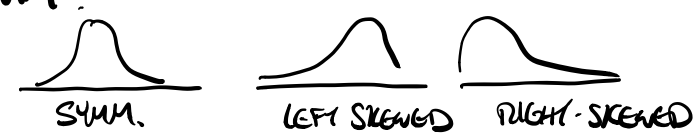
FINAL EXAM MON 12/14 3:30 - 5:45 PM

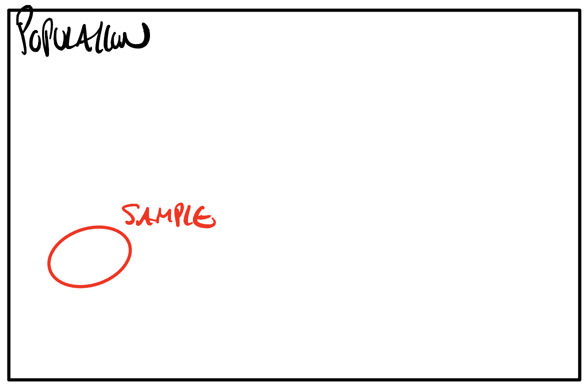
DESCRIPTIVE STATISTICS

HISTOGRAMS. INTERPRET A GIVEN HISTOGRAM.

SHAPES OF DISTRIBUTIONS



MEAN MEDIAN MODE



VARIANCE σ^2 POPULATION

s^2 SAMPLE

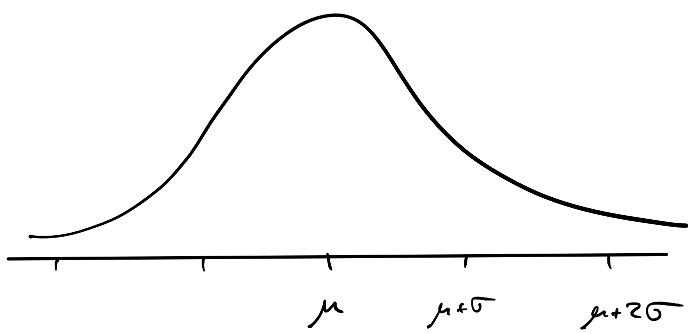
STANDARD DEV σ POPULATION

s SAMPLE

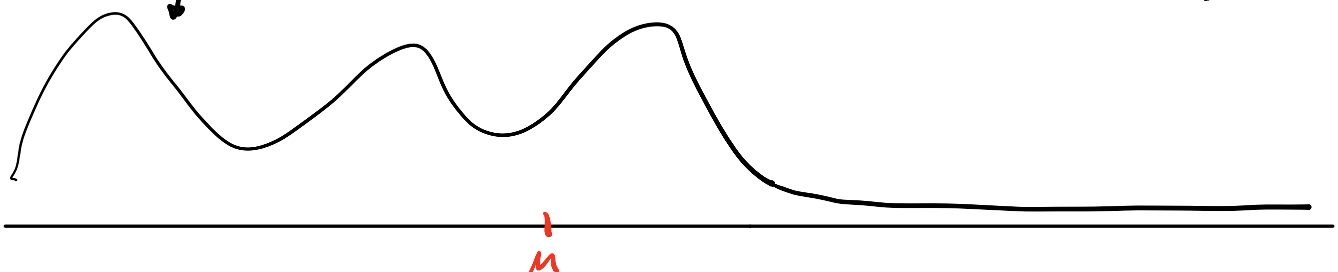
$$\sigma^2 = \frac{1}{n} \sum (x_i - \mu)^2 \qquad \sigma = \sqrt{\sigma^2}$$

$$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 \qquad s = \sqrt{s^2} \qquad \sigma \approx s$$

CHEVYCHEV'S THM:



APPLIES TO EVERY DISTRIBUTION.

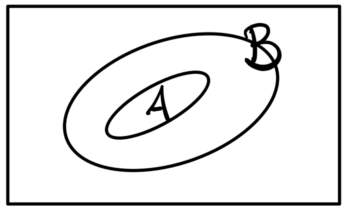


AT LEAST $1 - \frac{1}{k^2}$ (PROPORTION) OF DATA

LIES WITHIN k STANDARD DEV. OF MEAN

$$[\mu - k\sigma, \ \mu + k\sigma]$$

PROBABILITY



SETS, SUBSETS $A \subseteq B$

SET OPERATIONS: UNION $A \cup B$

INTERSECTION $A \cap B$

COMPLEMENT A^c

SET OF ALL POSSIBLE OUTCOMES OF EXPERIMENT, S SAMPLE SPACE

EVENT = SUBSET OF S, e.g. $A \subseteq S$.

$P(A)$ MEASURES LIKELIHOOD

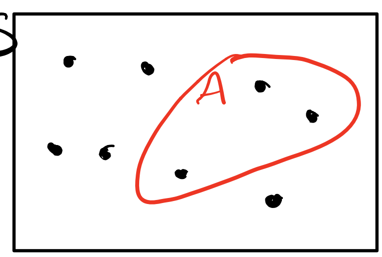
ONLY ONE WAY FOR IT TO OCCUR.

IF S IS FINITE & ALL SIMPLE EVENTS ALL EQUALLY LIKELY,

THEN

$$P(A) = \frac{\#(A)}{\#(S)}$$

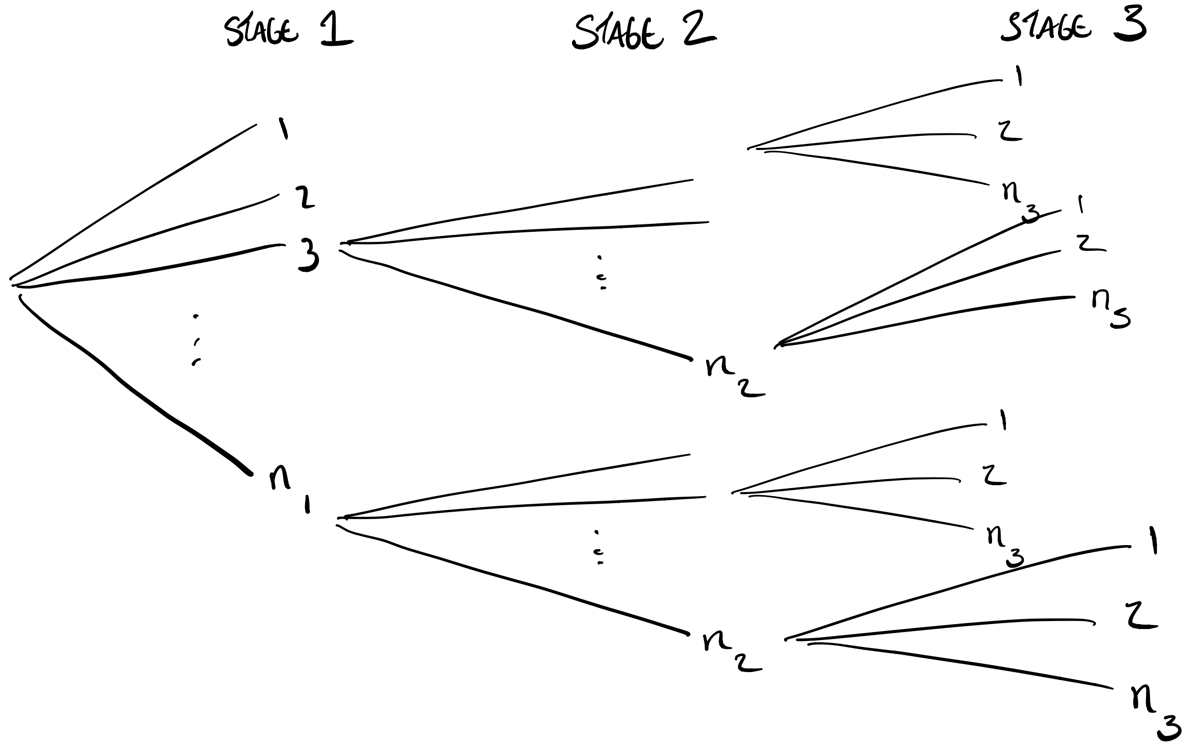
S



$P(A) = \frac{3}{8}$

COUNTING

PRODUCT RULE (GENERAL mn RULE)



TOTAL # POSSIBLE OUTCOMES $= n_1 n_2 n_3$

PERMUTATIONS $$P^n_r = \frac{n!}{(n-r)!}$$

COMBINATIONS $$C^n_r = \frac{n!}{r!(n-r)!}$$

ex. Suppose 18 club members must choose . ≤ 1 project per person

3 people to work on project A

4 people to work on project B

2 people to work on project C .

How many ways can they do this?

_ _ A _ B _ C A B

B _ _ A C B _ _ _

$${}_{18}C_3 \times {}_{15}C_4 \times {}_{11}C_2$$

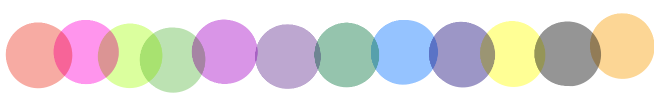
$$\cancel{{}_{18}C_3 \times {}_{18}C_4 \times {}_{18}C_2}$$

$${}_{18}P_3$$

You are given 12 tiles to arrange in a linear pattern.

How many ways can this be done if every tile is a different color? P^{12}_{12}

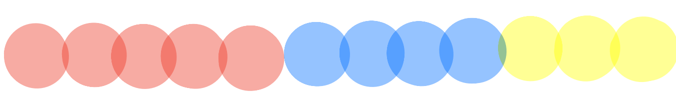
How many ways can this be done if there are 5 red tiles, 4 blue tiles, and 3 yellow tiles?



1 way

⋮ How many?

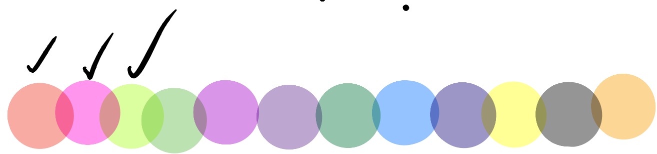
(OOOOO P^{12}_{5})

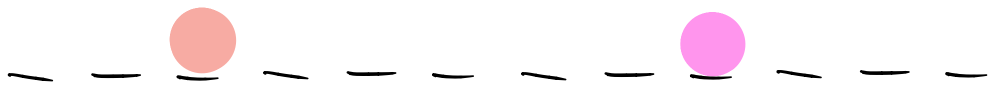


1 way

⋮ How many

$$C^{12}_{12} = \frac{12!}{12!\,0!} = 1$$



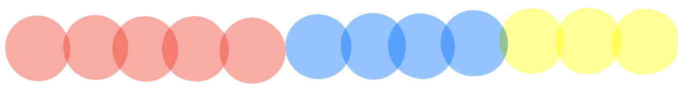


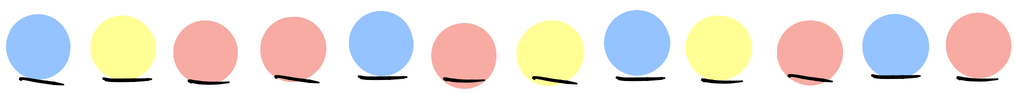
$${}_{12}C_1 \times {}_{11}C_1 \times {}_{10}C_1 \times \cdots \times {}_2C_1 \times {}_1C_1$$

$$12 \times 11 \times 10 \times \cdots \times 2 \times 1 = 12!$$

$$= P^{12}_{12}$$

479,001,600



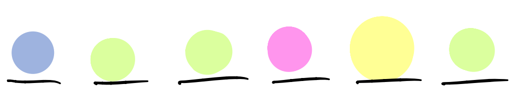


$${}_{12}C_5 \times {}_7C_4 \times \underbrace{{}_3C_3}_{1} = 27{,}720$$

HOW MANY 6-LETTER "WORDS" CAN YOU FORM FROM THE LETTERS WARBLE

$$P^6_6 = 6!$$

HOW MANY 6-LETTER "WORDS" CAN YOU FORM FROM THE LETTERS BUBBLE



$$\underset{B}{{}_6C_3} \times {}_3C_1 \times {}_2C_1 \times {}_1C_1$$

$$\underbrace{C^6_1}_{U}\,\underbrace{C^5_1}_{L}\,\underbrace{C^4_1}_{E}\,\underbrace{C^3_3}_{B} = \underbrace{{}_6C_3}_{B} \times \underbrace{{}_3C_1}_{U} \times \underbrace{{}_2C_1}_{L} \times \underbrace{{}_1C_1}_{E}$$

B $\boxed{U}$ B B $\boxed{L}$ $\boxed{E}$

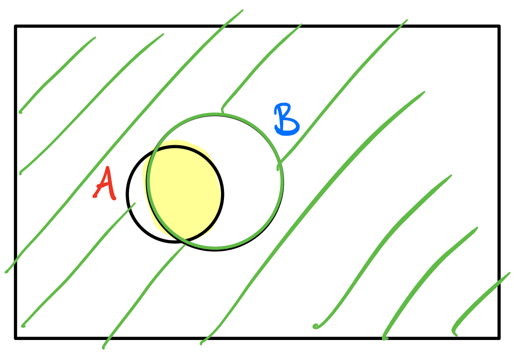
$$\frac{P^6_6}{P^3_3}$$

← # WAYS TO REARRANGE THE B's ?

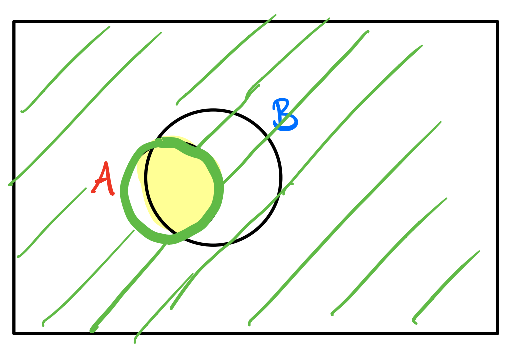
$$\frac{6!}{3!} = 6 \cdot 5 \cdot 4$$

CONDITIONAL PROB

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$



$$\frac{P(B)}{P(A)} \cdot P(A|B) = \frac{P(A \cap B)}{\cancel{P(B)}} \cdot \frac{\cancel{P(B)}}{P(A)} \rightarrow \frac{P(B)\,P(A|B)}{P(A)} = \frac{P(A \cap B)}{P(A)} = P(B|A)$$

BAYES' THM

BAYES' THM: $P(A|B) = \dfrac{P(A)P(B|A)}{P(B)}$

The definition of conditional probability, the general multiplication rule and using it to calculate probabilities, independent events, mutually exclusive events, Bayes' formula.

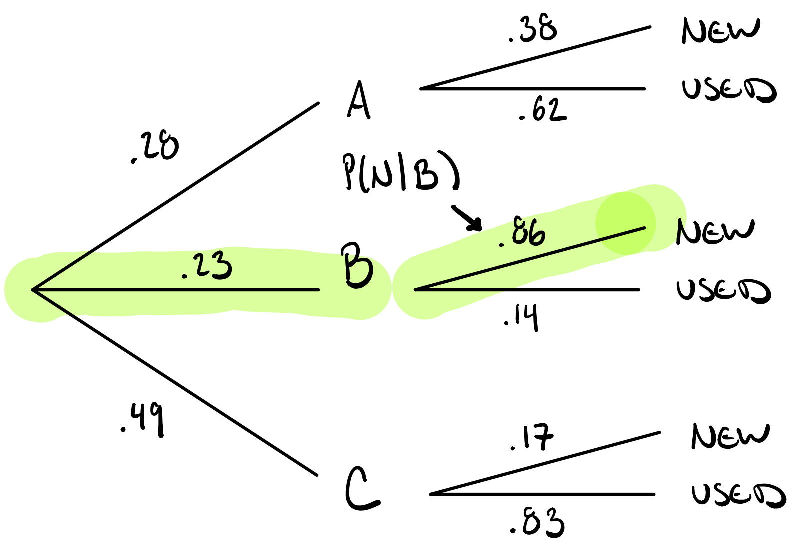
FROM SCRATCH

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \qquad P(A \cap B) = P(A)P(B|A) = P(B)P(A|B), \qquad P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

1. An experiment consists of flipping three fair 6-sided dice. One die has its faces labeled 1, 1, 1, 2, 2, 2; another die has its faces labeled 3, 3, 4, 4, 4, 4; and the third die has its faces labeled 5, 6, 6, 6, 6, 6.
   (a) What is the probability that all three dice show odd numbers?
   (b) What is the probability that exactly two of the dice show even numbers?
   (c) What is the probability that the sum of the three dice is odd?
   (d) What is the probability that two of the dice show an even number given that the sum of the three dice is odd?
   (e) Are the events "exactly two of the dice show an even number" and "the sum of the three dice is odd" independent events?
   (f) Are the events "exactly two of the dice show an even number" and "the sum of the three dice is odd" mutually exclusive events?
   (g) If the experiment is repeated 15 times, what is the probability that the sum of the dice is 11 exactly 9 times?

2. A car dealership that sells both new and used cars has three salespeople – Alexander, Brianna, and Christine.
   - 28% of all cars sold are sold by Alexander, and 38% of the cars sold by Alexander are new cars.
   - 23% of all cars sold are sold by Brianna, and 86% of the cars sold by Brianna are new cars. $P(N|B)$
   - 49% of all cars sold are sold by Christine, and 17% of the cars sold by Christine are new cars.

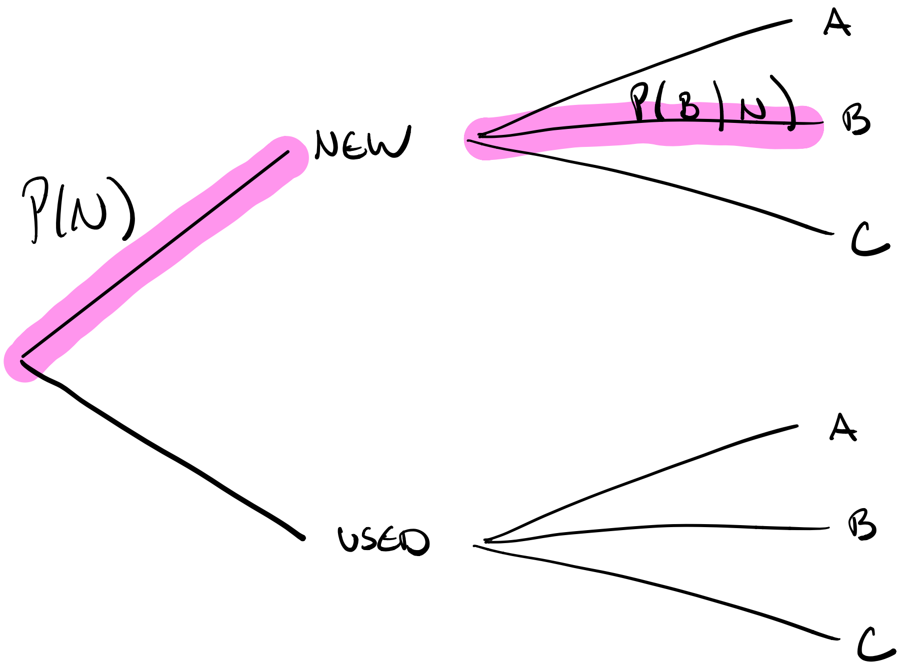
   If a customer purchases a new car from this dealership, what is the probability that Brianna sold them the car?



FIND $P(B|N)$

$P(N \cap B) = (.23)(.86) = .1978$

$$P(N \cap B) = P(N)P(B|N) = P(B)P(N|B)$$



$P(N \cap B) = .1978$

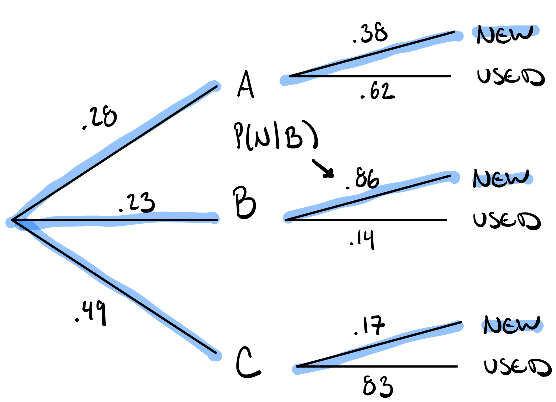
$$\frac{\cancel{P(N)}\,P(B|N)}{\cancel{P(N)}} = \frac{P(B)\,P(N|B)}{P(N)}$$

$$P(B|N) = \frac{P(B)\,P(N|B)}{P(N)}$$

(BAYES' FORMULA)

LAW OF TOTAL PROBABILITY.

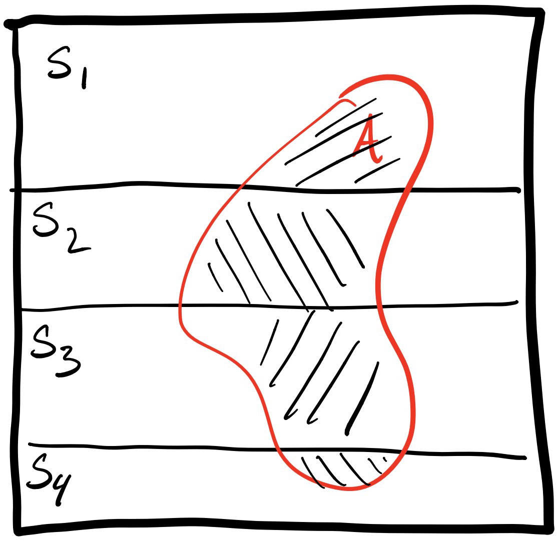
$$P(B|N) = \frac{(.23)(.86)}{(.28)(.38) + (.23)(.86) + (.49)(.17)} = \ldots$$



LAW OF TOTAL PROB.

$S_1, S_2, \ldots, S_K$

SUBSPACES / SUBPOPULATIONS



$$P(A) = P(A \cap S_1) + P(A \cap S_2) + P(A \cap S_3) + P(A \cap S_4)$$

$$P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \ldots + P(S_K)P(A|S_K)$$

Name:____ **5/22/2017**
Math 173 Introduction to Probability and Statistics **Final Exam**

Please box your final answers. Calculators are allowed, but not required. Answers may be left as fractions and/or expressions that may contain square-root ($\sqrt{\cdot}$), factorial (!), permutation (P_r^n), and combination (C_r^n) notation.

1. You are given a sample of $n = 12$ measurements: $\{5, 7, 2, 7, 2, 3, 6, 9, 3, 4, 5, 7\}$.

   (a) (4 points) What is the median m?

   (b) (4 points) What is the mean $\bar{x}$?

   (c) (4 points) What is the mode M?

   (d) (4 points) What is the variance s^2?

   (e) (4 points) What is the standard deviation s?

2. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c
B	.16	.04
B^c	.64	.16

(a) (4 points) Find $P(A \cup B)$.

(b) (4 points) Find $P(B|A^c)$

(c) (4 points) Are A and B independent? Why?

(d) (4 points) Are A and B mutually exclusive? Why?

3. Suppose that on a cold day 50% of children wear a hat, 40% of children wear mittens, and 30% of children wear neither a hat nor mittens. One child is selected randomly.

(a) (4 points) What is the probability that this child wears a hat and mittens?

(b) (4 points) What is the probability that this child wears mittens, given that this child wears a hat?

4. An urn contains 4 red marbles, 5 white marbles, and 6 blue marbles. 2 marbles are randomly selected from the urn, without replacement.
   (a) (4 points) Find the probability that the two marbles selected are the same color.

   (b) (4 points) Find the probability that the two marbles selected are different colors.

5. (8 points) An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability .4, whereas this probability decreases to .2 for a non-accident-prone person. If we assume that 30% of the population is accident prone, what is the probability that a new policyholder will have an accent within a year of purchasing a policy.

6. (8 points) A laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1% of the healthy persons tested. (That is, if a healthy person is tested, then, with probability .01, the result will imply he or she has the disease.) If 0.5% of the population actually has the disease, what is the probability a person has the disease given that the test result is positive?

7. A home security system is designed to have a 99% reliability rate. Suppose that 9 homes equipped with this system experience an attempted burglary. Find the probabilities of the following events.

   (a) (4 points) More than seven of the alarms are triggered.

   (b) (4 points) Eight or fewer of the alarms are triggered.

8. (8 points) 3 animals are to be randomly selected from a group of 3 goats and 4 sheep. Let x be the number of goats selected. Fill in the following chart with all possible values of the random variable x, along with the corresponding probabilities $p(x)$.

x	
$p(x)$	

9. (8 points) From experience, a shipping company knows that the cost of delivering a small package within 24 hours is \$14. The company charges \$16 for shipment but guarantees to refund the charge if delivery is not made within 24 hours. If the company fails to deliver only 2% of its packages within the 24-hour period, what is the expected gain/profit per package?

10. Suppose a fair die with faces labeled 1-6 is rolled n times. Let x be the number of times that a 6 is rolled.
    (a) (4 points) When $n = 8$, find $P(x \leq 1)$ exactly.
    (b) (4 points) When $n = 180$, use a normal distribution to approximate $P(x \geq 38)$.

11. (8 points) A random sample of 130 human body temperatures, provided by Allen Shoemaker in the Journal of Statistical Education, had a mean of 98.25° and a standard deviation of 0.73°. Construct a 99% confidence interval for the average body temperature of healthy people, and briefly explain what a 99% confidence interval is.

12. (8 points) The first day of baseball comes in late March, ending in October with the World Series. Does fan support grow as the season goes on? Two CNN/USA Today/Gallup polls, one conducted in March and one in November, both involved random samples of 1001 adults aged 18 and older. In the March sample, 45% of the adults claimed to be fans of professional baseball, while 51% of the adults in the November sample claimed to be fans. Construct a 99% confidence interval for the difference in the proportion of adults who claim to be fans in March versus November.

13. Suppose a scheduled airline flight must average at least 60% occupancy in order to be profitable to the airline. An examination of the occupancy rate for 120 10:00 A.M. flights from Atlanta to Dallas showed a mean occupancy per flight of 58% and a standard deviation of 11%.

    (a) (2 points) If μ is the mean occupancy per flight and if the company wishes to determine whether or not this scheduled flight is unprofitable, give the alternative and the null hypotheses for the test.

    (b) (2 points) Does the alternative hypothesis in part a imply a one- or two-tailed test? Explain.

    (c) (4 points) Do the occupancy data for the 120 flights suggest that this scheduled flight is unprofitable? Test using $\alpha = .05$.