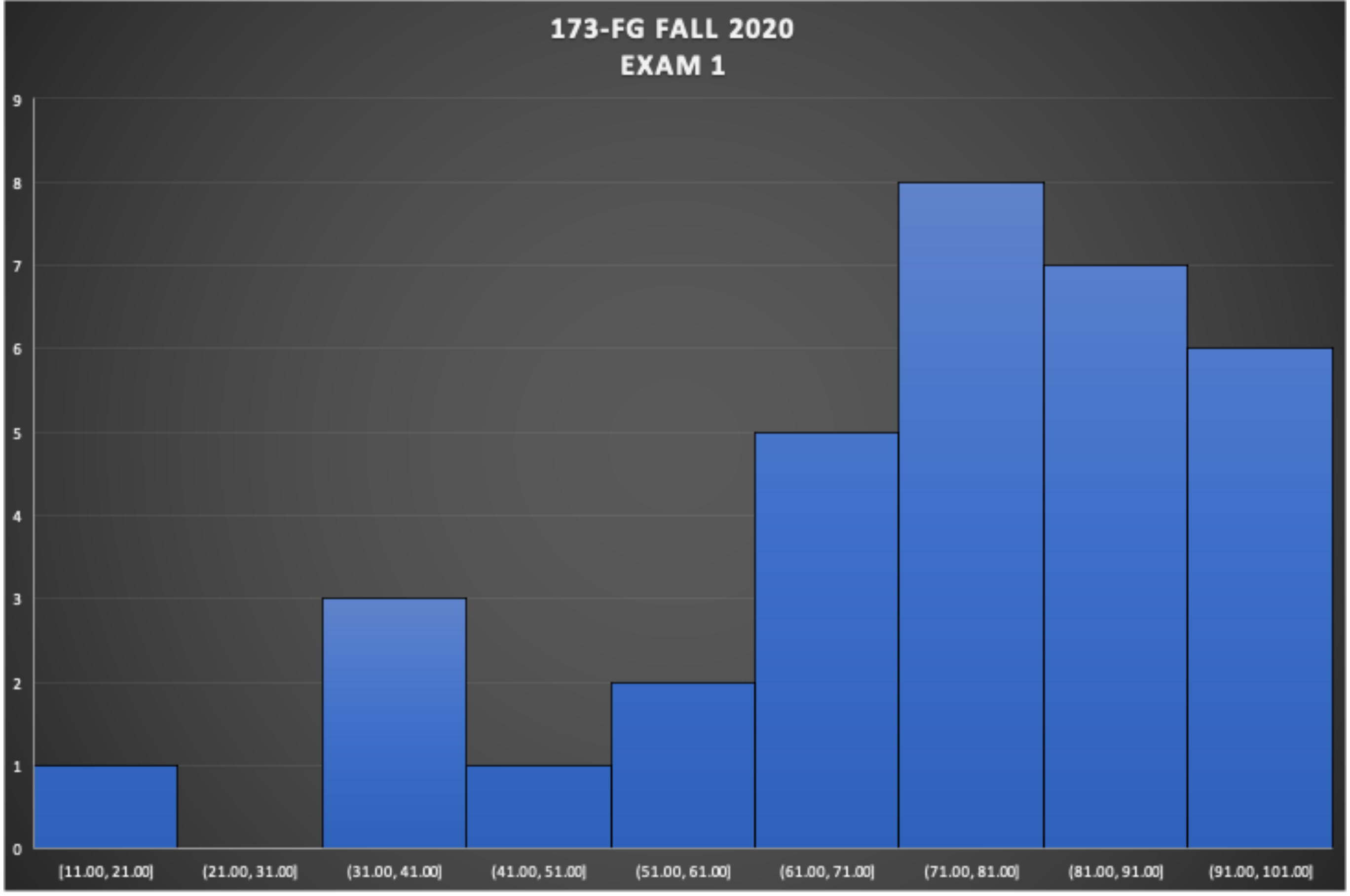
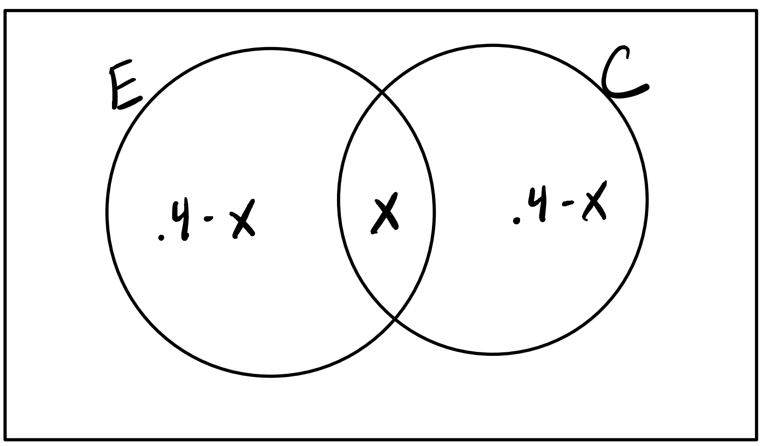


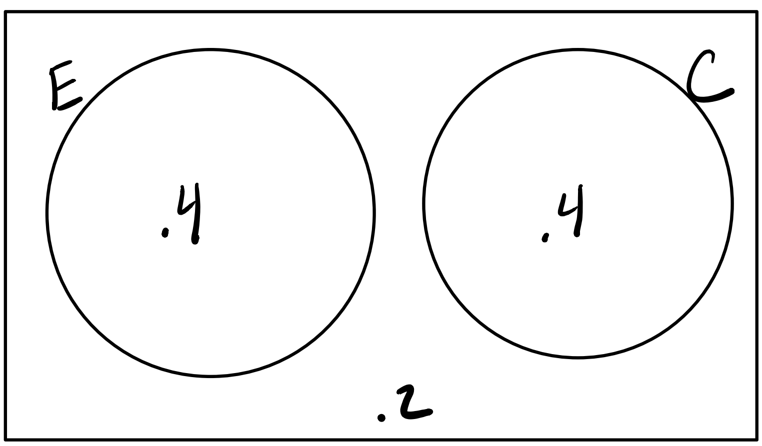
EXAM 1 Q15

$P(E) = .4$

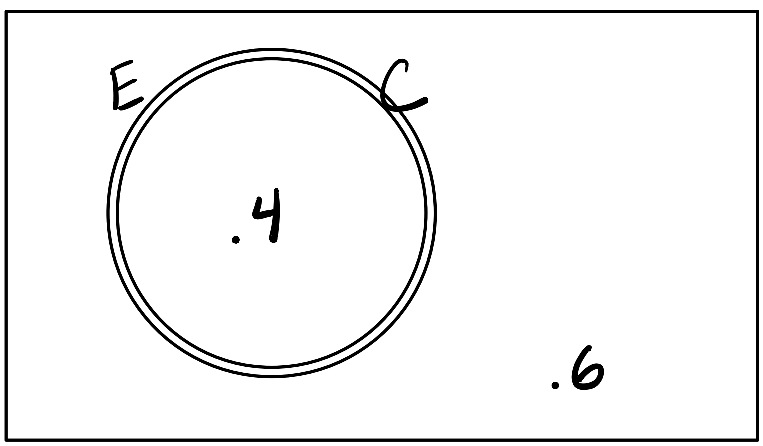
$P(C) = .4$

$P(E \cap C) = ?$





$P(E \cap C) = 0$



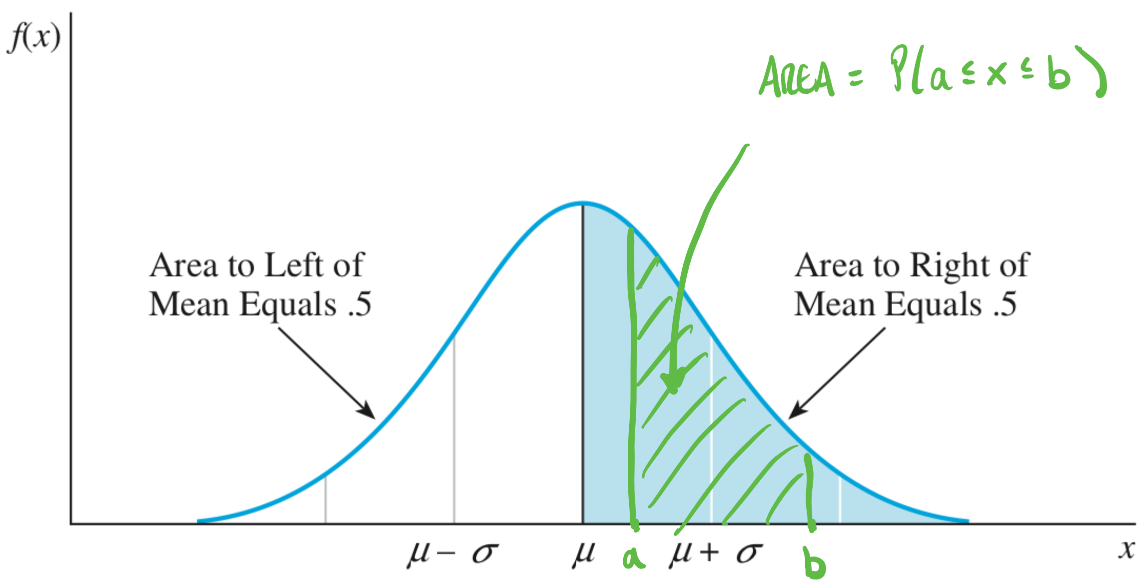
$P(E \cap C) = P(E) = P(C)$

NORMAL PROBABILITY DISTRIBUTION

§6.3 TABULATED AREAS OF THE NORMAL PROB. DISTR.

ALL NORMAL DISTRIBUTIONS (BELL CURVES) HAVE THE EXACT SAME SHAPE EXCEPT FOR

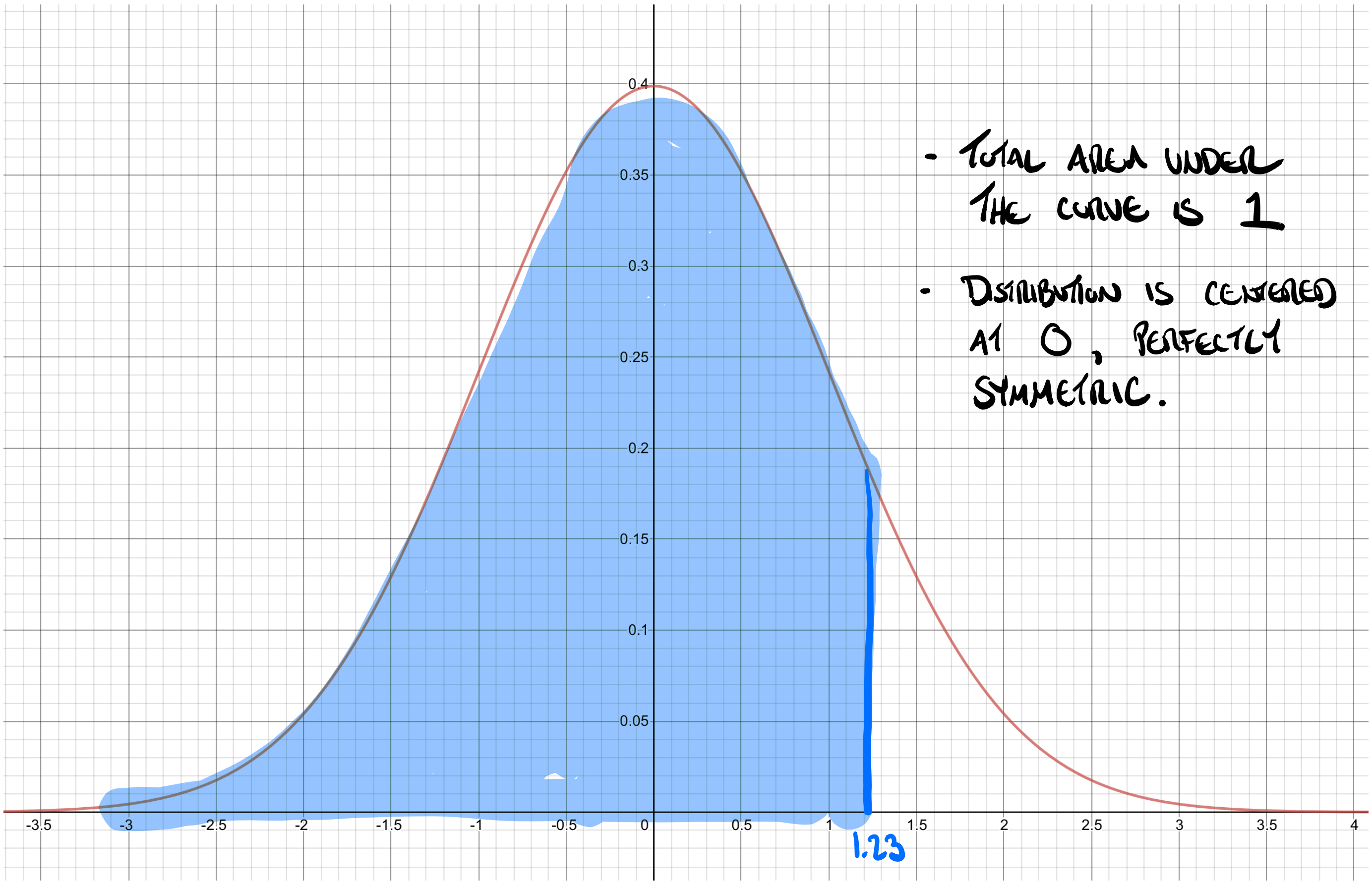
- HORIZONTAL / VERTICAL STRETCHING & SHRINKING
  CONTROLLED BY STAND. DEV. σ
- SHIFTING LEFT / RIGHT
  CONTROLLED BY MEAN μ.



SPECIAL CASE: STANDARD NORMAL DISTRIBUTION.

RANDOM VARIABLE X HAS NORMAL PROB. DISTR. WITH

$\mu = 0$, $\sigma = 1$



- TOTAL AREA UNDER THE CURVE IS 1
- DISTRIBUTION IS CENTERED AT 0, PERFECTLY SYMMETRIC.

FIND $P(X \leq 0)$. ← AREA UNDER CURVE, LEFT OF 0.

$\hookrightarrow .5$

FIND $P(X \leq 1.23)$ SEE TABLE 3 IN APPENDIX OF TEXT BOOK *

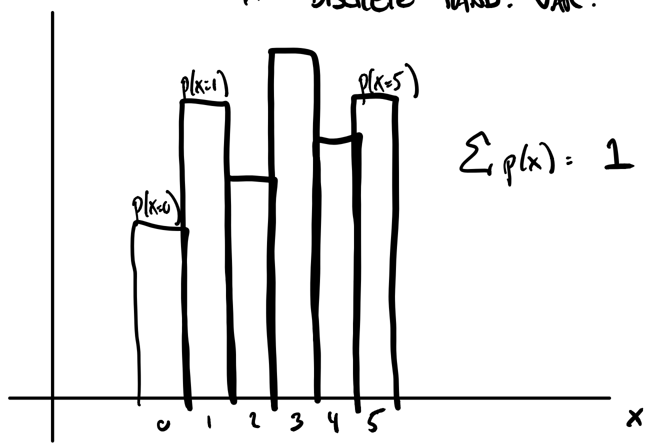
$P(-\infty < X < \infty) = 1$ MUST BE

$\therefore$ TOTAL AREA UNDER CURVE MUST BE 1.

X	0	1	2	3	4	5
$p(x)$	.1	.1	.2	.3	.1	.2

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{PROB. ADD TO } 1}$

X DISCRETE RAND. VAR.



$\sum p(x) = 1$

z	.00	.01	.02	.03	.04	.05
0.0	.5000	.5040	.5080	.5120	.5160	.5199
0.1	.5398	.5438	.5478	.5517	.5557	.5596
0.2	.5793	.5832	.5 871	.5910	.5948	.5987
0.3	.6179	.6217	.6255	.6293	.6331	.6368
0.4	.6554	.6591	.6628	.6664	.6700	.6736
0.5	.6915	.6950	.6985	.7019	.7054	.7088
0.6	.7257	.7291	.7324	.7357	.7389	.7422
0.7	.7580	.7611	.7642	.7673	.7704	.7734
0.8	.7881	.7910	.7939	.7967	.7995	.8023
0.9	.8159	.8186	.8212	.8238	.8264	.8289
1.0	.8413	.8438	.8461	.8485	.8508	.8531
1.1	.8643	.8665	.8686	.8708	.8729	.8749
1.2	.8849	.8869	.8888	.8907	.8925	.8944
1.3	.9032	.9049	.9066	.9082	.9099	.9115
1.4	.9192	.9207	.9222	.9236	.9251	.9265

$$P(x \le 1.23) = .8907$$

ROW 1.2 , COLUMN .03

$$\int_{-\infty}^{1.23} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx$$ (CALCULUS)

FACTORY HAS MACHINE THAT FILLS BOTTLES WITH MEDICINE. MACHINE IS SET TO FILL EACH BOTTLE WITH 35 mL OF MEDICINE. LET X = THE NUMBER OF mL ABOVE (+) OR BELOW (-) 35 IN A RANDOMLY SELECTED BOTTLE.

e.g. A BOTTLE FILLED TO 36.23 mL

THEN $X = 36.23 - 35 = 1.23$

A BOTTLE FILLED TO 33.87 mL

THEN $X = 33.87 - 35 = -1.13$

FIND THE PROB. OF A BOTTLE CONTAINING LESS THAN 36.23 mL.

$\Rightarrow P(X \leq 1.23) = .8907$.

FIND THE PROB. OF A BOTTLE CONTAINING MORE THAN 36.23 mL.

$\Rightarrow P(X > 1.23) = 1 - P(X \leq 1.23) = 1 - .8907$

$= .1093$

ex. FIND PROB $(-.53 \leq X \leq .48)$

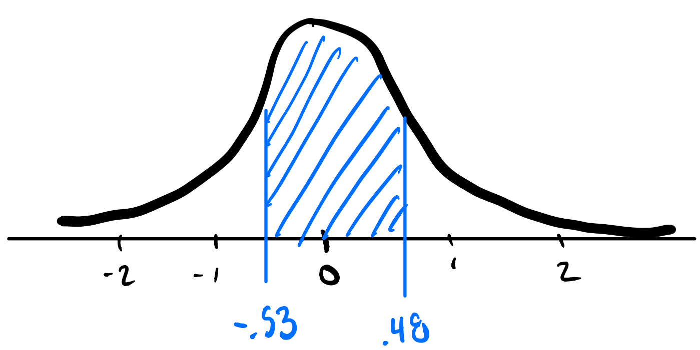
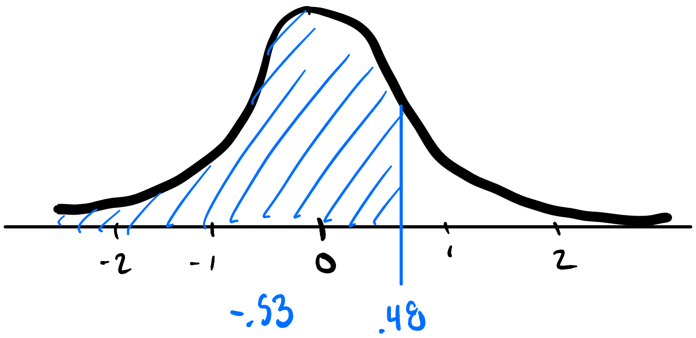
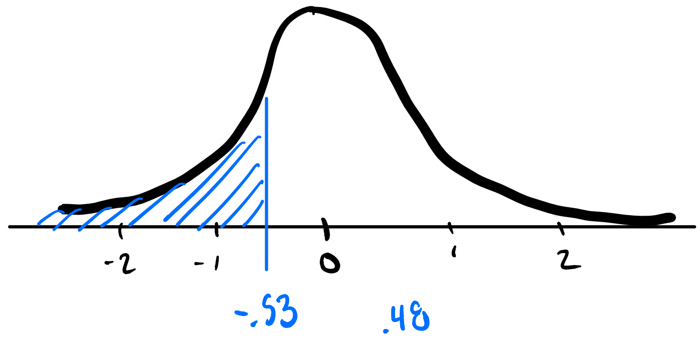


TABLE ONLY PROVIDES PROB. $P(x \leq \#)$





$P(x \leq .48) = .6844$

$P(x \leq -.53) = .2981$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5 871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

$$P(\underbrace{-.53 \leq x \leq .48}_{\text{EVENT}}) = P(x \leq .48) - P(x \leq -.53)$$

$$= .6844 - .2981$$

$$= .3863$$

MORE GENERALLY,

GIVEN AN NORMALLY DISTRIBUTED RANDOM VARIABLE X WITH MEAN μ & STANDARD DEVIATION σ,

THE R.V. IS STANDARDIZED BY EXPRESSING ITS VALUE AS THE NUMBER OF STANDARD DEVIATIONS σ IT LIES TO THE LEFT $(-)$ OR RIGHT $(+)$ OF THE MEAN. THE STANDARDIZED R.V. IS DENOTED Z.

e.g. X IS NORM. DISTR. WITH $\mu = 120$, $\sigma = 25$.

STANDARDIZE THE VALUE $X = 170$.

$X = 170 \Rightarrow X$ IS 50 MORE THAN 120

$\Rightarrow 2(25)$ MORE THAN 120

2σ MORE THAN $\mu \rightarrow Z = 2$

$X = 105 \Rightarrow X$ IS 15 LESS THAN 120

$\frac{3}{5}(25)$ LESS THAN 120

$\frac{3}{5}\sigma$ LESS THAN $\mu \rightarrow Z = -\frac{3}{5}$

$= -.6$

$$z = \frac{x - \mu}{\sigma}$$

Z IS THE STANDARD NORMALLY DISTRIBUTED R.V. $(\mu = 0,\ \sigma = 1)$.

THE VALUE OF Z THAT CORRESPONDS TO A VALUE FOR A (ARBITRARY) NORMALLY DISTR. R.V. X IS CALLED THE Z-SCORE FOR THAT VALUE.

e.g. X IS NORM. DISTR. RV. WITH
MEAN $\mu = 83$.
STAND. DEV. $\sigma = 4$.

(a) FIND Z-SCORE FOR $x = 88$.

(b) FIND Z-SCORE FOR $x = 75$.

(a) $z = \frac{x-\mu}{\sigma} = \frac{88-83}{4} = \frac{5}{4} = 1.25$

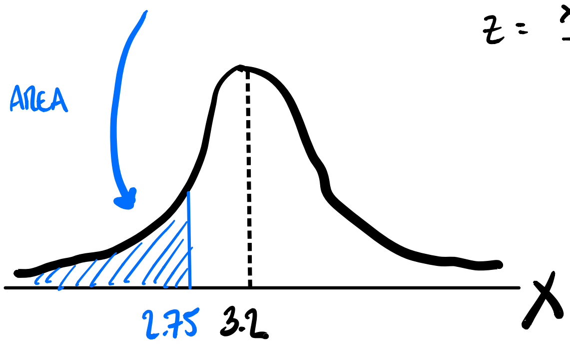
(b) $z = \frac{x-\mu}{\sigma} = \frac{75-83}{4} = \frac{-8}{4} = -2$

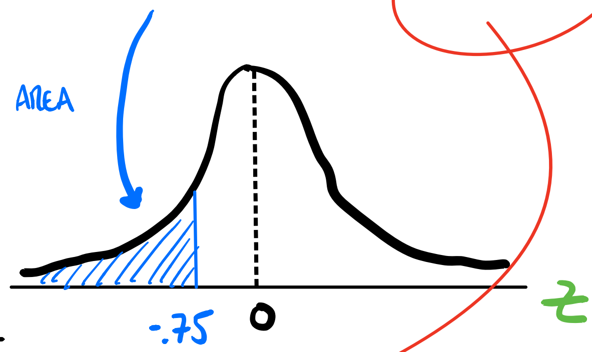
ex.

Suppose the birth weight x of wild boars is normally distributed with mean 3.2 lbs and standard deviation 0.6 lbs.
Find the probability that a wild boar is born weighing less than 2.75 lbs.

$$P(x \le 2.75) \xrightarrow[z = \frac{x-\mu}{\sigma}]{\text{STANDARDIZE}} P\left(z \le \frac{2.75 - 3.2}{0.6}\right)$$

$$= P(z \le -.75) = .2266$$

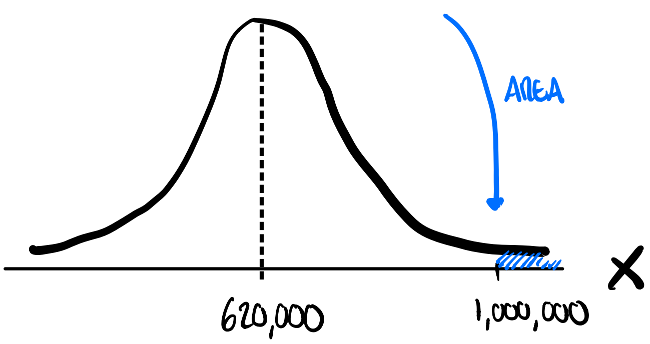




	.00	.01	.02	.03	.04	.05				
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776

Example. The monthly sales at a local car dealership is a random variable x. Assume that x is normally distributed with mean \$620,000 and standard deviation \$250,000.
Find the probability that the car dealership makes more than \$1,000,000 in a month.

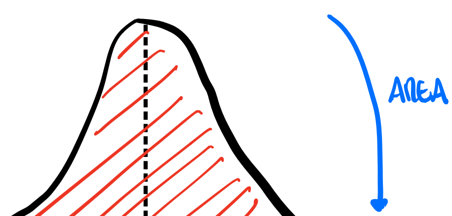
$$\text{FIND } P(x \ge 1{,}000{,}000) \xrightarrow[z = \frac{x-\mu}{\sigma}]{\text{STANDARDIZE}}$$

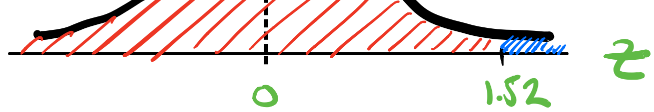


$$P\left(z \ge \frac{1{,}000{,}000 - 620{,}000}{250{,}000}\right)$$

$$= P(z \ge 1.52)$$

STANDARDIZE





$$P(z \geq 1.52) = 1 - P(z \leq 1.52) = 1 - .9357$$

$$= .0643$$

COMPLEMENTS

	.00	.01	.02							
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

6.26 **Breathing Rates** The number of times x an adult human breathes per minute when at rest has a probability distribution that is approximately normal, with the mean equal to 16 and the standard deviation equal to 4. If a person is selected at random and the number x of breaths per minute while at rest is recorded, what is the probability that x will exceed 22?

$$P(x \geq 22) = P\left(z \geq \frac{22 - 16}{4}\right)$$

$$z = \frac{x - \mu}{\sigma}$$

$$= P(z \geq 1.50)$$

$$= 1 - P(z \leq 1.50)$$

$$= 1 - .9332 = .0668$$

	.00									
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767