

§ 2.2 GRAPHS OF FUNCTIONS

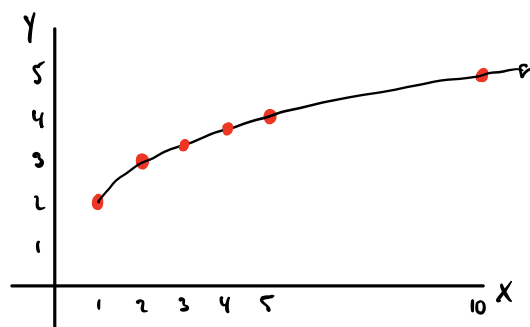
Def: THE GRAPH OF A FUNCTION f IS THE SET OF ALL POINTS (x, y) THAT SATISFY THE EQUATION $y = f(x)$.

$\Rightarrow$ POINTS $(x, f(x))$

ex. Let $f(x) = 2 + \sqrt{x-1}$.

SKETCH THE GRAPH BY PLOTTING

x	$y = 2 + \sqrt{x-1}$	(x, y)
1		$(1, 2)$
2		$(2, 3)$
3		$(3, 2 + \sqrt{2})$
4		$(4, 2 + \sqrt{3})$
5		$(5, 4)$
$\vdots$		
10		$(10, 5)$



THIS PROCESS OFTEN LEADS TO INSIGHTS INTO THE FUNCTION f .

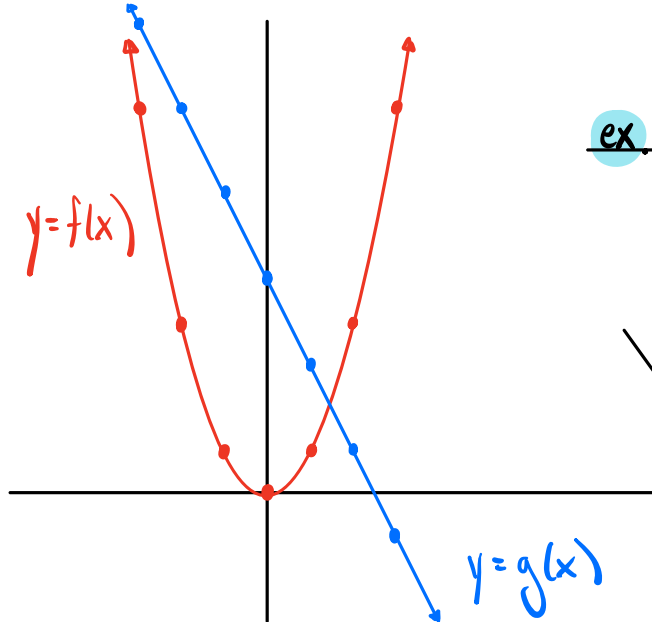
e.g. WHEN x INCREASES, y ALSO INCREASES.
(BY LESS & LESS)

ex. Let $f(x) = x^2$ & $g(x) = 5 - 2x$

SKETCH THE GRAPHS OF f & g BY PLOTTING.

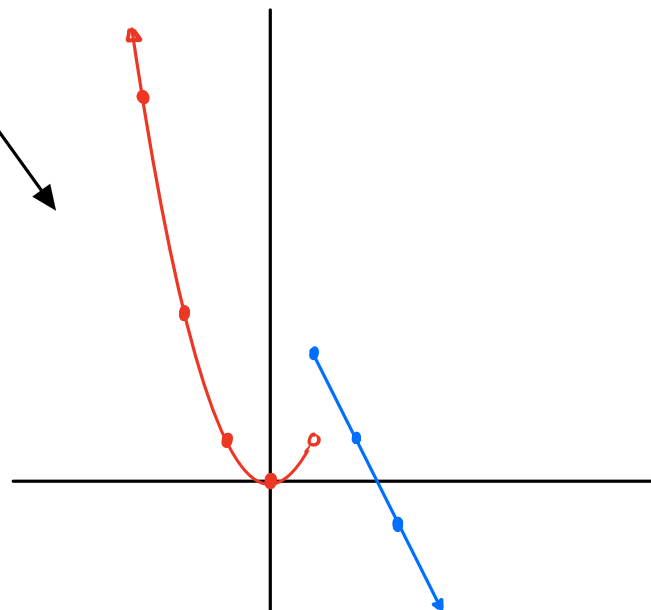
x	$f(x) = x^2$	$(x, f(x))$	$g(x) = 5 - 2x$	$(x, g(x))$
-3	9	$(-3, 9)$	11	$(-3, 11)$
-2	4	$(-2, 4)$	9	$(-2, 9)$
-1	1	$(-1, 1)$	7	$(-1, 7)$
0	0	$(0, 0)$	5	$(0, 5)$
1	1	$(1, 1)$	3	$(1, 3)$
2	4	$(2, 4)$	1	$(2, 1)$
3	9	$(3, 9)$	-1	$(3, -1)$

$\uparrow$ PLOT GRAPH OF f
 $\uparrow$ PLOT THE GRAPH OF



ex. SKETCH THE GRAPH OF **PIECEWISE DEFINED FUNC.**

$$p(x) = \begin{cases} x^2 & \text{IF } x < 1 \\ 5-2x & \text{IF } x \geq 1 \end{cases}$$



ex. SKETCH BY PLOTTING THE GRAPH OF

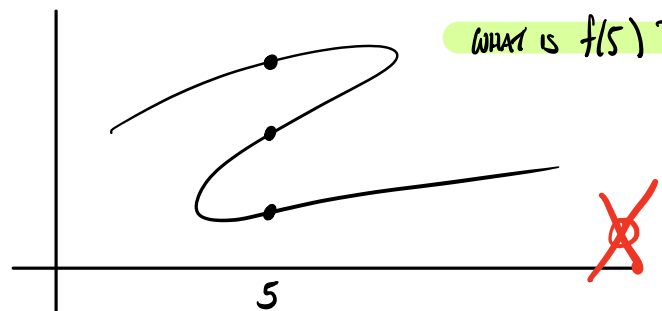
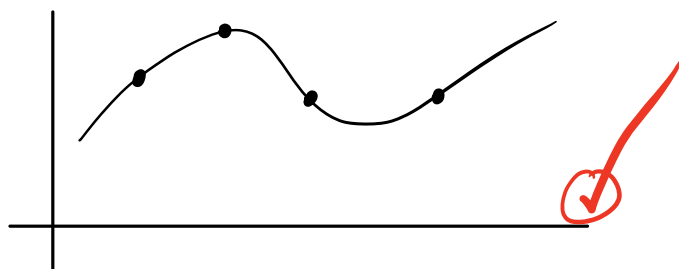
$$f(x) = |x|$$

$$\left(\text{Def: } |x| = \begin{cases} x & \text{IF } x \geq 0 \\ -x & \text{IF } x < 0 \end{cases} \right)$$

Vertical Line Test

THE GRAPH OF A FUNCTION IS A CURVE IN xy -PLANE.

BUT NOT EVERY CURVE IN xy -PLANE IS THE GRAPH OF A FUNCTION.



THE VERTICAL LINE TEST

A curve in the coordinate plane is the graph of a function if and only if no vertical line intersects the curve more than once.

NOTE: EVERY EQUATION IN x, y PRODUCES A GRAPH.

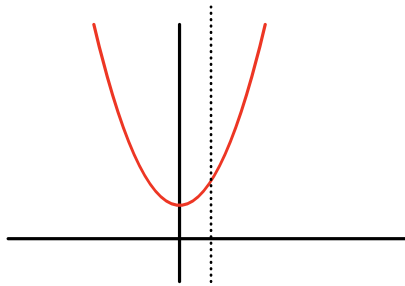


QUESTION:

WHEN IS THIS GRAPH OF AN EQUATION
THE GRAPH OF A FUNCTION?

WHEN DOES AN EQUATION DEFINE A FUNCTION

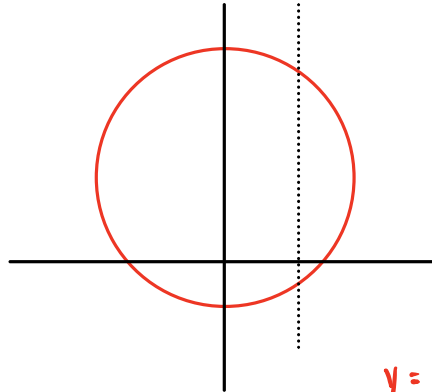
e.g. $y - x^2 = 3$



$$y = x^2 + 3$$

$$f(x) = x^2 + 3 \quad \checkmark$$

$$x^2 + (y - 2)^2 = 9$$



(APPLY VERTICAL
LINE TEST)

$$y = 2 \pm \sqrt{9 - x^2}$$

2 outputs?!
NOT A FUNCTION

ANSWER:

WHEN THE EQUATION CAN BE SOLVED FOR y

SUCH THAT FOR ANY $\# x$, THERE IS

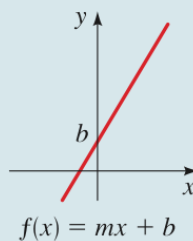
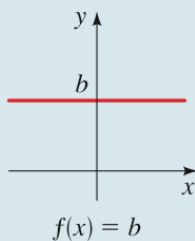
AT MOST 1 VALUE OF y THAT SATISFIES THE EQUATION

YOU SHOULD MEMORIZE THE SHAPES OF THE GRAPHS OF THE FOLLOWING FUNCTIONS:

SOME FUNCTIONS AND THEIR GRAPHS

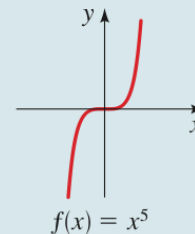
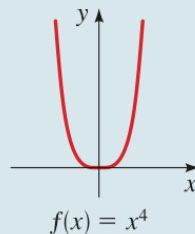
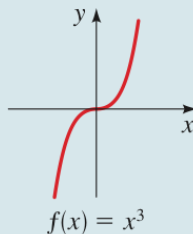
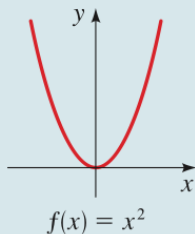
Linear functions

$$f(x) = mx + b$$



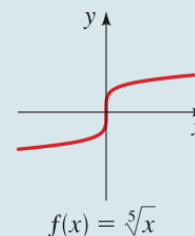
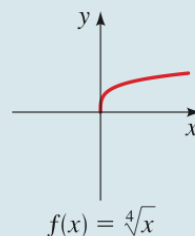
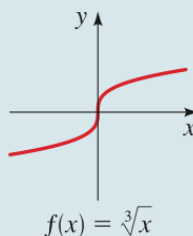
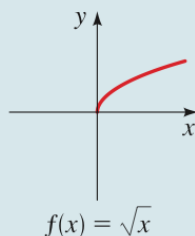
Power functions

$$f(x) = x^n$$



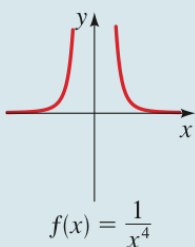
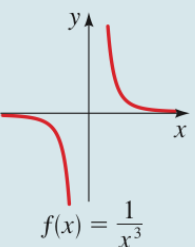
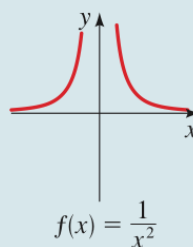
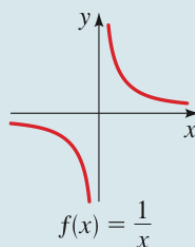
Root functions

$$f(x) = \sqrt[n]{x}$$



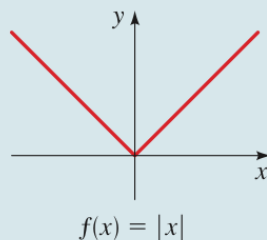
Reciprocal functions

$$f(x) = \frac{1}{x^n}$$



Absolute value function

$$f(x) = |x|$$



Greatest integer function

$$f(x) = \llbracket x \rrbracket$$

