

§1.7 INEQUALITIES

LINEAR (variable raised only to power 1, never in denominator)

e.g. Solve: $4x + 6 \leq 3x + 9$

* IS 0 A SOLUTION? 1?
10?

$$\begin{array}{ccc} & \swarrow & \searrow \\ x \leq 3 & \text{or} & -3 \leq -x \\ & \swarrow & \searrow \\ & \text{SAME } \checkmark & 3 \geq x \end{array}$$

SAME RULES AS EQUATIONS BUT
WITH 2 TWISTS!

* IF $A < B$ & C IS NEGATIVE
THEN $CA > CB$

* IF $0 < A < B$ (BOTH POSITIVE)
THEN $\frac{1}{A} > \frac{1}{B}$

* ... AND ONE NEW RULE

IF $A < B$
AND $C < D$

THEN $A + C < B + D$

ex. $\frac{2}{3} - \frac{1}{2}x \geq \frac{1}{6} + x$

ex. Simultaneous Ineq:

$$-3 \leq 3x + 7 \leq \frac{1}{2}$$

ex. $-\frac{1}{2} \leq \frac{4-3x}{5} \leq \frac{1}{4}$

Now Linear

POSITIVE OR NEGATIVE?
(COMPARE TO 0)

$$\frac{(0)(13)(-7)}{(-11)(102)(-29)}$$



OBSERVE:

IF THE # OF NEGATIVE FACTORS IN A PRODUCT/QUOTIENT IS

1) EVEN THEN THE PRODUCT/QUOTIENT IS POSITIVE

2) ODD THEN THE PRODUCT/QUOTIENT IS NEGATIVE

ex. $x^2 + 40 < 13x$

ex. $x^2 \geq 9$

ex. $5x^2 + 3x \geq 3x^2 + 2$

ex. $16x \leq x^3$

ex. $x^3(x+3)^2(x+1) > 0$

ex. $x^2(x^2-1) \geq 0$

ex. $\frac{4-x}{x+4} < 0$

ex. $\frac{x}{2} \geq \frac{5}{x+1} + 4$

ex. $-2 < \frac{x+1}{x-3}$

ex. $\frac{x+2}{x+3} < \frac{x-1}{x-2}$

WHY CAN'T WE CROSS-MULTIPLY?

How WOULD WE SOLVE THESE IF THEY WERE EQUATIONS?

WHY CAN'T WE JUST GET RID OF DENOMINATORS BY MULTIPLYING EVERYTHING BY LCD?



GUIDELINES FOR SOLVING NONLINEAR INEQUALITIES

- 1. Move All Terms to One Side.** If necessary, rewrite the inequality so that all nonzero terms appear on one side of the inequality sign. If the nonzero side of the inequality involves quotients, bring them to a common denominator.
- 2. Factor.** Factor the nonzero side of the inequality.
- 3. Find the Intervals.** Determine the values for which each factor is zero. These numbers will divide the real line into intervals. List the intervals that are determined by these numbers.
- 4. Make a Table or Diagram.** Use **test values** to make a table or diagram of the signs of each factor on each interval. In the last row of the table determine the sign of the product (or quotient) of these factors.
- 5. Solve.** Use the sign table to find the intervals on which the inequality is satisfied. Check whether the **endpoints** of these intervals satisfy the inequality. (This may happen if the inequality involves $\leq$ or $\geq$.)