Name: ANSWER KEY (72 POINTS TOTAL) 4/10/2019

Math 17300-CD Introduction to Probability and Statistics **Quiz 3**

1. Most African elephants grow tusks as adults. However, each elephant has a 20% chance of having a genetic trait that causes them to never grow tusks – they are tuskless. Suppose a random sample of 15 adult African elephants is collected.

   (a) (8 points) What is the probability that exactly 4 of the elephants are tuskless?

BINOMIAL : $n = 15$, $p = .2$, $x = \#$ TUSKLESS

$$P(x=4) = \underset{\uparrow\ 1365}{C^{15}_{4}} (.2)^4 (.8)^{11} = \boxed{.1876}$$

$$\left(P(x=k) = C^n_k\, p^k\, q^{n-k} \right)$$

   (b) (8 points) What is the probability that more than 1 of the elephants are tuskless?

$$P(x>1) = 1 - P(x \le 1)$$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - [C^{15}_0 (.2)^0 (.8)^{15} + C^{15}_1 (.2)^1 (.8)^{14}]$$

$$= 1 - [(.8)^{15} + 15(.2)(.8)^{14}] = 1 - [.0352 + .1319]$$

$$= \boxed{.8329}$$

2. Seeds are often treated with a fungicide for protection in poor-draining, wet envronments. In a small-scale experiment to determine what dilution of the fungicide to apply, 6 treated seeds and 8 untreated seeds were planted in clay soil and the number of plants emerging from the treated and untreated seeds were recorded. Suppose the dilution made no difference and a total of five plants emerged. Let x represent the number of plants that emerged from treated seeds.

(a) (8 points) Find $P(x = 4)$.

6 TREATED , 8 UNTREATED $\longrightarrow$ 14 TOTAL

$\downarrow$ $\downarrow$

CHOOSE 4 CHOOSE 1 $\longrightarrow$ CHOOSE 5 TOTAL

$$P(x=4) = \frac{C^6_4 \, C^8_1}{C^{14}_5} = \frac{(15)(8)}{2002}$$

$$= \boxed{.0599}$$

(b) (8 points) Find $P(2 \leq x < 4)$.

$$P(2 \leq 2 < 4) = P(x=2) + P(x=3)$$

$$= \frac{C^6_2 \, C^8_3}{C^{14}_5} + \frac{C^6_3 \, C^8_2}{C^{14}_5} = \frac{840}{2002} + \frac{560}{2002}$$

$$.4196 + .2797$$

$$= \frac{(15)(56) + (20)(28)}{2002}$$

$$= \boxed{.6993}$$

3. Let z be the *standard* normal random variable (with mean $\mu = 0$ and standard deviation $\sigma = 1$).

(a) (8 points) Find z_0 such that $P(z \le z_0) = .2578$.

FIND .2578 IN TABLE 3 : $\boxed{z_0 = -.65}$

(b) (8 points) Find z_0 such that $P(z \ge z_0) = .0207$.

$P(z \le z_0) = 1 - .0207 = .9793$

FIND .9793 IN TABLE 3 : $\boxed{z_0 = 2.04}$

4. Suppose x is a random variable with a normal probability distribution with mean $\mu = 42$ and standard deviation $\sigma = 3.5$.

(a) (8 points) Find $P(x \le 40)$.

$P(x \le 40) = P\left(z \le \frac{40-42}{3.5}\right)$ $z = \frac{x-\mu}{\sigma}$

$= P(z \le -.57)$

$= \boxed{.2843}$

(ON CALC: $P(z \le -\frac{4}{7}) \approx .2839$)

NOT ROUNDED TO NEAREST HUNDREDTH

(b) (8 points) Find $P(40 < x < 46)$.

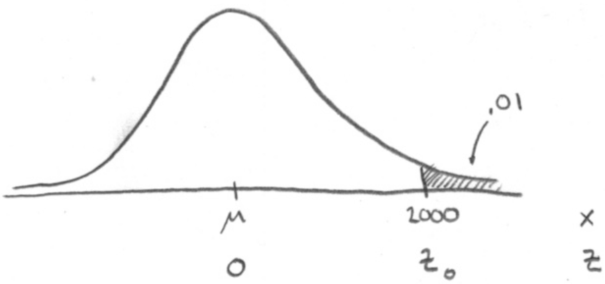
$P(40 < x < 46) = P(x < 46) - P(x < 40)$

$= P\left(z < \frac{46-42}{3.5}\right) - P\left(z < \frac{40-42}{3.5}\right)$ (ON CALL $= .5896$)

$= P(z < 1.14) - P(z < -.57)$

$= .8729 - .2843 = \boxed{.5886}$

5. (8 points) A grain loader can be set to discharge grain in amounts x that are normally distributed, with mean μ bushels and standard deviation $\sigma = 25.7$. bushels. If a company wishes to use the loader to fill containers that hold 2000 bushels of grain and wants to overfill only one container in 100, at what value of μ should the company set the loader? In other words, find μ such that $P(x > 2000) = .01$.



$$z_0 = \frac{2000 - \mu}{\sigma} = \frac{2000 - \mu}{25.7}$$

$P(x > 2000) = .01$

$\rightarrow P(z > z_0) = .01 \longrightarrow P(z \leq z_0) = .99$

TABLE 3: $z_0 = 2.33$

$$\therefore \quad 2.33 = \frac{2000 - \mu}{25.7}$$

$$2.33(25.7) = 2000 - \mu$$

$$\mu = 2000 - 2.33(25.7)$$

$$\boxed{\mu = 1940.119}$$