

§1.4 Solving Quadratic EQ's

$$ax^2 + bx + c = 0$$

Factoring

Zero-Product Property: If $AB = 0$

Then $A = 0$ or $B = 0$ (or both)

ex. solve $x^2 + 7x = x + 16$

$$(x+6)(x-2) = 0$$

ex. solve $3x^2 + 8 = 11x + 2$

$$(3x-2)(x-3) = 0$$

ex. solve $2x(x-5) = 7(x-5)$

$$(x-5)(2x-7) = 0$$

(1) Set Equal To 0

(2) Factor

(3) Set Factors Equal To 0
& Solve

Completing The Square

Suppose $x^2 + bx = c$

$$x^2 + 2\left(\frac{b}{2}\right)x = c$$

$$x^2 + 2\left(\frac{b}{2}\right)x + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$

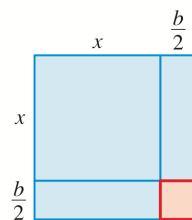
ADD $\left(\frac{b}{2}\right)^2$ TO BOTH SIDES OF EQ
TO "COMPLETE THE SQUARE"

Completing the Square

The area of the blue region is

$$x^2 + 2\left(\frac{b}{2}\right)x = x^2 + bx$$

Add a small square of area $(b/2)^2$ to
"complete" the square.



ex. $x^2 + 12x = 3$ (CANNOT FACTOR!)

$$(x+6)^2 = 3 + 36 \Rightarrow x = -6 \pm \sqrt{39}$$

ex. $x^2 - 5x - 1 = 0$ (CANNOT FACTOR!)

$$\left(x - \frac{5}{2}\right)^2 = 1 + \frac{25}{4} \Rightarrow x = \frac{5 \pm \sqrt{29}}{2}$$

THE QUADRATIC FORMULA

$$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\Rightarrow \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\Rightarrow x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

BACKUP
PLAN

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex. $10x^2 + 13x - 2 = 0$

$b^2 - 4ac$ IS CALLED THE DISCRIMINANT

•) IF $b^2 - 4ac > 0$ THEN 2 SOLUTIONS

•) IF $b^2 - 4ac = 0$ THEN 1 SOLUTION

•) IF $b^2 - 4ac < 0$ THEN NO SOLUTIONS

ex. SOLVE $x^2 + x + 1 = 0$ (NEG. DISCR. $\Rightarrow$ NO SOLUTIONS)

§ 1.6 SOLVING OTHER TYPES OF EQUATIONS

1.) SET EQUAL TO 0

2.) FACTOR

3.) SET FACTORS EQUAL TO 0
& SOLVE

ex. $6x^4 = 18x^2$

ex. $x^3 + 25 = x^2 + 25x$

4 TERMS $\Rightarrow$ TRY FACTOR BY GROUPING

2 GROUPS OF 2 TERMS $\rightarrow$
FACTOR OUT GCF FROM EACH GROUP
TRY TO LEAVE BEHIND COMMON FACTORS!

EXAMPLE 3 ■ An Equation Involving Fractional Expressions

Solve the equation $\frac{3}{x} - \frac{2}{x-3} = \frac{-12}{x^2-9}$.

MULTIPLY EACH TERM BY LCD
TO CANCEL ALL DENOMINATORS

EXAMPLE 4 ■ An Equation Involving a Radical

Solve the equation $2x = 1 - \sqrt{2-x}$.

ISOLATE RADICAL EXPRESSION,
SQUARE BOTH SIDES.

SOLUTIONS TO EQUATIONS
WITH DENOMINATORS OR
RADICALS NEED TO BE
CHECKED!