Name: *Answer Key* 3/27/2019

Math 17300-CD Introduction to Probability and Statistics **Midterm Exam**

1. You are given a population of $n = 5$ measurements: $1, 5, 7, 1, 6$.

   (a) (4 points) What is the median, m?

   $1 \quad 1 \quad \boxed{5} \quad 6 \quad 7$ (MIDDLE VALUE) $m = \boxed{5}$

   (b) (4 points) What is the mean, μ?

   $$\mu = \frac{\sum x_i}{n} = \frac{1+5+7+1+6}{5} = \frac{20}{5} = \boxed{4}$$

   (c) (4 points) What is/are the mode/modes, M?

   $M = \boxed{1}$

   (d) (4 points) What is the population variance, σ^2?

x	$x-\mu$	$(x-\mu)^2$
1	-3	9
5	1	1
7	3	9
1	-3	9
6	2	4

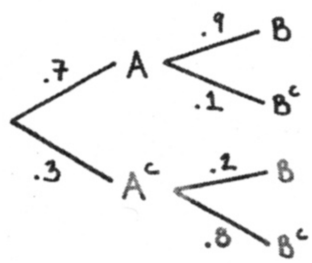
   $$\sigma^2 = \frac{\sum (x-\mu)^2}{N} = \frac{9+1+9+9+4}{5} = \frac{32}{5} = \boxed{6.4}$$

   (e) (4 points) What is the sample standard deviation, σ?

   $$\sigma = \sqrt{\sigma^2} = \sqrt{6.4} \approx \boxed{2.5298}$$

2. You have been kidnapped by an angry dragon. To escape, you must sneak past the dragon, who is blocking the exit. Luckily, dragons sleep a lot and there is a 70% chance that the dragon is sleeping. If the dragon is sleeping, there is a 90% chance that you will escape. But if the dragon is not sleeping, there is a only a 20% chance that you will escape. Let

$$A = \text{the event that the dragon is sleeping}$$
$$B = \text{the event that you escape.}$$



$P(A \cap B) = (.7)(.9) = .63$

$P(A \cap B^c) = (.7)(.1) = .07$

$P(A^c \cap B) = (.3)(.2) = .06$

$P(A^c \cap B^c) = (.3)(.8) = .24$

(a) (8 points) Use the given probabilities to fill in the chart below.

	A	A^C
B	.63	.06
B^C	.07	.24

(b) (4 points) Are A and B independent? Why?

No, $\left.\begin{array}{l} P(B|A) = .9 \\ P(B|A^c) = .2 \end{array}\right\}$ IF A & B WERE INDEPENDENT, THESE WOULD BOTH EQUAL $P(B) = .63 + .06 = .69$

(c) (4 points) Are A and B mutually exclusive? Why?

No, $P(A \cap B) \neq 0$

$.63 \neq 0$

(d) (4 points) What is the probability that you escape, i.e. what is $P(B)$?

$P(B) = P(B \cap A) + P(B \cap A^c) = .63 + .06 = \boxed{.69}$

(e) (4 points) If you do escape, what is probability that the dragon is sleeping, i.e. what is $P(A|B)$ $\cancel{P(B|A)}$?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.63}{.69} = \boxed{.9130}$$

3. On a particular day, a company recorded how its employees commuted to work and whether or not the arrived on time. It found that 55% of employees took the subway to work, 20% drove, 15% walked, and 10% biked. It also found that 80% of people who took the subway arrived on time, 85% of people who drove arrived on time, 98% of people who walked arrived on time, and 90% of people who biked arrived on time.

(a) (8 points) What percentage of employees arrived on time?

LET S = EVENT THAT EMPLOYEE TAKES SUBWAY
D = - - - DRIVES
W = - - - WALKS
B = - - - BIKES
(SUBPOPULATIONS)

A = - - - ARRIVES ON TIME

GIVEN: $P(S) = .55$, $P(D) = .20$, $P(W) = .15$, $P(B) = .10$ (ADD TO 1)

$P(A|S) = .80$
$P(A|D) = .85$
$P(A|W) = .98$
$P(A|B) = .90$

LAW OF TOTAL PROBABILITY:

$$\begin{aligned} P(A) &= P(S)P(A|S) + P(D)P(A|D) + P(W)P(A|W) + P(B)P(A|B) \\ &= (.55)(.8) + (.2)(.85) + (.15)(.98) + (.1)(.9) \\ &= .44 + .17 + .147 + .09 = \boxed{.847} \end{aligned}$$

(b) (8 points) If an employee arrived on time, what is the probability that she took the subway?

$$P(S|A) = \frac{P(S)P(A|S)}{P(A)}$$ (BAYE'S RULE

$$= \frac{(.55)(.80)}{.847} \approx \boxed{.5195}$$

(LEARNING THAT AN EMPLOYEE ARRIVED ON TIME DECREASES THE LIKELIHOOD THAT THEY TOOK THE SUBWAY FROM .55 TO .5195.)

4. (4 points) You are helping a friend choose which classes to take next semester. There are 3 math classes, 5 science classes, and 6 humanities classes available. She must register for 1 math class, 2 science classes, and 2 humanities classes. How many possible ways are there for your friend to choose her classes?

GENERAL mn RULE : # WAYS TO PICK MATH × # WAYS TO PICK SCIENCE × # WAYS TO PICK HUMANITIES

$$\to C^3_1 \; C^5_2 \; C^6_2 = (3)(10)(15) = \boxed{450}$$

5. A certain lottery ticket costs \$5 to buy, and each ticket has a 10% chance of winning \$5 (winning your money back), a 2% chance of winning \$50, and a 1% chance of winning \$250. Let x be the net gain (profit) from buying one lottery ticket.

(a) (8 points) Describe the probability distribution for x by filling out the following chart.

x	0	45	245	-5
$p(x)$	.1	.02	.01	.87

(b) (4 points) What is the expected value $E[x]$ (i.e. μ) for x?

$$E[x] = \sum x p(x) = (0)(.1) + (45)(.02) + (245)(.01) + (-5)(.87)$$

$$= 0 + .9 + 2.45 - 4.35$$

$$= \boxed{-1}$$

(IF YOU PLAY THE LOTTERY OVER & OVER, ON AVERAGE YOU LOSE \$1 EVERY TIME YOU PLAY.)

6. You are building a device that requires 5 working transistors. Company A will sell you a pack of 7 transistors, each of which has an 80% chance of working. Company B will sell you a pack of 8 transistors, each of which has a 70% chance of working.

$n = 7$

	p													
k	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	k
0	.932	.698	.478	.210	.082	.028	.008	.002	.000	.000	.000	.000	.000	0
1	.998	.956	.850	.577	.329	.159	.062	.019	.004	.000	.000	.000	.000	1
2	1.000	.996	.974	.852	.647	.420	.227	.096	.029	.005	.000	.000	.000	2
3	1.000	1.000	.997	.967	.874	.710	.500	.290	.126	.033	.003	.000	.000	3
4	1.000	1.000	1.000	.995	.971	.904	.773	.580	.353	.148	.026	.004	.000	4
5	1.000	1.000	1.000	1.000	.996	.981	.938	.841	.671	.423	.150	.044	.002	5
6	1.000	1.000	1.000	1.000	1.000	.998	.992	.972	.918	.790	.522	.302	.068	6
7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	7

$n = 8$

	p													
k	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	k
0	.923	.663	.430	.168	.058	.017	.004	.001	000	.000	.000	.000	.000	0
1	.997	.943	.813	.503	.255	.106	.035	.009	.001	.000	.000	.000	.000	1
2	1.000	.994	.962	.797	.552	.315	.145	.050	.011	.001	.000	.000	.000	2
3	1.000	1.000	.995	.944	.806	.594	.363	.174	.058	.010	.000	.000	000	3
4	1.000	1.000	1.000	.990	.942	.826	.637	.406	.194	.056	.005	.000	.000	4
5	1.000	1.000	1.000	.999	.989	.950	.855	.685	.448	.203	.038	.006	.000	5
6	1.000	1.000	1.000	1.000	.999	.991	.965	.894	.745	.497	.187	.057	.003	6
7	1.000	1.000	1.000	1.000	1.000	.999	.996	.983	.942	.832	.570	.337	.077	7
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	8

(a) (6 points) If you buy a pack of 7 transistors from company A, what is the probability that 5 or more of the transistors work?

BINOMIAL: $n = 7$, $p = .8$

$$P(x \geq 5) = 1 - P(x < 5)$$
$$= 1 - P(x \leq 4)$$
$$= 1 - .148 = \boxed{.852}$$

(b) (6 points) If you buy a pack of 8 transistors from company B, what is the probability that 5 or more of the transistors work?

BINOMIAL: $n = 8$, $p = .7$

$$P(x \geq 5) = 1 - P(x \leq 4)$$
$$= 1 - .194 = \boxed{.806}$$

7. (4 points) A group of 8 friends are out celebrating for the night. They decide to choose one person to pay for dinner, a second person to pay for concert tickets, and a third person to pay for drinks. How many different ways are there for them to do this?

CHOOSE 3 PEOPLE FROM 8 FOR DIFFERENT POSITIONS.

$\Rightarrow$ ORDER MATTERS.

$$P^8_3 = 8 \cdot 7 \cdot 6 = \boxed{336}$$

8. (8 points) Suppose a group of 15 friends gather to watch a basketball game. Nine of them are cheering for team A, and six of them are cheering for team B. Three friends are randomly chosen to go pick up some pizza. What is the probability that exactly two of the three friends are cheering for the same team? Hint: there are two types of ways this can happen, and we must add the corresponding probabilities.

HYPERGEOMETRIC : $N = 15$

$M = 9$ (SETTING TEAM A = "SUCCESS")

$n = 3$

x = # FRIENDS OUT OF n CHOSEN, THAT CHEER FOR TEAM A.

$$P(2A, 1B) = \frac{C^9_2 \, C^6_1}{C^{15}_3} = \frac{(36)(6)}{455} \approx .4747$$

$$P(1A, 2B) = \frac{C^9_1 \, C^6_2}{C^{15}_3} = \frac{(9)(15)}{455} \approx .2967$$

$$P(\text{TWO THE SAME}) = P(2A, 1B) + P(1A, 2B) = \frac{(36)(6) + (9)(15)}{455} \approx \boxed{.7714}$$