

## **Material**

Exam 2 will cover the following sections and topics.

1. Section 5.2 The Binomial Probability Distribution
2. Sections 6.1-3 The normal distribution
3. Section 6.4 Normal Approximation to the Binomial Probability Distribution
4. Sections 7.4-5 Central Limit Theorem and Sampling Distribution of the Sample Mean
5. Section 7.6 Sampling Distribution of the Sample Proportion

## Practice Problems

1. Imagine two different six-sided fair dice, called die A and die B. Die A has its faces labeled 1, 1, 1, 2, 2, 3. Die B has its faces labeled 1, 2, 2, 3, 3, 3. Which of the following events is more likely? Why?

- (a) • Roll die A 5 times and roll a 2 exactly 3 times.  
 (b) • Roll die B 5 times and roll a 3 exactly 1 time.  
 (c) • Roll both dice simultaneously 5 times and roll doubles exactly 4 times.

.1563

(a)

SINGLE ROLL

|            |               |               |               |
|------------|---------------|---------------|---------------|
| A: # SHOWN | 1             | 2             | 3             |
| PRBS       | $\frac{3}{6}$ | $\frac{2}{6}$ | $\frac{1}{6}$ |

BINOMIAL!  $X = \#$  SUCCESSES

DEFINE SUCCESS = ROLL A 2

FAILURE = NOT ROLL A 2

$n = 5$  TRIALS

$$p = \frac{2}{6} = \frac{1}{3}$$

$$q = 1 - p = \frac{2}{3}$$

FIND PROBABILITY OF ROLLING THREE 2's IN 5 ROLLS

$$= P(X=3)$$

$$P(X=K) = C_K^n p^K q^{n-K}$$

$$= P(X=3) = C_3^5 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 = .1646$$

( CALC :  $5 nCr 3 \times (1/3)^3 \times (2/3)^2$  )  
 $\boxed{\text{MATH}} \rightarrow \text{PRBS} \rightarrow 3: nCr$

(b)

SINGLE ROLL

|            |               |               |               |
|------------|---------------|---------------|---------------|
| A: # SHOWN | 1             | 2             | 3             |
| PRBS       | $\frac{1}{6}$ | $\frac{2}{6}$ | $\frac{3}{6}$ |

BINOMIAL!  $X = \#$  SUCCESSES

DEFINE SUCCESS = ROLL A 3

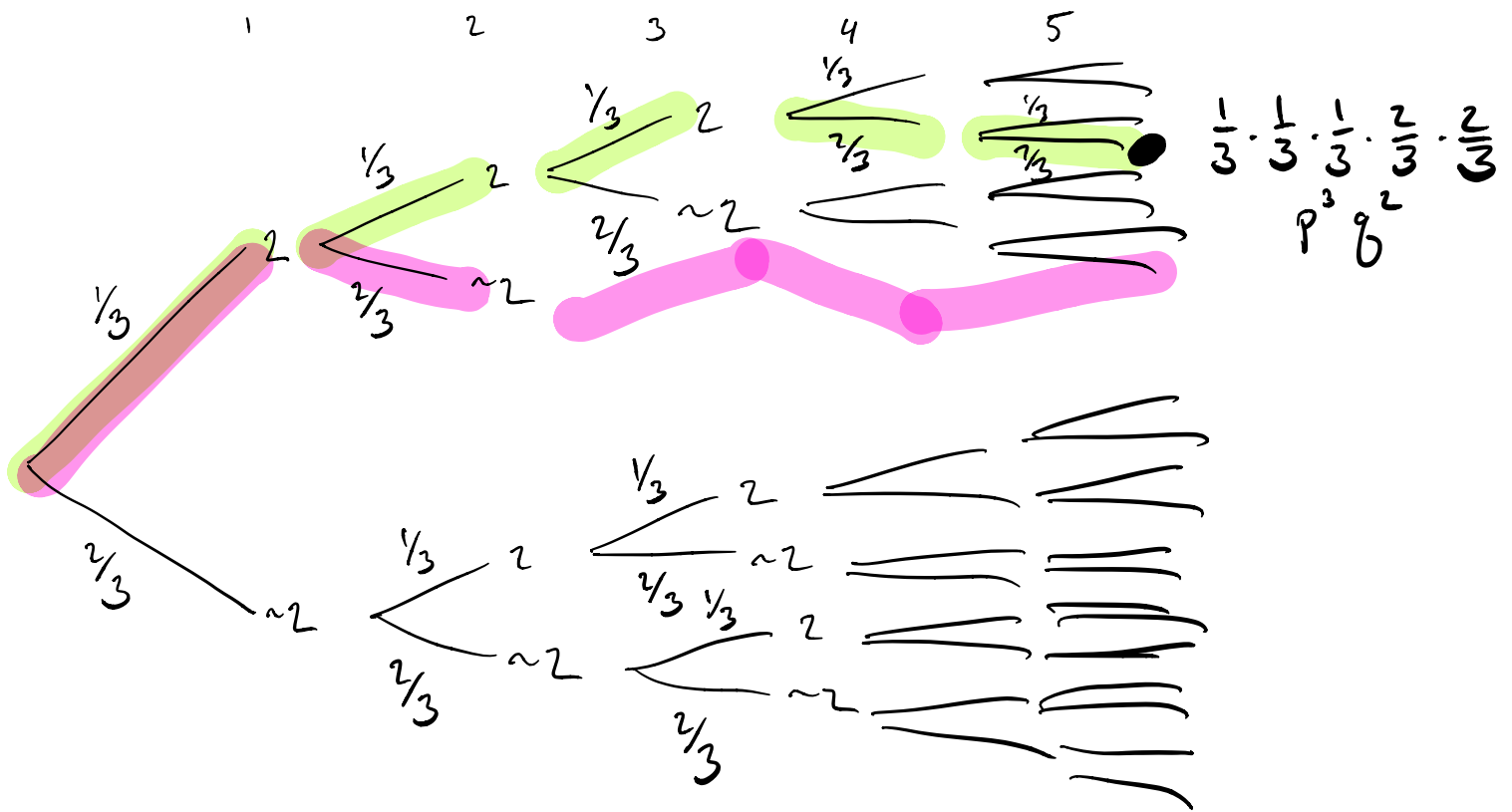
FAILURE = NOT ROLL A 3

$n = 5$  TRIALS

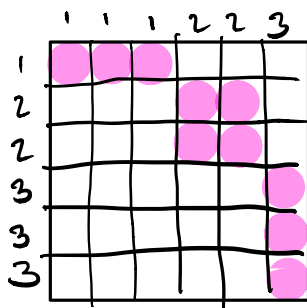
$$p = \frac{3}{6} = \frac{1}{2}$$

$$q = 1 - p = \frac{1}{2}$$

$$P(\text{ROLL ONE 3}) = P(X=1) = C_1^5 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^4 = .15625$$



(c)



$$P(\text{DOUBLES}) = \frac{10}{36} = \frac{5}{18}$$

$X = \# \text{ DOUBLES in } n=5 \text{ TRIALS}$

$$p = \frac{5}{18} \quad q = 1 - \frac{5}{18} = \frac{13}{18}$$

$$P(X=4) = C_4^5 \left(\frac{5}{18}\right)^4 \left(\frac{13}{18}\right)^1 = .0215$$

$$\text{Alt: } P(\text{DOUBLE}) = P(\{(1,1), (2,2), (3,3)\})$$

$$= P(1,1) + P(2,2) + P(3,3)$$

$$= \left(\frac{3}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{2}{6}\right)\left(\frac{2}{6}\right) + \left(\frac{1}{6}\right)\left(\frac{3}{6}\right) = \frac{5}{18}$$

2. Label each of the following random variables  $x$  as binomial, approximately binomial, or not binomial.

- (a) A single coin is flipped repeatedly until a head is observed and  $x$  is the number of flips.
- (b) Seven cards are dealt from a shuffled deck of 52 cards and  $x$  is the number of aces dealt.
- (c) Due to a pandemic, only 1 out of every 5 customers is allowed into a particular store. Sarah visits this store on 7 consecutive days and  $x$  is the number times she is allowed into the store.
- (d) A jar contains 20 marbles: 12 red and 8 blue. Jessica selects 5 marbles from the jar and  $x$  is the number of red marbles.
- (e) A jar contains 2000 marbles: 1200 red and 800 blue. Jessica selects 25 marbles from the jar and  $x$  is the number of red marbles.

(a) No, # of trials  $n$  undefined.

(b) No,

CARD BEING

ACE OR NOT

IS INDEPENDENT.

**Definition** A binomial experiment is one that has these five characteristics:

1. The experiment consists of  $n$  identical trials.
2. Each trial results in one of two outcomes. For lack of a better name, the one outcome is called a success,  $S$ , and the other a failure,  $F$ .
3. The probability of success on a single trial is equal to  $p$  and remains the same from trial to trial. The probability of failure is equal to  $(1 - p) = q$ .
4. The trials are independent.
5. We are interested in  $x$ , the number of successes observed during the  $n$  trials, for  $x = 0, 1, 2, \dots, n$ .

(c) Yes.

(d) No: trials not independent. (not identical)

(e)

#### RULE OF THUMB

If the sample size is large relative to the population size—in particular, if  $n/N \geq .05$ —then the resulting experiment is not binomial.

$$n = 25$$

$$N = 2000$$

$$\frac{25}{2000} = .0125 < .05 \quad \checkmark$$

APPROX. BINOMIAL.

$$1^{\text{st}} P(\text{red}) = \frac{1200}{2000}$$

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2<sup>nd</sup>

$$P(\text{red}) =$$

$$\begin{cases} \frac{1199}{1999} \\ \frac{1200}{1999} \end{cases}$$

IF 1<sup>st</sup> MARBLE RED

IF 1<sup>st</sup> MARBLE BLUE

NOT EXACTLY BINOMIAL

3. Use the cumulative binomial probability table below to answer this question ~~/////////~~. A small town has 1000 registered voters: 500 republican, 400 democrat, and 100 independent. Suppose 20 registered voters are randomly selected and the number of democrats  $x$  is counted.

- (a) What is the probability that at least half of them are deomrats?  
(b) What is the probability that  $10 < x \leq 16$

$$\frac{n}{N} = \frac{20}{1000} = .02 < .05 \Rightarrow \text{APPROX BINOM.}$$

$$n = 20 \quad x = \# \text{ DEMOCRATS}$$

$$p = \frac{400}{1000} = .4 \quad q = .6$$

$$\begin{aligned} (a) \quad P(x \geq 10) &= \cancel{P(x=10)} + \cancel{P(x=11)} + \dots + \cancel{P(x=20)} \\ &= 1 - P(x \leq 9) \\ &= 1 - .755 \\ &= .245 \end{aligned}$$

$$(b) \quad P(10 < x \leq 16) = P(x \leq 16) - P(x \leq 10)$$

~~0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16~~

$$= 1.000 - .872 = .128$$

TABLE 1 (continued)

 $n = 20$ 

| $k$ | $p$   |       |       |       |       |       |       |       |       |       |       |       |       | $k$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
|     | .01   | .05   | .10   | .20   | .30   | .40   | .50   | .60   | .70   | .80   | .90   | .95   | .99   |     |
| 0   | .818  | .358  | .122  | .012  | .001  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 0   |
| 1   | .983  | .736  | .392  | .069  | .008  | .001  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 1   |
| 2   | .999  | .925  | .677  | .206  | .035  | .004  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 2   |
| 3   | 1.000 | .984  | .867  | .411  | .107  | .016  | .001  | .000  | .000  | .000  | .000  | .000  | .000  | 3   |
| 4   | 1.000 | .997  | .957  | .630  | .238  | .051  | .006  | .000  | .000  | .000  | .000  | .000  | .000  | 4   |
| 5   | 1.000 | 1.000 | .989  | .804  | .416  | .126  | .021  | .002  | .000  | .000  | .000  | .000  | .000  | 5   |
| 6   | 1.000 | 1.000 | .998  | .913  | .608  | .250  | .058  | .006  | .000  | .000  | .000  | .000  | .000  | 6   |
| 7   | 1.000 | 1.000 | 1.000 | .968  | .772  | .416  | .132  | .021  | .001  | .000  | .000  | .000  | .000  | 7   |
| 8   | 1.000 | 1.000 | 1.000 | .990  | .887  | .596  | .252  | .057  | .005  | .000  | .000  | .000  | .000  | 8   |
| 9   | 1.000 | 1.000 | 1.000 | .997  | .952  | .755  | .412  | .128  | .017  | .001  | .000  | .000  | .000  | 9   |
| 10  | 1.000 | 1.000 | 1.000 | .999  | .983  | .872  | .588  | .245  | .048  | .003  | .000  | .000  | .000  | 10  |
| 11  | 1.000 | 1.000 | 1.000 | 1.000 | .995  | .943  | .748  | .404  | .113  | .010  | .000  | .000  | .000  | 11  |
| 12  | 1.000 | 1.000 | 1.000 | 1.000 | .999  | .979  | .868  | .584  | .228  | .032  | .000  | .000  | .000  | 12  |
| 13  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .994  | .942  | .750  | .392  | .087  | .002  | .000  | .000  | 13  |
| 14  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .998  | .979  | .874  | .584  | .196  | .011  | .000  | .000  | 14  |
| 15  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .994  | .949  | .762  | .370  | .043  | .003  | .000  | 15  |
| 16  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .999  | .984  | .893  | .589  | .133  | .016  | .000  | 16  |
| 17  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .996  | .965  | .794  | .323  | .075  | .001  | 17  |
| 18  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .999  | .992  | .931  | .608  | .264  | .017  | 18  |
| 19  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .999  | .988  | .878  | .642  | .182  | 19  |
| 20  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 20  |

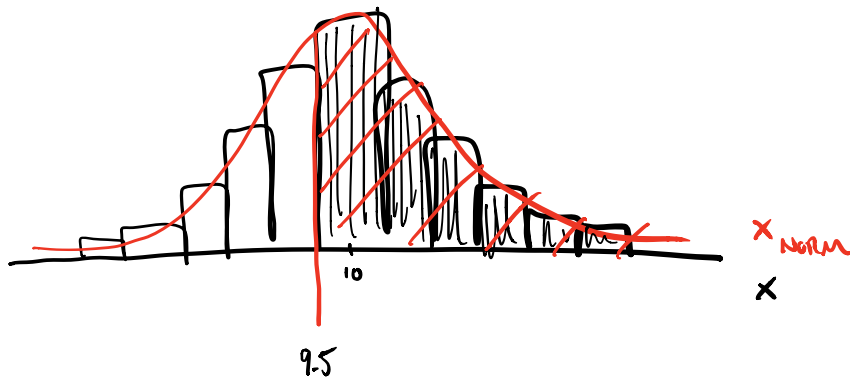
$$P(X \leq k)$$

4. Use a normal approximation to answer this question *approximately*. A small town has 1000 registered voters: 500 republican, 400 democrat, and 100 independent. Suppose 20 registered voters are randomly selected and the number of democrats  $x$  is counted.

- (a) What is the probability that at least half of them are democrats?  
 (b) What is the probability that  $10 < x \leq 16$

Justification:  $np > 5$   $ng > 5$

$$(20)(.4) = 8 > 5 \quad (20)(.6) = 12 > 5 \quad \checkmark$$



Binomial:  $\mu = np$   
 $\sigma = \sqrt{npq}$

$$Z = \frac{\text{RV.} - \text{MEAN}}{\text{STAND. DEV.}}$$

$$P(x \geq 10) \approx P(x_{\text{norm}} \geq 9.5)$$

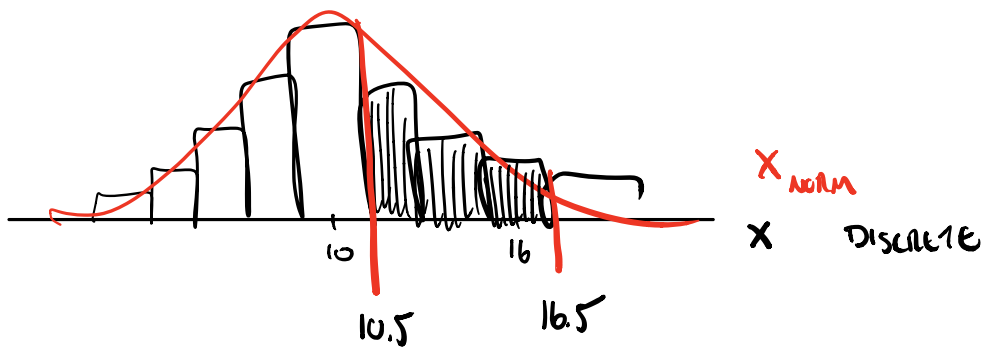
$$Z = \frac{x - np}{\sqrt{npq}}$$

$$= P\left(z \geq \frac{9.5 - (20)(.4)}{\sqrt{(20)(.4)(.6)}}\right)$$

.684653

$$= P(z \geq .68) = 1 - P(z \leq .68)$$

$$= 1 - .7517 = .2483 \quad \text{close to } .245!$$



$$P(10 < X \leq 16) \approx P(10.5 \leq X_{\text{NORM}} \leq 16.5) =$$

$$= P\left(z \leq \frac{16.5 - np}{\sqrt{npq}}\right) - P\left(z \leq \frac{10.5 - np}{\sqrt{npq}}\right)$$

$z \dots$



5. Each week, a particular charity collects  $x$  dollars in donations. The mean and standard deviation for  $x$  are 2,675 and 345, respectively. Assume  $x$  is normally distributed.

- (a) What is the probability that this charity raises more than 3,000? ~~next week~~
- (b) Assuming that the amount of money donated each week is independent of the amount of money donated every other week, what is the probability that this charity raises more than 3,000 each week for the next 3 weeks?
- (c) What is the probability that this charity averages more than 3,000 ~~in~~ <sup>WEEK</sup> donations over the next 3 weeks?

$$\begin{aligned}
 (a) \quad P(x \geq 3000) &= P\left(z \geq \frac{3000 - 2675}{345}\right) \\
 &= 1 - P\left(z \leq \frac{3000 - 2675}{345}\right) \\
 &= 1 - P(z \leq .94) \\
 &= 1 - .8654 = .1346
 \end{aligned}$$

$$(b) \quad (.1346)(.1346)(.1346) = .0024$$

$$2 \text{ out of } 3 \text{ weeks: } C_2^3 (.1346)^2 (.8654)^1 = .0470$$

(c)  $\bar{x}$  = SAMPLE MEAN

$n$  = SAMPLE SIZE 3

CENTRAL LIMIT THM:

$\bar{x}$  is Normal MEAN =  $\mu = 2675$

$$S.E. = \frac{\sigma}{\sqrt{n}} = \frac{345}{\sqrt{3}}$$

$$P(\bar{x} \geq 3000) = P\left(z \geq \frac{3000 - 2675}{345/\sqrt{3}}\right)$$

$$= P(z \geq 1.63) = 1 - P(z \leq 1.63) = 1 - .9484$$

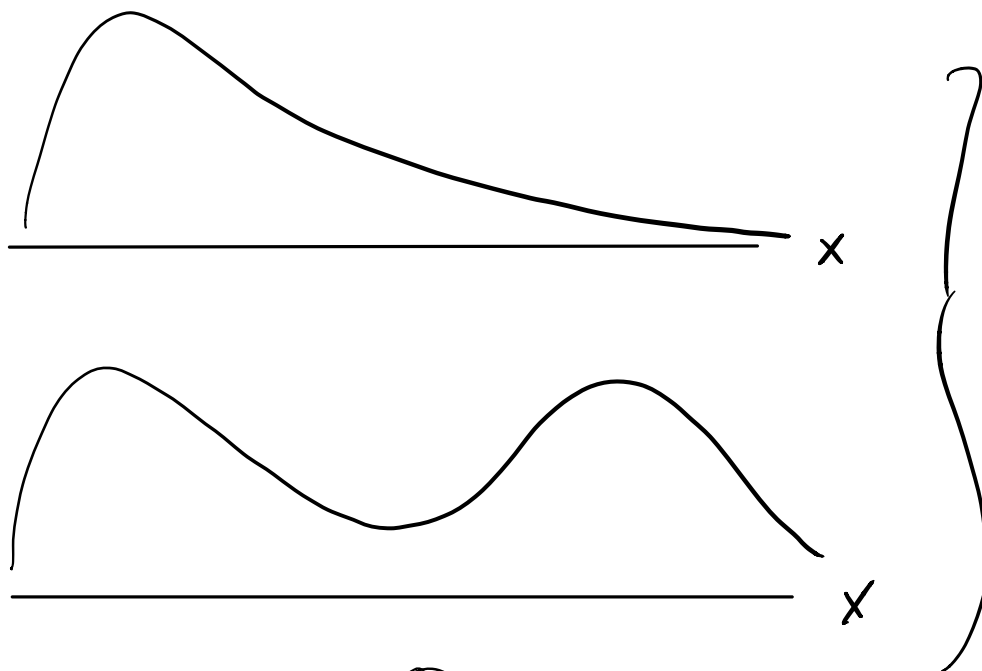
$$= .0516$$

6. John is a math professor and a big fan of data. Everytime he grades an exam he records the number of minutes  $x$  it takes to grade it. He finds that the mean and standard deviation for  $x$  are 8 min and 3 min, respectively. Answer one of the following questions and explain why the other question is impossible to answer.

- (a) What is the probability that the next exam takes longer than 10 minutes and 30 seconds to grade?  
 (b) What is the probability that the next 35 exams take longer than 5 hours to grade?

$$(a) P(x \geq 10.5) = P(z \geq \frac{10.5 - 8}{3})$$

$$= P(z \geq .83) = \dots$$



DISTRIBUTION MIGHT NOT  
LOOK LIKE THIS

IMPOSSIBLE TO ANSWER.

(b)

## THE SAMPLING DISTRIBUTION OF THE SAMPLE MEAN, $\bar{x}$

- If a random sample of  $n$  measurements is selected from a population with mean  $\mu$  and standard deviation  $\sigma$ , the sampling distribution of the sample mean  $\bar{x}$  will have mean  $\mu$  and standard deviation\*

$$\frac{\sigma}{\sqrt{n}}$$

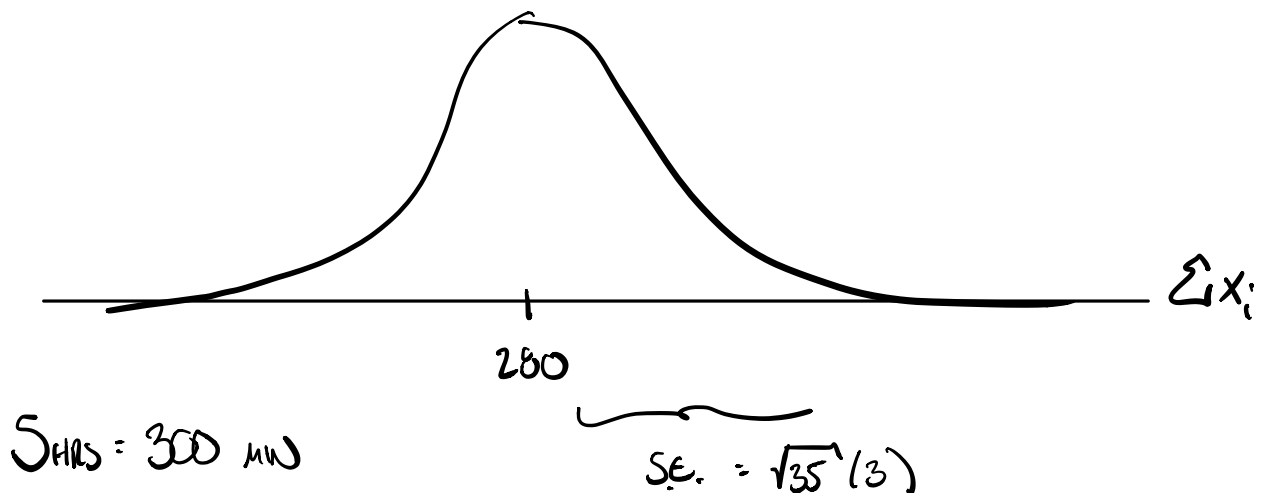
- If the population has a *normal* distribution, the sampling distribution of  $\bar{x}$  will be *exactly* normally distributed, *regardless of the sample size,  $n$* .
- If the population distribution is *nonnormal*, the sampling distribution of  $\bar{x}$  will be *approximately* normally distributed for large samples (by the Central Limit Theorem). Conservatively, we require  $n \geq 30$ .

? Q  
Sic

The Central Limit Theorem can be restated to apply to the **sum of the sample measurements  $\sum x_i$** , which, as  $n$  becomes large, also has an approximately normal distribution with mean  $n\mu$  and standard deviation  $\sigma\sqrt{n}$ .

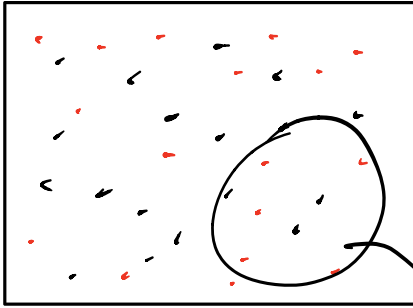
SAMPLE SIZE  $n = 35$

SAMPLE SUM  $\sum x_i \longrightarrow$  MEAN IS  $n\mu = 35(8) = 280$  mm  
S.E. IS  $\sqrt{n}\sigma = \sqrt{35}(3)$  mm



$$\begin{aligned} P(\sum x_i \geq 300) &= P\left(z \geq \frac{300 - 280}{3\sqrt{35}}\right) \\ &= P(z \geq 1.13) \\ &= 1 - P(z \leq 1.13) \\ &= 1 - .8708 = .1292 \end{aligned}$$

7. Use the central limit theorem to answer this question. In the United States, 23% of all adults have arthritis. If a sample of 363 American adults are randomly selected, what is the probability that fewer than 20% of the sample has arthritis? Note: Without the first sentence, there would actually be 3 legitimate ways to answer this question!



Proportion of Population with Arthritis  
is  $p = .23$

SAMPLE OF SIZE  $n = 363$

Proportion of SAMPLE with Arthritis  
is  $\hat{p}$ .

**PROPERTIES OF THE SAMPLING DISTRIBUTION OF THE SAMPLE PROPORTION,  $\hat{p}$**

- If a random sample of  $n$  observations is selected from a binomial population with parameter  $p$ , then the sampling distribution of the sample proportion

$$\hat{p} = \frac{x}{n}$$

will have a mean

$$p$$

and a standard deviation

$$SE(\hat{p}) = \sqrt{\frac{pq}{n}} \quad \text{where } q = 1 - p$$

- When the sample size  $n$  is large, the sampling distribution of  $\hat{p}$  can be approximated by a normal distribution. The approximation will be adequate if  $np > 5$  and  $nq > 5$ .

$$P(\hat{p} \leq .20) \approx P\left(z \leq \frac{.20 - .23}{\sqrt{\frac{(.23)(.77)}{363}}}\right)$$

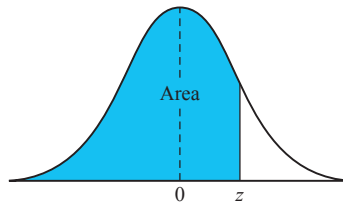
$$z = \frac{\text{N.V.} - \text{MEAN}}{\text{STAND. DEV.}} \rightarrow z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= P(z \leq -1.36) = .0869$$

$n = 25$

| $k$ | $p$   |       |       |       |       |       |       |       |       |       |       |       |       | $k$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|
|     | .01   | .05   | .10   | .20   | .30   | .40   | .50   | .60   | .70   | .80   | .90   | .95   | .99   |     |
| 0   | .778  | .277  | .072  | .004  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 0   |
| 1   | .974  | .642  | .271  | .027  | .002  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 1   |
| 2   | .998  | .873  | .537  | .098  | .009  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 2   |
| 3   | 1.000 | .966  | .764  | .234  | .033  | .002  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 3   |
| 4   | 1.000 | .993  | .902  | .421  | .090  | .009  | .000  | .000  | .000  | .000  | .000  | .000  | .000  | 4   |
| 5   | 1.000 | .999  | .967  | .617  | .193  | .029  | .002  | .000  | .000  | .000  | .000  | .000  | .000  | 5   |
| 6   | 1.000 | 1.000 | .991  | .780  | .341  | .074  | .007  | .000  | .000  | .000  | .000  | .000  | .000  | 6   |
| 7   | 1.000 | 1.000 | .998  | .891  | .512  | .154  | .022  | .001  | .000  | .000  | .000  | .000  | .000  | 7   |
| 8   | 1.000 | 1.000 | 1.000 | .953  | .677  | .274  | .054  | .004  | .000  | .000  | .000  | .000  | .000  | 8   |
| 9   | 1.000 | 1.000 | 1.000 | .983  | .811  | .425  | .115  | .013  | .000  | .000  | .000  | .000  | .000  | 9   |
| 10  | 1.000 | 1.000 | 1.000 | .994  | .902  | .586  | .212  | .034  | .002  | .000  | .000  | .000  | .000  | 10  |
| 11  | 1.000 | 1.000 | 1.000 | .998  | .956  | .732  | .345  | .078  | .006  | .000  | .000  | .000  | .000  | 11  |
| 12  | 1.000 | 1.000 | 1.000 | 1.000 | .983  | .846  | .500  | .154  | .017  | .000  | .000  | .000  | .000  | 12  |
| 13  | 1.000 | 1.000 | 1.000 | 1.000 | .994  | .922  | .655  | .268  | .044  | .002  | .000  | .000  | .000  | 13  |
| 14  | 1.000 | 1.000 | 1.000 | 1.000 | .998  | .966  | .788  | .414  | .098  | .006  | .000  | .000  | .000  | 14  |
| 15  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .987  | .885  | .575  | .189  | .017  | .000  | .000  | .000  | 15  |
| 16  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .996  | .946  | .726  | .323  | .047  | .000  | .000  | .000  | 16  |
| 17  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .999  | .978  | .846  | .488  | .109  | .002  | .000  | .000  | 17  |
| 18  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .993  | .926  | .659  | .220  | .009  | .000  | .000  | 18  |
| 19  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .998  | .971  | .807  | .383  | .033  | .001  | .000  | 19  |
| 20  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .991  | .910  | .579  | .098  | .007  | .000  | 20  |
| 21  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .998  | .967  | .766  | .236  | .034  | .000  | 21  |
| 22  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .991  | .902  | .463  | .127  | .002  | 22  |
| 23  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .998  | .973  | .729  | .358  | .026  | 23  |
| 24  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | .996  | .928  | .723  | .222  | 24  |
| 25  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 25  |

Figure 1: Cumulative binomial probability distribution,  $P(x \leq k)$



**TABLE 3** Areas under the Normal Curve

| z    | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0017 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0722 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

TABLE 3 (continued)

| z   | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |