Name: * Answer Key * 10/15/2018

Math 173 Introduction to Probability and Statistics 50 POINTS **Quiz 1**

+4 BONUS POINTS

1. Provide formulas for the following.

2 (a) The number of ways to choose and arrange r distinct objects from a collection of n distinct objects, i.e. P_r^n.

$$P_r^n = \frac{n!}{(n-r)!}$$

2 (b) The number of ways to choose r distinct objects from a collection of n distinct objects, i.e. C_r^n.

$$C_r^n = \frac{n!}{r!(n-r)!}$$

4 2. How many different ways can a committee of 7 people choose a president, vice-president, and secretary?

$$P_3^7 = \frac{7!}{(7-3)!} = 7 \cdot 6 \cdot 5 = \boxed{210}$$

(ORDER MATTERS : PRES. = ALICE, V.PRES. = BOB, SEC. = CINDY) DIFFERENT ORDER => DIFFERENT ASSIGNMENTS

4 3. How many ways can Noah select 2 elephants and 2 mice from a group of 5 elephants and 7 mice?

2-STAGE EVENT : CHOOSE ELEPHANTS (1) THEN CHOOSE MICE (2)

$$C_2^5 \times C_2^7 = \frac{5!}{2!3!} \times \frac{7!}{2!5!} = \frac{5 \cdot 4}{2} \times \frac{7 \cdot 6}{2}$$

$$= 10 \times 21 = \boxed{210}$$

4. A teacher has given her class a list of 8 problems to study, and a student knows how to answer 6 of these problems. The teacher will randomly select 4 of the 8 problems to make the exam, with each problem being worth the same number of points.

4 (a) How many distinct exams can the teacher possibly make?

$$C_4^8 = \frac{8!}{4!4!} = \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2} = \boxed{70}$$

(ORDER OF PROBLEMS DOESN'T MATTER)

4 (b) What is the probability that the student can solve all 4 problems on the exam?

EXAMS SUCH THAT STUDENT CAN SOLVE ALL PROBLEMS $= C_4^6 = \frac{6!}{4!2!} = \frac{6\cdot5}{2} = 15$

$$\therefore \quad \frac{15}{70} = \boxed{\frac{3}{14} \approx .2143}$$

BONUS (+4) (c) What is the probability that the student can solve *at least* 3 problems on the exam?

$$P(\text{AT LEAST } 3) = P(4) + P(3) = \frac{C_4^6 \cdot C_0^2}{C_4^8} + \frac{C_3^6 \cdot C_1^2}{C_4^8}$$

↑ PART (b)

$$= 15/70 + 40/70 = 55/70 = \boxed{11/14 \approx .7857}$$

5. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c	
B	.22	.38	.6
B^c	.18	.22	.4
	.4	.6	1

4 (a) (4 points) Find $P(A|B)$.

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \boxed{\frac{.22}{.6} \approx .3667}$$

4 (b) (4 points) Find $P(B|A)$.

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \boxed{\frac{.22}{.4} = .55}$$

4 (c) (2 points) Are A and B independent events? Explain briefly.

$\boxed{\text{No}: \quad P(A\cap B) \neq P(A)P(B) \quad .22 \neq (.4)(.6)}$ OR $P(A|B) \neq P(A)$ $\quad .3667 \neq .4$ OR $P(B|A) \neq P(B)$ $\quad .55 \neq .6$

4 (d) (2 points) Are A and B mutually exclusive events? Explain briefly. (ANY OF THESE IS SUFFICIENT)

$\boxed{\text{No}: \quad P(A\cap B) \neq 0 \quad .22 \neq 0}$

6. City crime records show that 15% of all crimes are violent and 85% are nonviolent, involving theft, forgery, and so on. Additionally, 90%of violent crimes are reported versus 60% of nonviolent crimes.

4 (a) What is the overall reporting rate for crimes in the city?

LET V = CRIME IS VIOLENT

V^c = CRIME IS NON-VIOLENT

R = CRIME IS REPORTED

LAW OF TOTAL PROBABILITY: $P(R) = P(V)P(R|V) + P(V^c)P(R|V^c)$

$= (.15)(.9) + (.85)(.6)$

$= .135 + .51 = \boxed{.645}$

4 (b) If a crime in progress is reported to the police, what is the probability that the crime is violent?

BAYE'S RULE: $P(V|R) = \dfrac{P(V)P(R|V)}{P(R)}$

$= \dfrac{(.15)(.9)}{(.645)} = \boxed{.2093}$

$P(R)$ FROM PART (a)

6 7. Suppose a lottery ticket costs \$5 to purchase. 5% of these tickets win \$10, 1% of these tickets win \$100, and the rest of the tickets do not win anything. Fill out the following chart for the probability distribution of the random variable x =*expected gain from buying one lottery ticket*, and calculate the *expected value* (i.e. *mean value*) for x.

	x	$p(x)$
$10-5=$	5	.05
$100-5=$	95	.01
$0-5=$	-5	.94

$\mu = E[x] = \sum x\,p(x)$

$= (5)(.05) + (95)(.01) + (-5)(.94)$

$= \boxed{-3.5 \text{ (DOLLARS)}}$