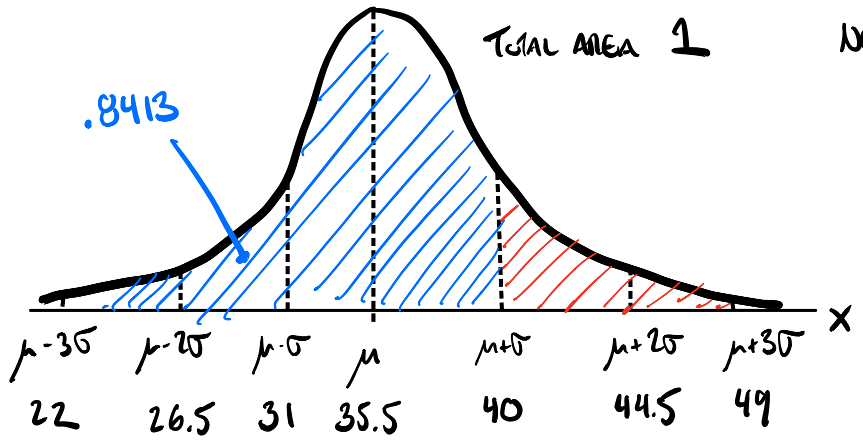
§6.3 CONTINUED

Example. Studies show that gasoline efficiently x for compact cars is normally distributed with mean 35.5 mpg and a standard deviation 4.5 mpg.

1. What percentage of compact cars get 40 mpg or more?



TOTAL AREA 1

NORMAL WITH $\mu = 35.5$

$$z = \frac{x-\mu}{\sigma}$$

COMPLEMENTARY EVENTS

$$P(x \geq 40) = P(z \geq 1) = 1 - P(z \leq 1)$$

$$z = \frac{40 - 35.5}{4.5}$$

$$= 1 - .8413 = .1587$$

15.87%

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5 871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

2. Find the percent of compact cars with gas mileage above 50mpg.

Z NORMAL WITH $\mu=0$, $\sigma=1$

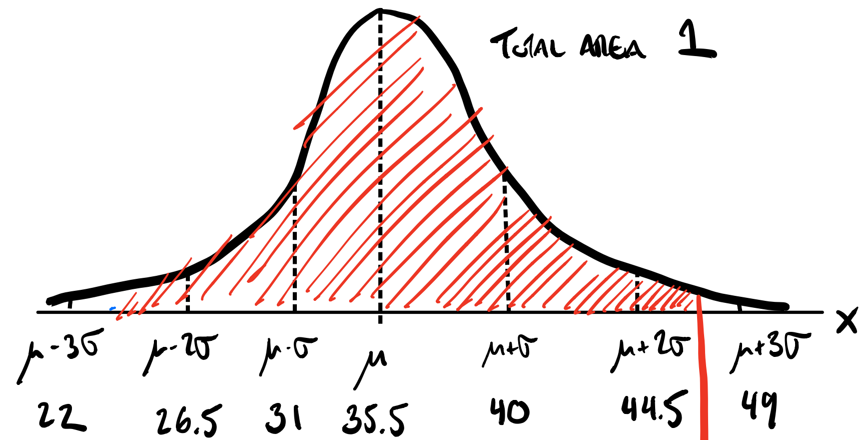
$$P(x \geq 50) \xrightarrow[z = \frac{x-\mu}{\sigma}]{\text{STANDARDIZE}} P\left(z \geq \frac{50-35.5}{4.5}\right)$$

$$= P(z \geq 3.2\overline{2}) = 1 - P(z \leq 3.22)$$

$$= 1 - .9994 = .0006 \rightsquigarrow 0.06\%$$

	.00	.01	.02							
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Still assume that gas efficiency x for compact cars is normally distributed with mean 35.5 and standard deviation 4.5. What gas mileage must a compact car get in order to get better gas mileage than 95% of compact cars?



$x_0 = 42.9025$

FIND x_0 SUCH THAT

$P(x \le x_0) = .95$

UNIT FOR x IS mpg

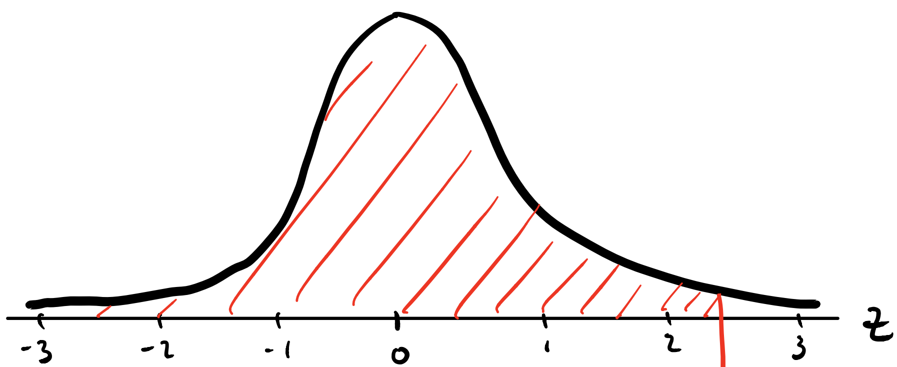
$z = \frac{x - \mu}{\sigma} \quad \frac{mpg}{mpg}$

z = # STANDARD DEV ABOVE OR BELOW THE MEAN

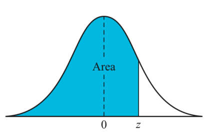
FIND z_0 SUCH THAT

$P(z \le z_0) = .95$

FIND THIS PROB. IN BODY OF THE TABLE
READ OFF z_0.



$z_0 = 1.645$



Z-VALUES HEADERS

TABLE 3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048

PROBABILITIES IN BODY

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5 871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	[.9495 .95	.9505]	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

$P(z \le 1.64) = .9495$ $\qquad$ $P(z \le 1.65) = .9505$

.95 IS EXACTLY IN BETWEEN
$z = 1.64$ & $z = 1.65$

$P(z \le z_0) = .95 \leadsto \boxed{z_0 = 1.645}$

$P(z \le 1.645) \approx .95$

$z = \frac{x - \mu}{\sigma}$

$P(x \le x_0) = .95$

$$1.645 = \frac{x - 35.5}{4.5}$$

$$(4.5)(1.645) = x - 35.5$$

$$35.5 + (4.5)(1.645) = x$$

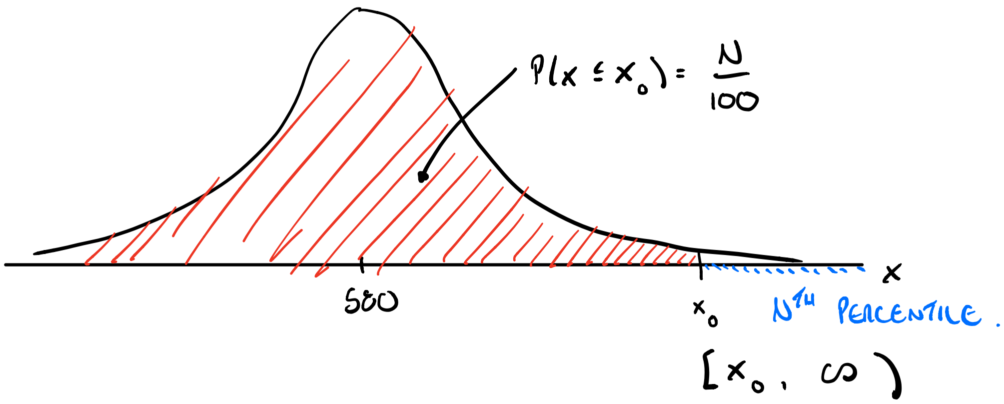
$$42.9025 = x$$

$P(x \le 42.9025) = .95$

SAT scores are normally distributed with mean 580 and standard deviation 50. How high of a score must you get to be in the 99th percentile of SAT scores?

IN GENERAL A NUMBER X IS SAID TO BE IN THE N^{TH} PERCENTILE

IF $x \geq x_0$ & $P(x \leq x_0) = N\%$.



CUT OFF FOR 99^{TH} PERCENTILE

FIND x_0 SUCH THAT $P(x \leq x_0) = .99$

FIND z_0 SUCH THAT $P(z \leq z_0) = .99$

GIVEN PROBABILITY (FIND IN BODY OF TABLE)

FIND Z-SCORE (LOOK AT COLUMN & ROW)

	.00	.01	.02	.03						
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

PROBABILITY

$z_0 = 2.33$

$P(z \leq 2.33) \approx .99$

$$z = \frac{x - \mu}{\sigma}$$

$z_0 = 2.33$

$$z_0 = \frac{x_0 - \mu}{\sigma}$$

$$\sigma z_0 = x_0 - \mu$$

$$x_0 = \mu + z_0 \sigma$$

$$x_0 = 580 + 2.33(50) = 696.5$$

§6.4 The Normal Approximation to the Binomial Distribution

Let x be a binomial random variable with n trials,

probability of success p on each trial (independent, identical)

probability of failure $q = 1 - p$.

X = # successes in n trials.

$$P(x = k) = C^n_k \, p^k q^{n-k}, \qquad 0 \le k \le n$$

ex. Suppose you flip a coin $n = 25$ times.

Find the prob of observing no more than 8 heads.

$$P(x \le 8) = P(x=0) + P(x=1) + P(x=2) + \ldots + P(x=8)$$

↳ use table $\sim .054$

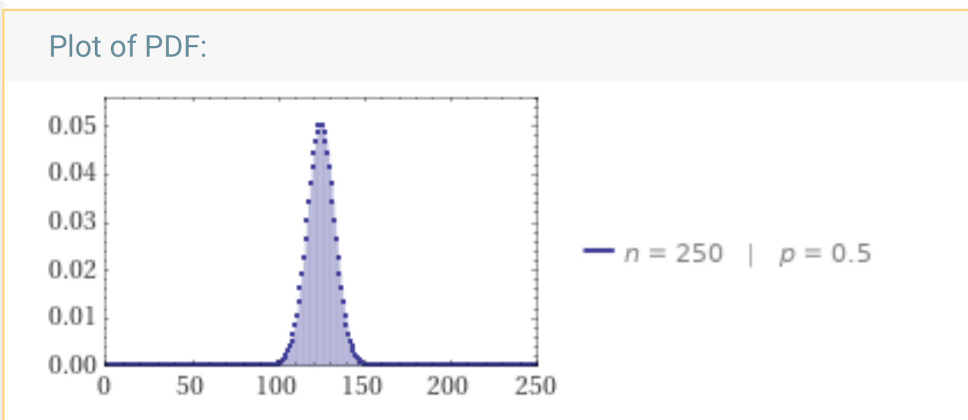
TABLE 1 *(continued)*

$n = 25$

						p							
k	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	k
0	.778	.277	.072	.004	.000	.000	.000	.000	.000	.000	.000	.000	.000	0
1	.974	.642	.271	.027	.002	.000	.000	.000	.000	.000	.000	.000	.000	1
2	.998	.873	.537	.098	.009	.000	.000	.000	.000	.000	.000	.000	.000	2
3	1.000	.966	.764	.234	.033	.002	.000	.000	.000	.000	.000	.000	.000	3
4	1.000	.993	.902	.421	.090	.009	.000	.000	.000	.000	.000	.000	.000	4
5	1.000	.999	.967	.617	.193	.029	.002	.000	.000	.000	.000	.000	.000	5
6	1.000	1.000	.991	.780	.341	.074	.007	.000	.000	.000	.000	.000	.000	6
7	1.000	1.000	.998	.891	.512	.154	.022	.001	.000	.000	.000	.000	.000	7
8	1.000	1.000	1.000	.953	.677	.274	.054	.004	.000	.000	.000	.000	.000	8
9	1.000	1.000	1.000	.983	.811	.425	.115	.013	.000	.000	.000	.000	.000	9
10	1.000	1.000	1.000	.994	.902	.586	.212	.034	.002	.000	.000	.000	.000	10
11	1.000	1.000	1.000	.998	.956	.732	.345	.078	.006	.000	.000	.000	.000	11
12	1.000	1.000	1.000	1.000	.983	.846	.500	.154	.017	.000	.000	.000	.000	12
13	1.000	1.000	1.000	1.000	.994	.922	.655	.268	.044	.002	.000	.000	.000	13
14	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.006	.000	.000	.000	14
15	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.017	.000	.000	.000	15
16	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.047	.000	.000	.000	16
17	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.109	.002	.000	.000	17
18	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.220	.009	.000	.000	18
19	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.383	.033	.001	.000	19
20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.579	.098	.007	.000	20
21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.766	.236	.034	.000	21
22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.902	.463	.127	.002	22
23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.973	.729	.358	.026	23
24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.928	.723	.222	24
25	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	25

SUPPOSE YOU FLIP A COIN $n = 250$ TIMES.

FIND THE PROB OF OBSERVING NO MORE THAN 110 HEADS?



MY GRADES

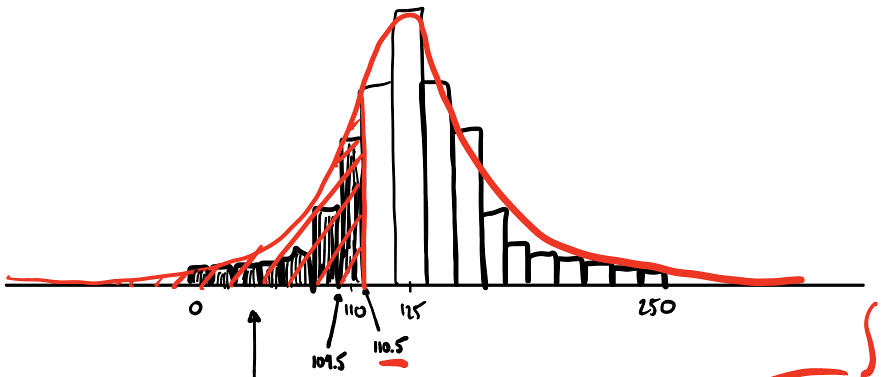
CLICK → SCORE

THE BINOMIAL DISTRIBUTION IS APPROXIMATELY NORMAL WITH MEAN STANDARD DEVIATION

§5.2

$\mu = np$

$\sigma = \sqrt{npq}$



X_{BIN}, n TRIALS, $P(\text{SUCCESS}) = p$

X_{NORM} $\left\{ \begin{array}{l} \mu = np \\ \sigma = \sqrt{npq} \end{array} \right.$

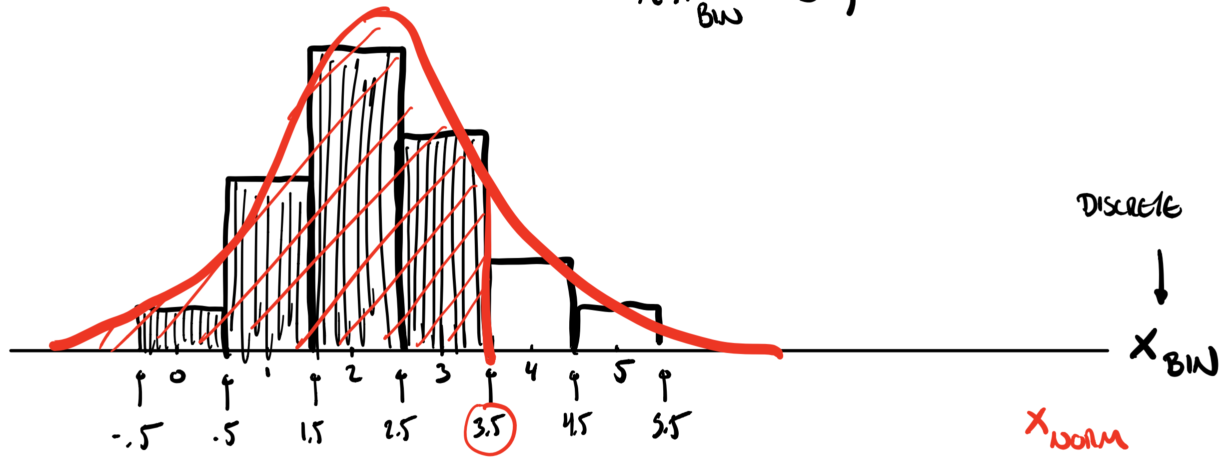
$P(X_{BIN} \le 110) =$

$$\approx P(X_{NORM} \le 110.5) = P\left(z \le \frac{110.5 - (250)(.5)}{\sqrt{(250)(.5)(.5)}}\right)$$

$$= P(z \le -1.83) = .0336$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

$P(X_{BIN} \le 3)$



EDGES OF RECTANGLE OCCUR AT INTEGERS $\pm .5$

Suppose 15% of the apples in a particular orchard are infected with mold. In a random sample of 50 apples, find the probability that more than 10 of them are infected.

LET X = # INFECTED APPLES IN A SAMPLE OF SIZE $n = 50$.

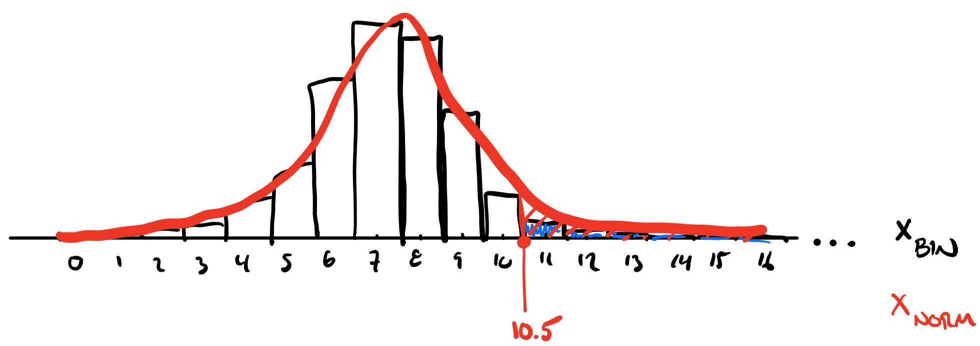
$P(\text{INFECTED}) = p = .15$

$P(\text{OK}) = q = .85$

FIND $P(X > 10)$

$\mu = np = (50)(.15) = 7.5$

$\sigma = \sqrt{npq} = \sqrt{(50)(.15)(.85)} =$



BLUE AREA $\approx$ RED AREA

DISCRETE CONTINUOUS, NORMAL

$$P(X_{BIN} > 10) \approx P(X_{NORM} \ge 10.5)$$

$$P\left(z \ge \frac{10.5 - \mu}{\sigma}\right)$$

$$P\left(z \ge \frac{10.5 - (50)(.15)}{\sqrt{(50)(.15)(.85)}}\right)$$

$$= 1 - P(z \le 1.19) = 1 - .8830 = .1170$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5 871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319