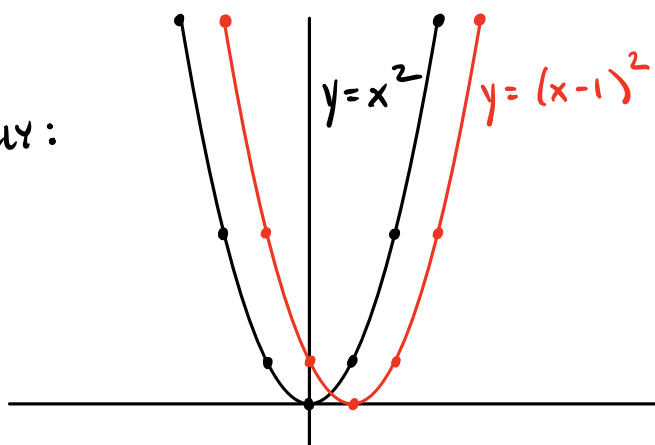
(IF WE LET $f(x) = x^2$ THEN THESE GRAPHS ARE $y = f(x) + 2$ & $y = f(x) - 4$)
VERTICAL SHIFTS!

ex. COMPLETE THE TABLE OF VALUES:

	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$(x-1)^2$	16	9	4	1	0	1	4

HORIZONTAL SHIFT
 1 UNIT TO THE RIGHT!

GRAPHICALLY:



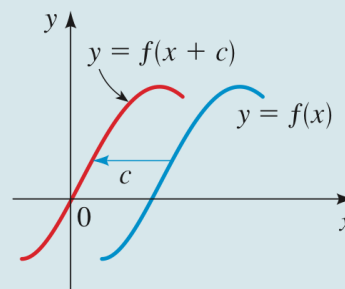
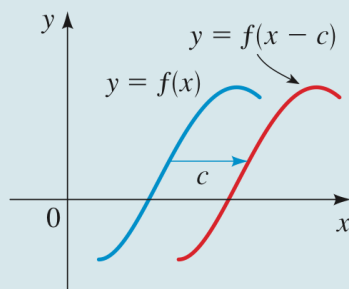
IN GENERAL:

HORIZONTAL SHIFTS OF GRAPHS

Suppose $c > 0$.

To graph $y = f(x - c)$, shift the graph of $y = f(x)$ to the right c units.

To graph $y = f(x + c)$, shift the graph of $y = f(x)$ to the left c units.



ex. Sketch $y = \sqrt{x}$ (WITHOUT PLOTTING - IT DOESN'T HAVE TO BE PERFECT).

THEN SKETCH $y = \sqrt{x - 4}$ & $y = \sqrt{x + 3}$

IF WE LET $f(x) = \sqrt{x}$

THEN THESE GRAPHS ARE $y = f(x - 4)$ $y = f(x + 3)$

HORIZONTAL SHIFTS!

ex. Sketch $y = |x|$ (WITHOUT PLOTTING - IT DOESN'T HAVE TO BE PERFECT).

THEN SKETCH (a) $y = |x - 5| + 2$

(b) $y = |x + 2| - 5$

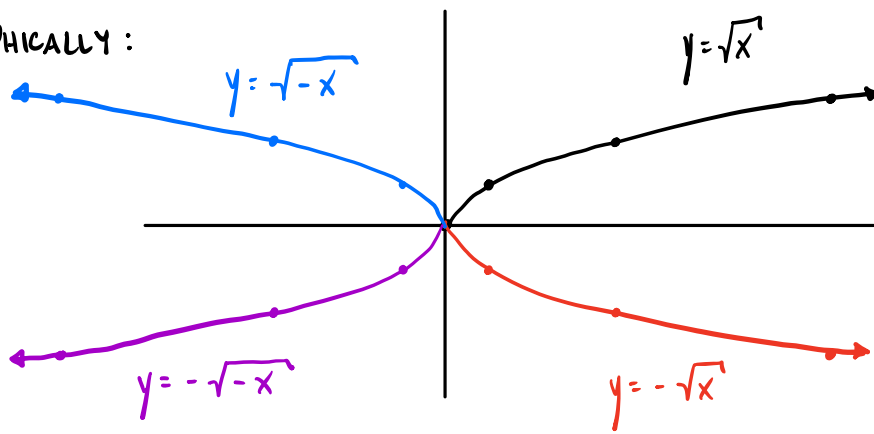
ex. COMPLETE THE TABLE OF VALUES

	-4	-3	-2	-1	0	1	2	3	4
$\sqrt{x}$	UND	UND	UND	UND	0	1	$\sqrt{2}$	$\sqrt{3}$	2
$-\sqrt{x}$	UND	UND	UND	UND	0	-1	$-\sqrt{2}$	$-\sqrt{3}$	-2
$\sqrt{-x}$	2	$\sqrt{3}$	$\sqrt{2}$	1	0	UND	UND	UND	UND
$-\sqrt{-x}$	-2	$-\sqrt{3}$	$-\sqrt{2}$	-1	0	UND	UND	UND	UND

"SYMMETRY"

"REFLECTIONS"

GRAPHICALLY :



" SYMMETRY "

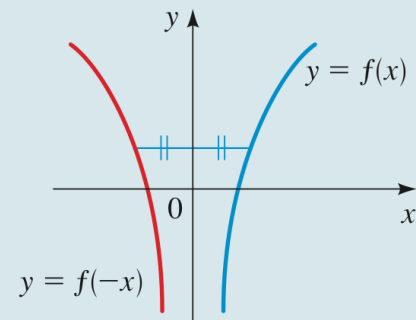
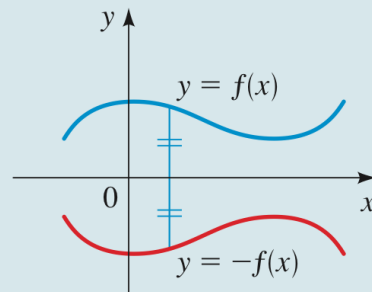
" REFLECTIONS "

IN GENERAL :

REFLECTING GRAPHS

To graph $y = -f(x)$, reflect the graph of $y = f(x)$ in the x -axis.

To graph $y = f(-x)$, reflect the graph of $y = f(x)$ in the y -axis.

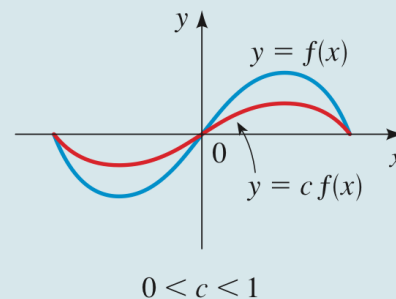
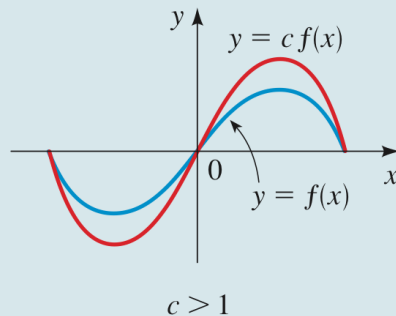


VERTICAL STRETCHING AND SHRINKING OF GRAPHS

To graph $y = cf(x)$:

If $c > 1$, stretch the graph of $y = f(x)$ vertically by a factor of c .

If $0 < c < 1$, shrink the graph of $y = f(x)$ vertically by a factor of c .



ex. use the graph of $y = x^2$ to sketch

(a) $y = \frac{1}{2} x^2$

(b) $y = 2(x-3)^2 + 1$

EX. USE THE GIVEN VALUES OF $f(x)$ TO COMPLETE THE TABLE OF VALUES
AS MUCH AS POSSIBLE

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$f(x)$	1	4	1	5	9	2	6	5	3	5	8	9	7
$f(2x)$				1	1	9	0	3	8	7			
$f(\frac{1}{2}x)$	5		9		2		6		5		3		5

"SHRINK"

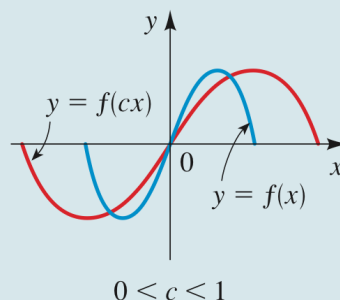
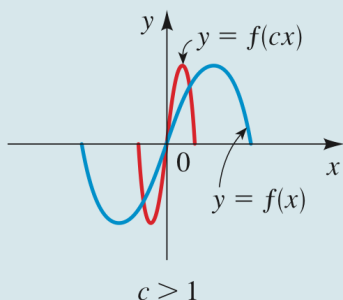
"STRETCH"

HORIZONTAL SHRINKING AND STRETCHING OF GRAPHS

To graph $y = f(cx)$:

If $c > 1$, shrink the graph of $y = f(x)$ horizontally by a factor of $1/c$.

If $0 < c < 1$, stretch the graph of $y = f(x)$ horizontally by a factor of $1/c$.



EX. USE THE GIVEN GRAPH $y = f(x)$ TO SKETCH THE INDICATED GRAPH (TRANSFORMATION).

73. (a) $y = f(2x)$

(b) $y = f(\frac{1}{2}x)$

71. (a) $y = f(x - 2)$

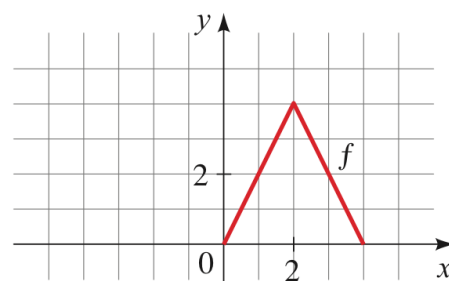
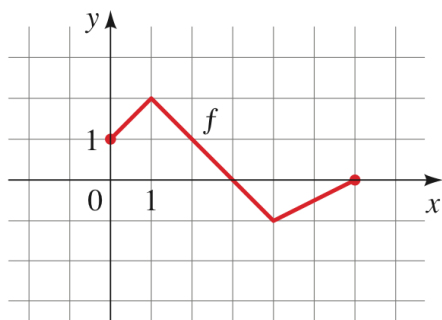
(b) $y = f(x) - 2$

(c) $y = 2f(x)$

(d) $y = -f(x) + 3$

(e) $y = f(-x)$

(f) $y = \frac{1}{2}f(x - 1)$



ADDITIONAL EXERCISES :



45. $y = (x - 3)^2 + 5$



47. $y = 3 - \frac{1}{2}(x - 1)^2$

49. $y = |x + 2| + 2$

51. $y = \frac{1}{2}\sqrt{x + 4} - 3$

46. $y = \sqrt{x + 4} - 3$

48. $y = 2 - \sqrt{x + 1}$

50. $y = 2 - |x|$

52. $y = 3 - 2(x - 1)^2$