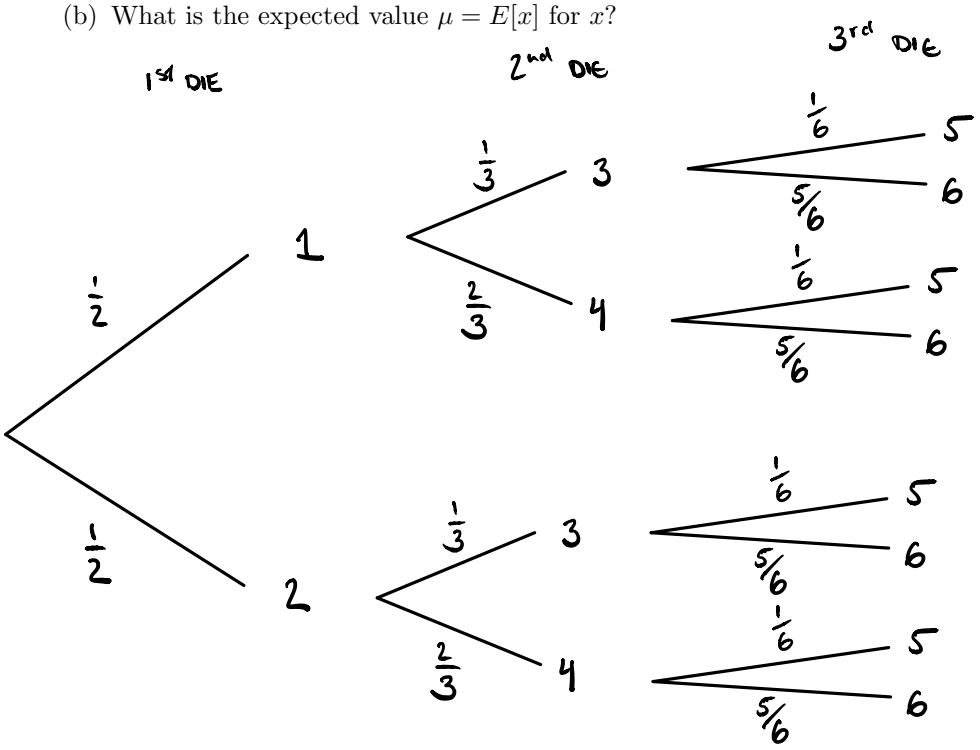
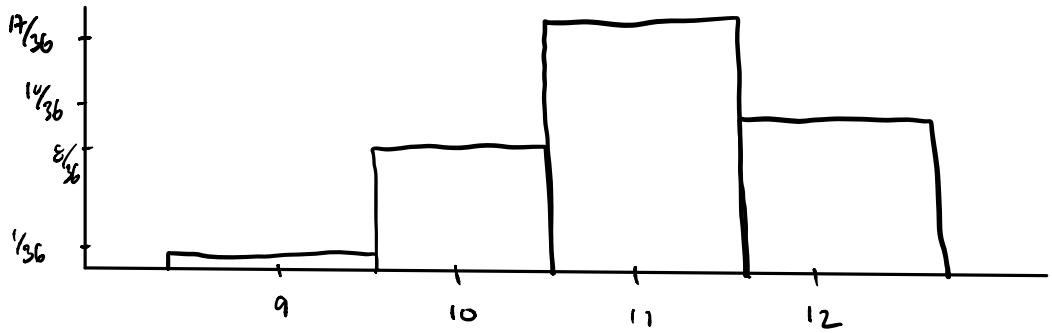
1. An experiment consists of flipping three fair 6-sided dice. One die has its faces labeled 1, 1, 1, 2, 2, 2; another die has its faces labeled 3, 3, 4, 4, 4, 4; and the third die has its faces labeled 5, 6, 6, 6, 6, 6. Let x be the sum of the three dice shown.
   (a) Describe the probability distribution for the discrete random variable x.
   (b) What is the expected value $\mu = E[x]$ for x?



X	Prob.
9	$\frac{1}{36}$
10	$\frac{5}{36}$
10	$\frac{1}{18} = \frac{2}{36}$
11	$\frac{5}{18} = \frac{10}{36}$
10	$\frac{1}{36}$
11	$\frac{5}{36}$
11	$\frac{1}{18} = \frac{2}{36}$
12	$\frac{5}{18}$

x	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{8}{36}$	$\frac{17}{36}$	$\frac{10}{36}$



(b) What is the expected value $\mu = E[x]$ for x?

$$\mu = E[x] = \sum x\,p(x) = \left(\tfrac{1}{36}\right)(9) + \left(\tfrac{8}{36}\right)(10) + \left(\tfrac{17}{36}\right)(11) + \left(\tfrac{10}{36}\right)(12)$$

$$= \frac{9 + 80 + 187 + 120}{36} = 11$$

2. Weights of chicken eggs are normally distributed with mean $\mu = 2.53$ oz and standard deviation $\sigma = .24$ oz.

   (a) What is the probability that a randomly selected chicken egg weighs more than 2.75 pounds?

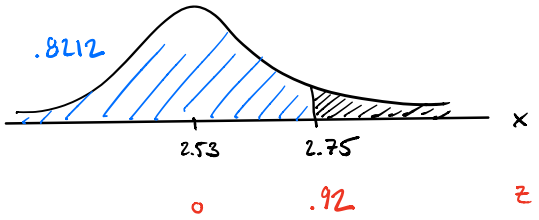
   (b) What is the probability that exactly 4 out of 10 randomly selected chicken eggs weigh more than 2.75 lbs?

   (c) What is the probability that 10 randomaly selected chicken eggs have a total weight less than 24 oz (i.e. have an average weight less than 2.4 oz)?

(a) LET X = WEIGHT OF RAND. SEL. CHICKEN EGG.

$P(x > 2.75) = P(z > .92)$

$z = \frac{x - \mu}{\sigma}$ $\quad z = \frac{2.75 - 2.53}{.24} \approx .92$



NEVER EVER SUBTRACT A Z-SCORE FROM 1!

$P(z > .92) = 1 - P(z \leq .92)$

$= 1 - .8212 = .1788$

(b) What is the probability that exactly 4 out of 10 randomly selected chicken eggs weigh more than 2.75 lbs?

BINOMIAL EXP. $\longrightarrow$ $P(x = k) = C_k^n \, p^k \, q^{n-k}$ $\swarrow$ INCLUDE ON YOUR 2 PAGES OF NOTES!

$n = 10$

$p = .1788$

$q = .8212$

x = # SUCCESSES

$P(x = 4) = C_4^{10} (.1788)^4 (.8212)^6$

$= .0658$

(c) What is the probability that 10 randomaly selected chicken eggs have a total weight less than 24 oz (i.e. have an average weight less than 2.4 oz)?

CENTRAL LIMIT THM!

LET Σx = SAMPLE SUM (TOTAL WEIGHT OF 10 EGGS)

SAMPLE SIZE $n = 10$

POPULATION MEAN $\mu = 2.53$ oz

POPULATION STAND DEV. $\sigma = .24$ oz

MEAN FOR $\Sigma x = n\mu = 10(2.53) = 25.3$

STANDERD ERROR FOR $\Sigma x = \sqrt{n}\,\sigma = \sqrt{10}(.24)$

$$P(\Sigma x < 24) = P\left(z < \frac{24 - 25.3}{\sqrt{10}(.24)}\right)$$

$$= P(z < -1.71) = .0436$$

LESS VARIATION FOR SAMPLE MEANS $\bar{x}$ (STAND ERROR $= \frac{\sigma}{\sqrt{n}}$)

MORE VARIATION FOR SAMPLE SUMS Σx (STAND. ERROR $= \sqrt{n}\,\sigma$)

$$P\left(\frac{\Sigma x}{10} < \frac{24}{10}\right) = P(\bar{x} < 2.4) = P\left(z < \frac{2.4 - 2.53}{.24/\sqrt{10}}\right)$$

$\bar{x}$: SAMPLE MEAN.

MEAN FOR $\bar{x} = \mu = 2.53$

STANDERD ERROR FOR $\bar{x} = \frac{\sigma}{\sqrt{n}} = \frac{.24}{\sqrt{10}}$

$$= P(z < -1.71) = .0436$$

3. A printer is broken in such a way that every time a page is printed, there is a 10% chance that that printed page will contain a defect.
   (a) If 20 pages are printed, what is the probability that more than 1 page contains a defect?
   (b) If 300 pages are printed, use a normal distribution to approximate the probability that more than 15 pages contain a defect.

(a) BINOMIAL EXPERIMENT: $n = 20$, $p = .1$, $q = .9$, x = # DEFECTS

$P(x > 1) = P(x=2) + P(x=3) + \ldots + P(x=20)$

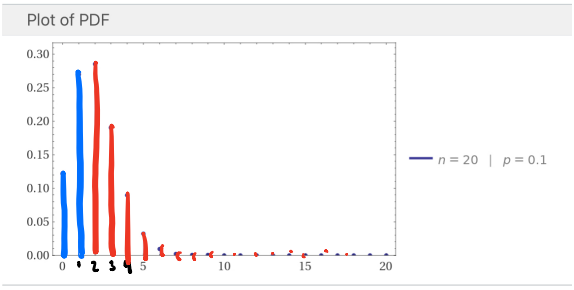
COMPLEMENTS!

$= 1 - P(x \leq 1)$

$= 1 - (P(x=0) + P(x=1))$

We can only approximate a binomial distribution with a normal distribution when $np>5$ and $nq>5$

$(20)(.1) = 2 \not> 5$ → NOT WELL APPROX. BY NORMAL DIST.

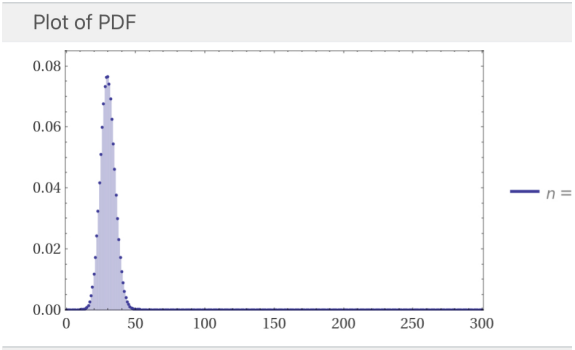


$1 - (P(x=0) + P(x=1))$

$= 1 - \left(C_0^{20}(.1)^0(.9)^{20} + C_1^{20}(.1)^1(.9)^{19} \right)$

$= .6083$

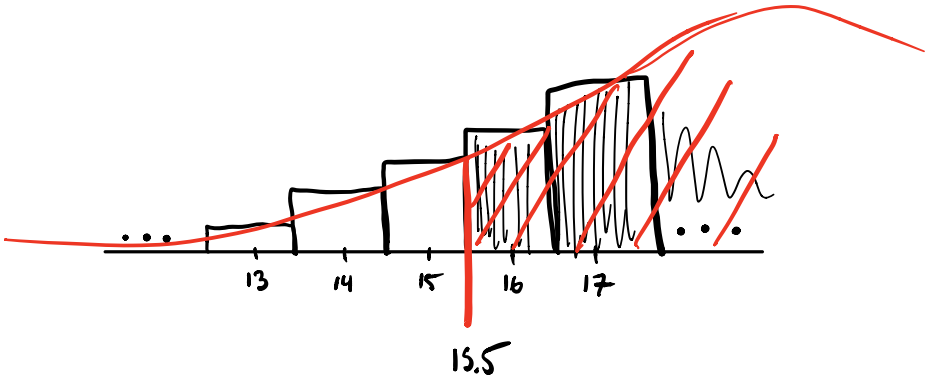
(b) If 300 pages are printed, use a normal distribution to approximate the probability that more than 15 pages contain a defect.



Note: $np = (300)(.1) = 30 > 5$

$nq = (300)(.9) = 270 > 5$

↳ Well approx. by normal distr.



$P(x > 15)$

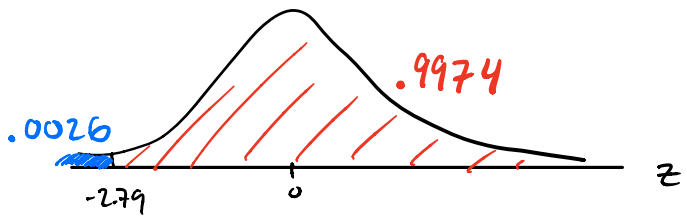
$\approx P(x_{Norm} \geq 15.5)$

↳ Mean $= np = (300)(.1) = 30$

Stand. Dev. $= \sqrt{npq} = \sqrt{(300)(.1)(.9)} = \sqrt{27}$

$= P\left(z \geq \frac{15.5 - 30}{\sqrt{27}}\right) = P(z \geq -2.79) = 1 - .0026$

$= .9974$



4. A researcher gives a survey to 456 randomly selected New Yorkers over the age of 75 and finds that 61% of them rely on public transportation to get around.
   (a) Create a 99% confidence interval for the true proportion of New Yorkers over age 75 that rely on public transportation to get around.
   (b) Give two ways the researcher could narrow her confidence interval, i.e. increase her accuracy.

(a) SAMPLE SIZE $n = 456$

SAMPLE PROPORTION $\hat{p} = .61$

$\hat{q} = .39$

$\hat{p}$ IS NORMALLY DISTRIBUTED WITH

MEAN p

STANDARD ERROR $\sqrt{\frac{pq}{n}} \approx \sqrt{\frac{\hat{p}\hat{q}}{n}}$

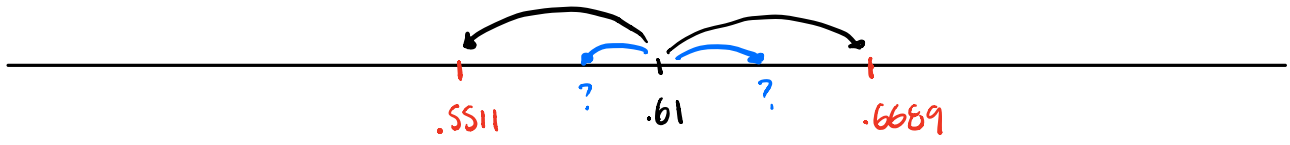
CONF. INT: SAMPLE STAT. $\pm z_{\alpha/2}$ S.E.

$$p \approx \hat{p} \pm 2.58\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$p \approx .61 \pm 2.58\sqrt{\frac{(.61)(.39)}{456}}$$

HOW COULD WE MAKE THIS # SMALLER

$$[\ .5511\ ,\ .6689\ ]$$



(1) INCREASE SAMPLE SIZE!

(2) DECREASE CONFIDENCE LEVEL

Suppose you want to estimate the proportion of American citizens that would agree to receive a particular vaccine. How many people would you need to sample in order to answer this question within an error of .5 percentage points, with a confidence level of 99%?

$$\hat{p} \pm \underbrace{2.58\sqrt{\frac{pq}{n}}}_{\text{LESS THAN } .5\%}$$

$$2.58\sqrt{\frac{pq}{n}} < .005$$

Assume worst case: $p = q = .5$

$$2.58\sqrt{\frac{(.5)(.5)}{n}} < .005$$

$$\sqrt{\frac{.25}{n}} < \frac{.005}{2.58}$$

$$\frac{.25}{n} < \left(\frac{.005}{2.58}\right)^2$$

$$\frac{.25}{\left(\frac{.005}{2.58}\right)^2} < n \qquad 66,564 < n$$

AT LEAST 66,565

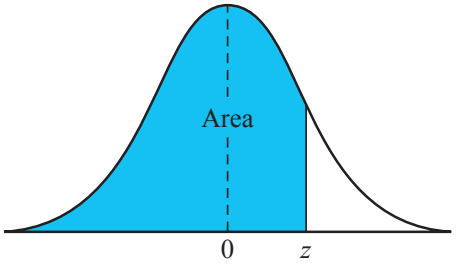


TABLE 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

TABLE 3 *(continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998