Name: *ANSWER KEY*

Math 173 Introduction to Probability and Statistics **11/7/2018**

Midterm Exam

1. A population consists of the following five measurements.

$$5, 7, 1, 2, 4$$

(a) (4 points) What is the median m for the population?

1 , 2 , 4 , 5 , 7

↑

$m = \boxed{4}$

(b) (4 points) What is the mean μ for the population?

$$\mu = \frac{5+7+1+2+4}{5} = \frac{19}{5} = \boxed{3.8}$$

(c) (4 points) What is the variance σ^2 for the population?

x	$x - \mu$	$(x-\mu)^2$
5	1.2	1.44
7	3.2	10.24
1	-2.8	7.84
2	-1.8	3.24
4	.2	.04

↓ TOTAL (SUM)

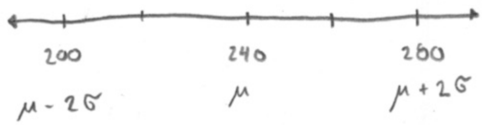
22.8

$$\sigma^2 = \frac{\Sigma (x-\mu)^2}{n} = \frac{22.8}{5} = \boxed{4.56}$$

(d) (4 points) What is the standard deviation σ for the population?

$$\sigma = \sqrt{\sigma^2} = \sqrt{4.56} = \boxed{2.1354}$$

2. (8 points) Jess, a music publisher, studies the Billboard Top 100 songs and finds that the average song length is 240 sec, with standard deviation 20 sec. Using Tchebysheff's rule, she can be sure that at least how many songs have lengths that lie between 200 and 280 sec?



AT LEAST $1 - \frac{1}{2^2} = \frac{3}{4}$ OF SONG LENGTHS LIE WITHIN 2 STANDARD DEVIATIONS OF μ (i.e. BETWEEN 200 & 280 SEC).

$\frac{3}{4}$ of $100 = \boxed{75}$

3. (8 points) Suppose you own 5 pairs of pants, 6 shirts, and 3 jackets. You are packing for a weekend out of town and decide to bring 2 pairs of pants, 3 shirts, and 1 jacket. How many possible ways are there to pack for this trip?

3 STAGE EVENT : # WAYS TO CHOOSE PANTS × # WAYS TO CHOOSE SHIRTS × # WAYS TO CHOOSE JACKET

$$C^5_2 \times C^6_3 \times C^3_1 = 10 \times 20 \times 3 = \boxed{600}$$

$$\left(\frac{5!}{2!3!} \times \frac{6!}{3!3!} \times \frac{3!}{1!2!} \right)$$

4. Serial numbers for a particular product consist of 4 *distinct* digits (out of ten digits, 0-9) arranged in a particular order.

(a) (4 points) How many distinct serial numbers are there?

$$P^{10}_4 = \frac{10!}{6!} = 10 \times 9 \times 8 \times 7 = \boxed{5040}$$

(b) (4 points) How many distinct serial numbers contain only odd digits?

5 ODD DIGITS : 1, 3, 5, 7, 9

$$P^5_4 = \frac{5!}{1!} = 5 \times 4 \times 3 \times 2 = \boxed{120}$$

(c) (4 points) Assuming all serial numbers are equally likely, what is the probability that a serial number contains only odd digits?

$$\boxed{\frac{120}{5040} = \frac{1}{42} \approx .0238}$$

5. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c	
B	.42	.18	.60
B^c	.24	.~~3~~16	.40
	.66	.34	1

(a) (4 points) Find $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.42}{.60} = \boxed{.7}$$

(b) (4 points) Find $P(B|A)$.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{.42}{.66} \approx \boxed{.6364}$$

(c) (4 points) Are A and B independent events? Explain briefly.

$\boxed{\text{No.}}$

$P(A \cap B) \neq P(A)P(B)$ $\quad .42 \neq (.6)(.66)$

$P(A|B) \neq P(A)$ $\quad .7 \neq .66$

$P(B|A) \neq P(B)$ $\quad .6364 \neq .6$

(d) (4 points) Are A and B mutually exclusive events? Explain briefly.

$\boxed{\text{No.}}$ $P(A \cap B) \neq 0$ $\quad .42 \neq 0$

6. The following table summarizes data about a population of student commuters on the first day of school.

	Walk	Bike	Train/Bus	Car
Proportion of population	.06	.09	.72	.13
Proportion that arrive late	.02	.04	.18	.08

(a) (8 points) Overall, what is the probability that a student commuter from this population arrives late on the first day of school?

LAW OF TOTAL PROBABILITY : $P(A) = P(S_1)P(A|S_1) + \dots + P(S_n)P(A|S_n)$

LET L = ARRIVE LATE
W = WALK
B = BIKE
TB = TRAIN/BUS
C = CAR

THEN $P(L) = P(W)P(L|W) + P(B)P(L|B) + P(TB)P(L|TB) + P(C)P(L|C)$

$= .06(.02) + .09(.04) + .72(.18) + .13(.08)$

$= .0012 + .0036 + .1296 + .0104 = \boxed{.1448}$

(b) (8 points) Given that a student commuter is late on the first day of school, what is the probability that they took the train/bus?

BAYE'S RULE : $P(A|B) = \dfrac{P(A)P(B|A)}{P(B)}$

$\therefore \quad P(TB|L) = \dfrac{P(TB)P(L|TB)}{P(L)} = \dfrac{(.72)(.18)}{.1448} = \dfrac{.1296}{.1448} \approx \boxed{.8950}$

7. (8 points) From experience, a shipping company knows that the cost of delivering a small package is \$12. The company charges \$16 for shipment but guarantees to refund the charge if delivery is not made within 24 hours. Suppose the company fails to deliver 2% of its packages within the 24-hour period. Fill out the following chart for the probability distribution of the random variable x = *profit for delivering a package* and calculate the *expected value* for x.

	x	$p(x)$
$-12 + 16 =$	4	.98
$-12 + 16 - 16 =$	-12	.02

$\mu = E[x] = \sum x\,p(x)$

$= (4)(.98) + (-12)(.02)$

$= 3.92 - .24$

$= \boxed{\$3.68}$

8. (8 points) A bowl of nuts contains 8 almonds and 10 peanuts. If you randomly select 5 nuts from the bowl, what is the probability that you select exactly 3 almonds and 2 peanuts?

HYPERGEOMETRIC DISTRIBUTION: IF X = # ALMONDS (ARBITRARY) IN THE 5 NUTS SELECTED, THEN $P(X=k) = \dfrac{C_k^M \, C_{n-k}^{N-M}}{C_n^N}$ WHERE $N = 18$, $M = 8$, $n = 5$.

$$\therefore \quad P(X=3) = \frac{C_3^8 \, C_2^{10}}{C_5^{18}} = \frac{(56)(45)}{8568} \approx \boxed{.2941}$$

9. When a professional bowler plays a frame, he gets a strike 85% of the time.

(a) (8 points) Find the probability that this bowler gets exactly 7 strikes in 10 frames.

BINOMIAL DISTRIBUTION: IF X = # STRIKES IN 10 FRAMES, THEN

$P(X=k) = C_k^n \, p^k q^{n-k}$, WHERE $n = 10$, $p = .85$, $q = .15$.

$$\therefore \quad P(X=7) = \underbrace{C_7^{10}}_{(120)} (.85)^7 (.15)^3 \approx \boxed{.1298}$$

(b) (8 points) Use a normal approximation to estimate the probability that this bowler gets less than 415 strikes in 500 frames, and state the "rule of thumb" THAT allows us to use this approximation?

WHEN $np > 5$ & $nq > 5$, THE DISTRIBUTION FOR X IS APPROXIMATELY NORMAL. ($np = (500)(.85) = 425$, $nq = (500)(.15) = 75$ ✓)

$P_{BIN}(X < 415) \approx P_{NORM}(X \leq 414.5)$ WHERE $\mu = np = (500)(.85) = 425$

$\sigma = \sqrt{npq} = \sqrt{(500)(.85)(.15)} \approx 7.9844$

$$\left(z = \frac{x - \mu}{\sigma} \right)$$

$$= P\left(z \leq \frac{414.5 - 425}{7.9844}\right) = P(z \leq -1.3151) \approx \boxed{.0942}$$

IS NORMALLY DISTRIBUTED WITH

10. The distribution of salaries of all American high school teachers ~~has~~ a mean of $\mu = \$58{,}030$ and a standard deviation of $\sigma = \$4{,}500$.

(a) (8 points) If an American high school teacher is randomly selected, what is the probability that their salary is above \$60,000?

$$P(x > 60{,}000) = 1 - P(x \le 60{,}000)$$

$$= 1 - P\left(z \le \frac{60{,}000 - 58{,}030}{4{,}500}\right)$$

$$= 1 - P(z \le .4378)$$

$$= 1 - .6692$$

$$\approx \boxed{.3308}$$

(b) (8 points) If a random sample of $n = 25$ American high school teachers is collected, what is the probability that the sample mean salary $\bar{x}$ is above \$60,000?

DISTRIBUTION OF $\bar{x}$ IS NORMAL (SINCE DISTRIBUTION OF x IS NORMAL)

WITH $\mu = 4500$ & $S.E. = \frac{\sigma}{\sqrt{n}} = \frac{4500}{\sqrt{25}} = 900$

$$P(\bar{x} > 60{,}000) = 1 - P(x \le 60{,}000) = 1 - P\left(z \le \frac{60{,}000 - 58{,}030}{900}\right)$$

$$= 1 - P(z \le 2.1889)$$

$$\approx 1 - .9857 = \boxed{.0143}$$