STANDARD DEVIATION: MEASURE THE VARIATION

$x_1, x_2, x_3, \dots, x_n$ (SAMPLE) [POPULATION]

1. SUB. MEAN

SAMPLE MEAN $\bar{x}$ | POP MEAN μ

$x_1 - \bar{x}, \quad x_2 - \bar{x}, \quad x_3 - \bar{x}, \quad \dots, \quad x_n - \bar{x}$

$\mu \qquad \mu \qquad \mu \qquad \mu$

2. SQUARE

$(x_1 - \bar{x})^2, \quad (x_2 - \bar{x})^2, \quad (x_3 - \bar{x})^2, \quad \dots, \quad (x_n - \bar{x})^2$

3. ADD 'EM UP, DIVIDE BY $n-1$ (DIV. BY n)

$\longrightarrow$ VARIANCE $$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{n}$$

SAMPLE STANDARD DEVIATION: $s = \sqrt{s^2}$

POPULATION STANDARD DEVIATION: $\sigma = \sqrt{\sigma^2}$

1st MEASUREMENT $x_1 = 0$

2nd MEASUREMENT $x_2 = 5$

$\vdots$ $x_3 = 1$

$x_4 = 1$

5TH MEASUREMENT $x_5 = 3$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{0+5+1+1+3}{5} = 2$$

$$= \sum_{i=1 \text{ (FIRST)}}^{5 \text{ (LAST)}} x_i$$

$$\sum_{i=1}^{5} x_i = x_1 + x_2 + x_3 + x_4 + x_5$$

$$\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5$$

$$\sum_{i=1}^{5} 2^i = 2 + 4 + 8 + 16 + 32$$

§4.2 EVENTS & THE SAMPLE SPACE

EXPERIMENT - PROCESS BY WHICH AN OBSERVATION IS MADE.

SIMPLE EVENT - OUTCOME OBSERVED ON SINGLE REP. OF EXP.

ALL SIMPLE EVENTS TOGETHER SHOULD FORM ALL POSSIBLE OUTCOMES.

EVENT - BUILT FROM SIMPLE EVENTS :

A COLLECTION OF SIMPLE EVENTS.

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ex. EXPERIMENT: DRAW A CARD FROM A STANDARD DECK. OF 52

SIMPLE EVENTS: 52 SIMPLE EVENTS

A♠, 2♠, ETC..

EVENTS: SEVEN = { 7♠, 7♣, 7♡, 7♢ } ← 4

NOT SEVEN = { 48 SIMPLE EVENTS }

Def: TWO EVENTS ARE MUTUALLY EXCLUSIVE IF THEY CANNOT BOTH HAPPEN AT ONCE – IF ONE OCCURS, THEN THE OTHER DOES NOT.

ex. EVENTS: DRAW 8, DRAW JACK $\Big\}$ MUTUALLY EXCLUSIVE.

Note: SIMPLE EVENTS ARE ALL MUTUALLY EXCLUSIVE.

LISTS BETWEEN CURLY BRACKETS { , , ... }

ORDER DOESN'T MATTER

Def: THE SET OF ALL SIMPLE EVENTS IS CALLED THE SAMPLE SPACE.

ex. EXPERIMENT: FLIP A COIN & ROLL A DIE.

DESCRIBE THE SAMPLE SPACE, S.

$$S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$$

12 SIMPLE EVENTS THAT MAKE UP THE SAMPLE SPACE.

DESCRIBE THE EVENT: ROLL AT LEAST A 5.

$$E = \{H5, H6, T5, T6\}$$

THE SAMPLE SPACE CONTAINS 12 SIMPLE EVENTS.

§4.3 CALCULATING PROBABILITIES USING SIMPLE EVENTS

Def: THE PROBABILITY OF AN EVENT A, P(A), IS A MEASURE OF OUR BELIEF THAT A WILL OCCUR.

0 (0 ≤ ALL PROBABILITIES ≤ 1) 1

GUARANTEED TO NOT OCCUR. GUARANTEED TO OCCUR

IF AN EXPERIMENT IS PERFORMED n TIMES, & WE COUNT THE FREQUENCY WITH WHICH A OCCURS, THEN

$$\text{RELATIVE FREQUENCY OF EVENT A} = \frac{\text{FREQUENCY OF A}}{n}$$

$$\text{THEN } P(A) = \lim_{n \to \infty} \frac{\text{FREQUENCY OF A}}{n}.$$

AS WE PERFORM THE EXPERIMENT OVER AND OVER MORE & MORE TIMES, THE RELATIVE FREQUENCY APPROACHES A NUMBER, P(A).

e.g. ROLL A DIE n TIMES & COUNT THE NUMBER OF 6'S.

n	# 6's	RELATIVE FREQUENCY
300	49	.1633
600	101	.1683...
1200	202	.1683...
30 000	4998	.1683

.1683 - PROBABILITY OF ROLLING A 6.

$P(6) = \frac{1}{6} =$

n	# 6's	RELATIVE FREQ.
1	0	0
10	3	.3
100	20	.2
1000	183	.183
10000	1683	.1683

PROBABILITIES FOR SIMPLE EVENTS

1. EACH PROBABILITY MUST BE BETWEEN 0 & 1.
2. SUM OF PROBABILITIES FOR SIMPLE EVENTS MUST EQUAL 1.

e.g. SUPPOSE AN EXPERIMENT RESULTS IN 1 OF 5 SIMPLE EVENTS E_1, E_2, E_3, E_4, E_5 WITH PROBABILITIES

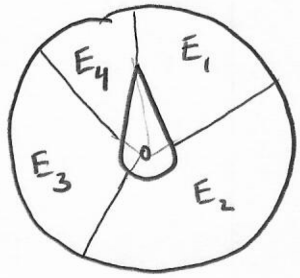
$$\boxed{P(E_1) = .25 \quad , \quad P(E_2) = .1 \quad , \quad P(E_3) = .2 \\ P(E_4) = .35 \quad , \quad P(E_5) = .1}$$

EVENT $A = \{E_1, E_2\}$

$$P(A) = P(E_1) + P(E_2)$$
$$= .25 + .1 = .35$$

Def: GIVEN AN EVENT A, THE PROBABILITY OF A, $P(A)$ = SUM OF PROBABILITIES OF SIMPLE EVENTS CONTAINED IN A.

ex. SUPPOSE SAMPLE SPACE $S = \{E_1, E_2, E_3, E_4\}$



$P(E_1) = .2 \quad P(E_2) = .4$

$P(E_3) = .3 \quad P(E_4) = .1$

$A = \{E_1, E_3\}$, $B = \{E_2, E_3\}$

$C = \{E_2, E_3, E_4\}$

$P(A) = P(E_1) + P(E_3) = .2 + .3 = .5$

$P(B) = P(E_2) + P(E_3) = .4 + .3 = .7$

$P(C) = P(E_2) + P(E_3) + P(E_4) = .4 + .3 + .1 = .8$

NOTE: A SPECIAL CASE OF EXPERIMENTS IS THAT IN WHICH ALL SIMPLE EVENTS ARE EQUALLY LIKELY.

ex. JAR CONTAINS 3 BLUE & 4 WHITE MARBLES.

EXPERIMENT: SELECT 1 MARBLE. (AT RANDOM)

DEFINE SAMPLE SPACE S.

$S = \{ \text{BLUE}, \text{WHITE} \}$ ← ONE WAY TO LIST SAMPLE SPACE.

SAMPLE SPACE $S = \{ \underbrace{B_1, B_2, B_3, W_1, W_2, W_3, W_4}_{} \}$

7 SIMPLE EVENTS, ALL EQUALLY LIKELY.

EACH SIMPLE EVENT HAS SAME PROBABILITY,

& THEY ADD UP TO 1 : $\Rightarrow$ ALL HAVE PROB. $\frac{1}{7}$

EVENT $BLUE = \{ B_1, B_2, B_3 \} \rightarrow P(BLUE) = \frac{3}{7}$

$WHITE = \{ W_1, W_2, W_3, W_4 \} \rightarrow P(WHITE) = \frac{4}{7}$

ex. EXPERIMENT : FLIPPING A COIN 4 TIMES.

FIND PROBABILITY OF OBSERVING 3 HEADS.

SAMPLE SPACE : $S = \{ \underbrace{0H, 1H, 2H, 3H, 4H}_{} \}$

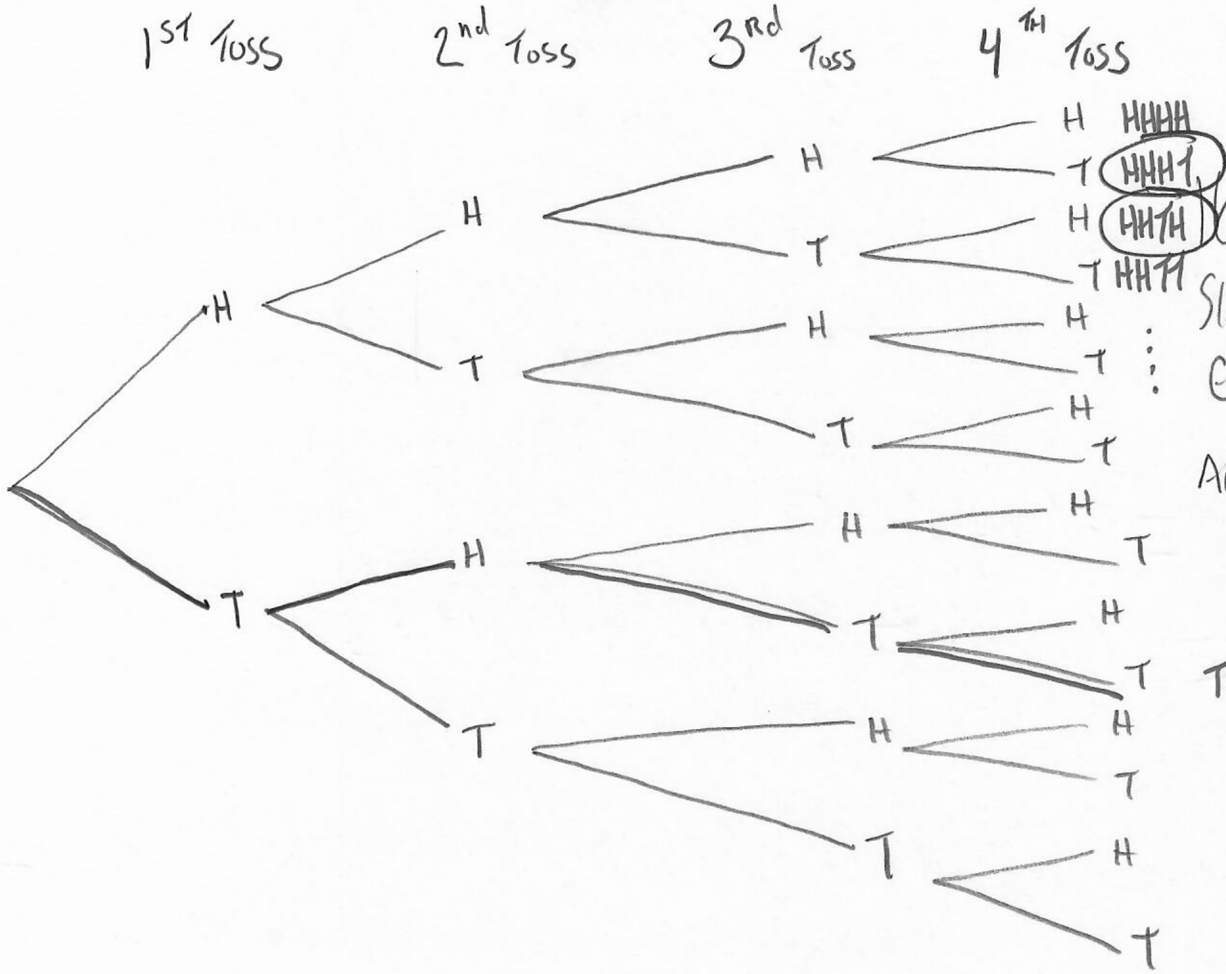
NOT ALL EQUALLY LIKELY.

UNKNOWN PROBABILITIES.

LET'S BREAK IT DOWN FURTHER!

$S = \{ TTTT, TTTH, \ldots \}$

TREE DIAGRAM:



SIMPLE EVENTS:

ALL EQUALLY LIKELY.

THTT

EACH PATH FROM LEFT TO RIGHT THROUGH THE TREE CORRESPONDS TO ONE SIMPLE EVENT (POSSIBLE OUTCOME OF THE EXPERIMENT).

DEFINE A = EVENT OF OBSERVING EXACTLY 3 HEADS.

$$A = \{HHHT, HHTH, HTHH, THHH\}$$

$$P(A) = P(HHHT) + P(HHTH) + P(HTHH) + P(THHH)$$

$$= \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \boxed{.25}$$

16 SIMPLE EVENTS, ALL EQUALLY LIKELY.

SUM OF PROB OF SIMPLE EVENTS MUST BE 1.

IF ALL SIMPLE EVENTS ARE EQUALLY LIKELY,

AND THERE ARE n SIMPLE EVENTS,

THEN THE PROBABILITY OF EACH SIMPLE EVENT

IS $\frac{1}{n}$.

ex. EXPERIMENT: SELECT ONE STUDENTS FROM A SHOOL AT RANDOM.

SCHOOL CONTAINS: 150 FRESHMEN, 175 JUNIORS, 220 SOPHOMORES, 155 SENIORS

SIMPLE EVENTS: $150 + 220 + 175 + 155 = 700$

EVENT A: FRESHMAN

$$P(A) = \left(\frac{1}{700}\right)150 = \frac{150}{700} = .2143$$

B: JUNIOR OR SENIOR

$$P(B) = \left(\frac{1}{700}\right)(175 + 155) = \frac{330}{700}$$

HOW TO CALCULATE THE PROBABILITY OF AN EVENT:

1. LIST ALL SIMPLE EVENTS IN SAMPLE SPACE
   (MAKE THEM ALL EQUALLY LIKELY IF POSSIBLE)

2. ASSIGN APPROPRIATE PROB. TO EACH SIMPLE EVENT.

3. DETERMINE WHICH SIMPLE EVENTS RESULT IN
   THE EVENT OF INTEREST

4. SUM UP THE PROBABILITIES OF THESE SIMPLE EVENTS.

ex. $\{E_1, \ldots, E_{10}\} = S$

$P(E_1) = 3P(E_2) = .45 \quad ; \quad P(E_3) = P(E_4) = \ldots = P(E_{10})$

$P(E_1) = .45$

$$\frac{3P(E_2)}{3} = \frac{.45}{3} \Rightarrow P(E_2) = .15$$

$$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_{10}) = 1$$

$$.45 + .15 + P(E_3) + \ldots + P(E_{10}) = 1$$

$$P(E_3) + \ldots + P(E_{10}) = .4$$

Let $P(E_3) = P(E_4) = \dots = P(E_{10}) = x$

$8x = .4 \Rightarrow x = .05$

$P(E_1) = .45$, $P(E_2) = .15$

$P(E_i) = .05$ For $i = 3, 4, \dots, 10$