Name: *ANSWER KEY* 7/3/2017

Math 173 Introduction to Probability and Statistics Quiz 4

1. You are given a *sample* of $n = 8$ measurements: 1, 7, 9, 1, 7, 1, 5, 1.

(a) (2 points) Find the median m.

$1, 1, 1, \boxed{1, 5}, 7, 7, 9$ $\qquad m = \frac{1+5}{2} = \boxed{3}$

(b) (4 points) Find the mean $\bar{x}$.

$$\bar{x} = \frac{1}{n}\sum x_i = \frac{1+7+9+1+7+1+5+1}{8} = \frac{32}{8} = \boxed{4}$$

(c) (2 points) Find the mode M.

$M = \boxed{1}$

(d) (4 points) Find the variance s^2 and the standard deviation s.

x	$x-\mu$	$(x-\mu)^2$
1	-3	9
7	3	9
9	5	25
1	-3	9
7	3	9
1	-3	9
5	1	1
1	-3	9

$$s^2 = \frac{\sum(x-\mu)^2}{n-1}$$

$$= \frac{9+9+25+9+9+9+1+9}{7}$$

$$= \frac{80}{7} \approx \boxed{11.4286}$$

$$s = \sqrt{s^2} \approx \boxed{3.3806}$$

2. (4 points) A sample of measurements are collected from a population with unknown distribution. The sample mean and sample standard deviation are computed. According to Tchebysheff's theorem, at least what proportion of measurements lie within 3 standard deviations of the mean?

$$1 - \frac{1}{3^2} = \boxed{\frac{8}{9} \text{ or } .8889}$$

3. An experiment can result in events A, B, both A and B, and neither A nor B with the following probabilities.

	B	B^c
A	.5	.1
A^c	.4	0

(a) (4 points) Find $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.5}{.5 + .4} = \boxed{\frac{5}{9} \text{ or } .5556}$$

(b) (4 points) Are events A and B independent? Why/why not?

$\boxed{\text{No.}}$ $P(A) = .5 + .1 = .6 \neq P(A|B) = .5556$

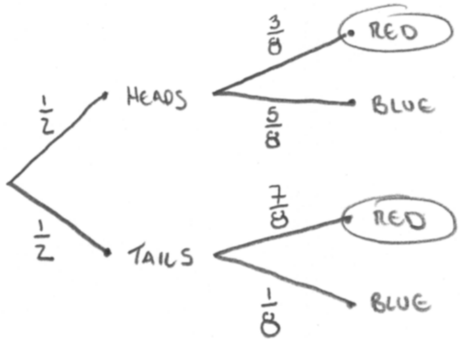
Also, $P(B) = .5 + .4 = .9$, so $P(A)P(B) = (.6)(.9) = .54$
$\neq P(A \cap B) = .5$

(c) (4 points) Are events A and B mutually exclusive? why/why not?

$\boxed{\text{No. } P(A \cap B) \neq 0.}$

4. Suppose there are two boxes labeled Box A and Box B. Box A contains 3 red and 5 blue marbles. Box B contains 7 red and 1 blue marble. You flip a fair coin and if it lands heads you select one marble from Box A; if it lands tails you select one marble from Box B.

(a) (4 points) Find the probability of selecting a red marble.



$$P(\text{RED}) = P(\text{HEADS} \cap \text{RED}) + P(\text{TAILS} \cap \text{RED})$$

$$= P(\text{HEADS})P(\text{RED} \mid \text{HEADS}) + P(\text{TAILS})P(\text{RED} \mid \text{TAILS})$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{3}{8}\right) + \left(\tfrac{1}{2}\right)\left(\tfrac{7}{8}\right) = \boxed{\tfrac{10}{16} \text{ OR } .625}$$

(b) (4 points) Find the probability that the coin landed heads, given that you selected a red marble.

$$P(\text{HEADS} \mid \text{RED}) = \frac{P(\text{HEADS} \cap \text{RED})}{P(\text{RED})} = \frac{P(\text{HEADS})P(\text{RED} \mid \text{HEADS})}{P(\text{RED})}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{8}\right)}{\left(\frac{10}{16}\right)} = \boxed{\tfrac{3}{10} \text{ OR } .3}$$

5. (8 points) Under the "no pass, no play" rule for athletes, an athlete who fails a course is disqualified from participating in sports activities during the next grading period. Suppose the probability that an athlete who has not previously been disqualified will be disqualified is .15 and the probability that an athlete who has been disqualified will be disqualified again in the next time period is .5. If 30% of the athletes have been disqualified before, what is the unconditional probability that an athlete will be disqualified during the next grading period?

LET P = EVENT THAT ATHLETE HAS BEEN PREVIOUSLY DISQUALIFIED

D = EVENT THAT ATHLETE WILL BE DISQUALIFIED

GIVEN: $P(D \mid P^c) = .15$

$P(D \mid P) = .5$

$P(P) = .3 \quad \Rightarrow \quad P(P^c) = .7$

FIND $P(D)$.

$$P(D) = P(P)P(D \mid P) + P(P^c)P(D \mid P^c)$$ (LAW OF TOTAL PROBABILITY)

$$= (.3)(.5) + (.7)(.15)$$

$$= \boxed{.255}$$

6. A random variable x has the following probability distributuon.

x	1	3	5	7	9
$p(x)$	.1	.2	.4	w	.15

(a) (2 points) Solve for w , i.e. find $p(7)$.

SINCE $\sum p(x) = 1$, $W = \boxed{.15}$

(b) (4 points) Find the mean μ, i.e. expected value $E[x]$.

$$\mu = E[x] = \sum x\,p(x)$$

$$= 1(.1) + 3(.2) + 5(.4) + 7(.15) + 9(.15)$$

$$= .1 + .6 + 2 + 1.05 + 1.35$$

$$= \boxed{5.1}$$

(c) (4 points) Find the variance σ^2 and standard deviation σ.

x	$x-\mu$	$(x-\mu)^2$	$p(x)$
1	-4.1	16.81	.1
3	-2.1	4.41	.2
5	-.1	.01	.4
7	1.9	3.61	.15
9	3.9	15.21	.15

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

$$= (16.81)(.1) + (4.41)(.2) + (.01)(.4) + (3.61)(.15) + (15.21)(.15)$$

$$= \boxed{5.39}$$

$$\sigma = \sqrt{\sigma^2} \approx \boxed{2.3216}$$

7. (8 points) From experience, a shipping company knows that the cost of delivering a small package within 24 hours is \$18.75. The company charges \$22.50 for shipment but guarantees to refund the charge if delivery is not made within 24 hours. If the company fails to deliver only 2% of its packages within the 24-hour period, what is the expected gain (i.e. profit) per package?

LET X = GAIN (PROFIT) TO THE SHIPPING COMPANY

(RANDOM VARIABLE)

x	$p(x)$
$22.50 - 18.75$	$.98$
-18.75	$.02$

$$\mu = E[x] = \sum x\,p(x)$$

$$= (22.50 - 18.75)(.98) + (-18.75)(.02)$$

$$= 3.675 - .375$$

$$= \boxed{\$3.30}$$

8. (8 points) According to the Humane Society of the United States, there are approximately 77.5 million owned dogs in the United States, and approximately 40% of all U.S. households own at least one dog. Suppose that 15 households are randomly selected for a pet ownership survey. What is the probability that exactly eight of the households have at least one dog?

BINOMIAL RANDOM VARIABLE X.

$n = 15$

$p = .4$

$q = .6$

$$P(x = k) = C^n_k\, p^k\, q^{n-k}$$

$$P(x = 8) = C^{15}_{8}\, (.4)^8 (.6)^7 = 6435\,(.00065536)(.0279936)$$

$$\approx \boxed{.1181}$$

9. A student prepares for an exam by studying a list of 10 problems. She can solve 6 of them. For the exam, the instructor selects 5 questions at random from the list of 10.

(a) (4 points) What is the probability that the student can solve all 5 problems on the exam?

HYPERGEOMETRIC RANDOM VARIABLE X.

$N = 10$

$M = 6$

$n = 5$

$$P(X = k) = \frac{C^M_k \, C^{N-M}_{n-k}}{C^N_n}$$

$$P(X = 5) = \frac{C^6_5 \, C^4_0}{C^{10}_5} = \frac{(6)(1)}{252} = \boxed{\frac{6}{252} \text{ OR } \frac{1}{42} \text{ OR } .0238}$$

(b) (4 points) What is the probability that the students can solve at least 4 problems on the exam?

$$P(X \geq 4) = P(X = 4) + P(X = 5)$$

$$= \frac{C^6_4 \, C^4_1}{C^{10}_5} + \frac{C^6_5 \, C^4_0}{C^{10}_5}$$

$$= \frac{(15)(4) + (6)(1)}{252} = \boxed{\frac{66}{252} \text{ OR } \frac{11}{42} \text{ OR } .2619}$$

10. (8 points) Suppose that you must establish regulations concerning the maximum number of people who can occupy an elevator. A study indicates that if eight people occupy the elevator, the probability distribution of the total weight of the eight people is approximately normally distributed with a mean equal to 1200 pounds and a standard deviation of 99 pounds. What is the probability that the total weight of eight people exceeds 1500 pounds?

$$P(X \geq 1500) = P(Z \geq 3.03) = 1 - P(Z \leq 3.03)$$

$$\left(Z = \frac{X - \mu}{\sigma} = \frac{1500 - 1200}{99} \approx 3.03 \right)$$

$$= 1 - .9988 = \boxed{.0012}$$