Name: *Answer Key* **3/13/2019**

Math 17300-CD Introduction to Probability and Statistics **Quiz 2**

1. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c
B	.34	.16
B^c	.36	.14

(a) (4 points) Find $P(A)$.

$$P(A) = P(A \cap B) + P(A \cap B^c)$$
$$= .34 + .36$$
$$= \boxed{.7}$$

(b) (4 points) Find $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(B \cap A) + P(B \cap A^c)}$$
$$= \frac{.34}{.34 + .16} = \frac{.34}{.5} = \boxed{.68}$$

(c) (2 points) Are A and B independent events? Explain briefly.

$\boxed{\text{No. } P(A) \neq P(A|B) \quad .7 \neq .68}$

OR

No. $P(A \cap B) \neq P(A)P(B)$

$.34 \neq (.7)(.5)$

$.34 \neq .35$

OR

$P(B) \neq P(B|A)$

$.5 \neq \frac{.34}{.7} = .4857$

(d) (2 points) Are A and B mutually exclusive events? Explain briefly.

$\boxed{\text{No. } P(A \cap B) \neq 0 \quad .34 \neq 0}$

2. City crime records show that 20% of all crimes are violent and 80% of all crimes are nonviolent. Ninety percent of violent crimes are reported, and 70% of nonviolent crimes are reported.

   (a) (4 points) What proportion of crimes (overall) are reported?

LET V = EVENT THAT A CRIME IS VIOLENT

R = EVENT THAT A CRIME IS REPORTED

GIVEN: $P(V) = .2$ $\quad P(R|V) = .9$

$P(V^c) = .8$ $\quad P(R|V^c) = .7$

LAW OF TOTAL PROBABILITY:

$$P(R) = P(V)P(R|V) + P(V^c)P(R|V^c)$$
$$= (.2)(.9) + (.8)(.7)$$
$$= .18 + .56 = \boxed{.74}$$

   (b) (4 points) If a crime is reported, what is the probability that the crime is violent?

BAYE'S RULE:

$$P(V|R) = \frac{P(V)P(R|V)}{P(R)}$$
$$= \frac{(.2)(.9)}{.74} = \frac{.18}{.74} = \boxed{.2432}$$

3. (6 points) You are tested for a medical condition that occurs in 1 in 1000 people. You are told that if you have the condition then you are guaranteed to test positive, and if you do not have the disease then you will still test positive 5% of the time. You test positive – what is the probability you have the condition?

LET C = EVENT THAT YOU HAVE THE CONDITION

P = EVENT THAT YOU TEST POSITIVE

GIVEN: $P(C) = \frac{1}{1000} = .001$

$P(P|C) = 1$

$P(P|C^c) = .05$

BAYE'S RULE

LAW OF TOTAL PROBABILITY

$$P(C|P) = \frac{P(C)P(P|C)}{P(P)} = \frac{P(C)P(P|C)}{P(C)P(P|C) + P(C^c)P(P|C^c)}$$

$$= \frac{(.001)(1)}{(.001)(1) + (.999)(.05)} = \frac{.001}{.001 + .04995} = \frac{.001}{.05095} \approx \boxed{.0196}$$

LESS THAN 2% !!

4. A random variable x can equal 0, 1, 2, 3, 4, or 5. A portion of the probability distribution is shown here.

x	0	1	2	3	4	5
$p(x)$	.16	.20	.35	0.10	$p(4)$	.04

(a) (2 points) Find $p(4)$.

$$\sum p(x) = 1 \implies p(4) = 1 - (.16 + \ldots + .04) = 1 - .85 = \boxed{.15}$$

(b) (4 points) Find the expected value $E[x]$, i.e. the mean μ.

$$E[x] = \mu = \sum x\,p(x) = (0)(.16) + (1)(.20) + (2)(.35)$$
$$+ (3)(.10) + (4)(.15) + (5)(.04)$$
$$= 0 + .2 + .7 + .3 + .6 + .2$$
$$= \boxed{2}$$

(c) (4 points) Find the standard deviation σ for the random variable x.

x	$x-\mu$	$(x-\mu)^2$	$p(x)$
0	-2	4	.16
1	-1	1	.20
2	0	0	.35
3	1	1	.10
4	2	4	.15
5	3	9	.04

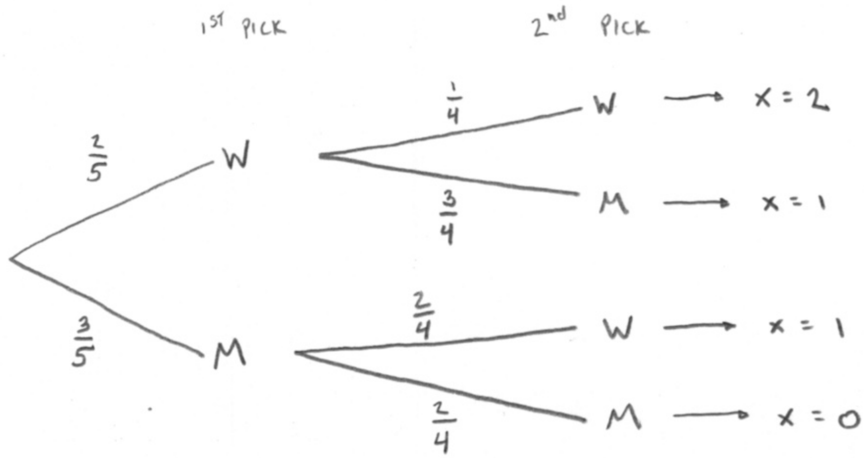
$$\sigma^2 = \sum (x-\mu)^2 p(x)$$
$$= (4)(.16) + (1)(.20) + (0)(.35)$$
$$+ (1)(.10) + (4)(.15) + (9)(.04)$$
$$= .64 + .2 + 0 + .1 + .6 + .36$$
$$= 1.9$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{1.9} \approx \boxed{1.3784}$$

5. (6 points) A company has 5 applicants for 2 positions: 2 women and 3 men. Suppose that the 5 applicants are equally likely to be hired. Let x equal the number of women chosen to fill the two positions. Describe the probability distribution for the discrete random variable x.

x	0	1	2
$p(x)$	$\frac{3}{10}$	$\frac{3}{5}$	$\frac{1}{10}$

P(MW) + P(WM)



$P(WW) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$

$P(WM) = \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} = \frac{3}{10}$

$P(MW) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$

$P(MM) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$

6. (6 points) It costs a shipping company \$11 to ship a package. The company charges \$18 to ship a package, but will refund the charge if the package does not arrive on time. If 90% of packages arrive on time, what is the expected net gain (profit) for this company per package?

	x	$p(x)$
$18 - 11 =$	7	.9
$18 - 11 - 18 =$	−11	.1

$$E[x] = \mu = \sum x\,p(x) = (7)(.9) + (-11)(.1)$$

$$= 6.3 - 1.1$$

$$= 5.2 \rightarrow \boxed{\$5.20}$$