5.2 Consider a binomial random variable with $n = 9$ and $p = .3$. Let x be the number of successes in the sample.

a. Find the probability that x is exactly 2.

b. Find the probability that x is less than 2.

c. Find $P(x > 2)$.

d. Find $P(2 \le x \le 4)$.

BINOMIAL EXPERIMENTS

↳ n IDENTICAL & INDEPENDENT TRIALS

EACH HAS $P(\text{SUCCESS}) = p$

$P(\text{FAILURE}) = q = 1 - p$

R.V. x = # SUCCESSES IN n TRIALS

$$P(x = k) = C_k^n \, p^{k} \, q^{n-k}$$

$(0 \le k \le n)$ k = # SUCCESSES , $n - k$ = # FAILURES

$$k \le n$$
$$-k \quad -k$$
$$0 \le n - k$$

(a) $P(x=2) = C_2^9 (.3)^2 (.7)^7$

$= 36 (.09)(.0823543)$

$= .2668$

(b) $P(x < 2) = P(x = 0 \text{ or } x = 1)$

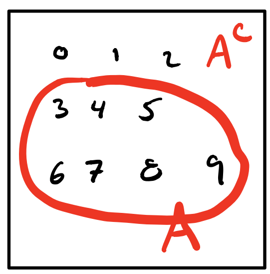
$= P(x = 0 \cup x = 1)$

$= P(x=0) + P(x=1)$

$= C_0^9 (.3)^0 (.7)^9 + C_1^9 (.3)^1 (.7)^8$

$= .0404 + .1556$

$= .1960$



(c) $P(x > 2) = P(x=3) + P(x=4) + \ldots + P(x=9) = \ldots$

$P(x > 2) = P\left((x \le 2)^c\right) = 1 - P(x \le 2)$

$= 1 - [\underbrace{P(x=0) + P(x=1)} + \underbrace{P(x=2)}]$

$1 - [.1960 + .2668] = 1 - .4628$

$= .5372$

(d) $P(2 \le x \le 4) = P(x=2) + P(x=3) + P(x=4) = \dots$

$P(x=k) = C^9_k (.3)^k (.7)^{9-k}$

5.9 Let x be a binomial random variable with $n = 10$ and $p = .4$. Find these values:

a. $P(x = 4)$ **b.** $P(x \ge 4)$ **c.** $P(x > 4)$

d. $P(x \le 4)$ **e.** $\mu = np$ **f.** $\sigma = \sqrt{npq}$

5.34 Man's Best Friend According to the Humane Society of the United States, there are approximately 77.5 million owned dogs in the United States, and approximately 40% of all U.S. households own at least one dog.[4] Suppose that the 40% figure is correct and that 15 households are randomly selected for a pet ownership survey.

a. What is the probability that exactly eight of the households have at least one dog?

b. What is the probability that at most four of the households have at least one dog?

c. What is the probability that more than 10 households have at least one dog?

ASSUME BINOMIAL WHEN

$\frac{\text{SIZE OF SAMPLE}}{\text{SIZE OF POP}} = \frac{n}{N} < .05$

$\frac{15}{\approx \text{BIG}} < .05$ ✓

$P(x=k) = C^n_k p^k q^{n-k}$

BINOMIAL EXP. $n = 15$ $p = .4$ $q = .6$, $x = \#$ SUCCESSES

$0 \le x \le 15$

(a) $P(x=8) = C^{15}_8 (.4)^8 (.6)^7$

$= 6435 \times .4^8 \times .6^7 = .1181$

(b) $P(x \le 4) \rightarrow$ TABLE 1 IN APPENDIX $n=15$

$n = 15$

k	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	k
0	.860	.463	.206	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000	0
1	.990	.829	.549	.167	.035	.005	.000	.000	.000	.000	.000	.000	.000	1
2	1.000	.964	.816	.398	.127	.027	.004	.000	.000	.000	.000	.000	.000	2
3	1.000	.995	.944	.648	.297	.091	.018	.002	.000	.000	.000	.000	.000	3
4	1.000	.999	.987	.836	.515	.217	.059	.009	.001	.000	.000	.000	.000	4
5	1.000	1.000	.998	.939	.722	.403	.151	.034	.004	.000	.000	.000	.000	5
6	1.000	1.000	1.000	.982	.869	.610	.304	.095	.015	.001	.000	.000	.000	6
7	1.000	1.000	1.000	.996	.950	.787	.500	.213	.050	.004	.000	.000	.000	7
8	1.000	1.000	1.000	.999	.985	.905	.696	.390	.131	.018	.000	.000	.000	8
9	1.000	1.000	1.000	1.000	.996	.966	.849	.597	.278	.061	.002	.000	.000	9
10	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.164	.013	.001	.000	10
11	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.352	.056	.005	.000	11
12	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.602	.184	.036	.000	12
13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.833	.451	.171	.010	13
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.794	.537	.140	14
15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	15

$$P(x \le 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= .217$$

$n = 123 \quad p = .32$

(c) $P(x > 10) = 1 - P(x \le 10)$

$$= 1 - .991 = .009$$

$$P(x=7) = \begin{cases} C^{15}_{7} (.4)^7 (.6)^8 = .1771 \\ P(x \le 7) - P(x \le 6) = .787 - .610 = .177 \end{cases}$$

§6.1 Probability Distributions for Continuous Random Variables

Continuous random variables take on infinitely many (uncountable many) values. So we can't assign positive probabilities p(x) for every possible value of the random variable x, because the probabilities would no longer sum to 1.

DISCRETE :

ALL POSSIBLE VALUES FOR X

$P(x=0) + P(x=1) + P(x=2) + P(x=3) = 1$

CONTINUOUS :

$0 \leq X \leq 3$

ALL POSSIBLE VALUES FOR X

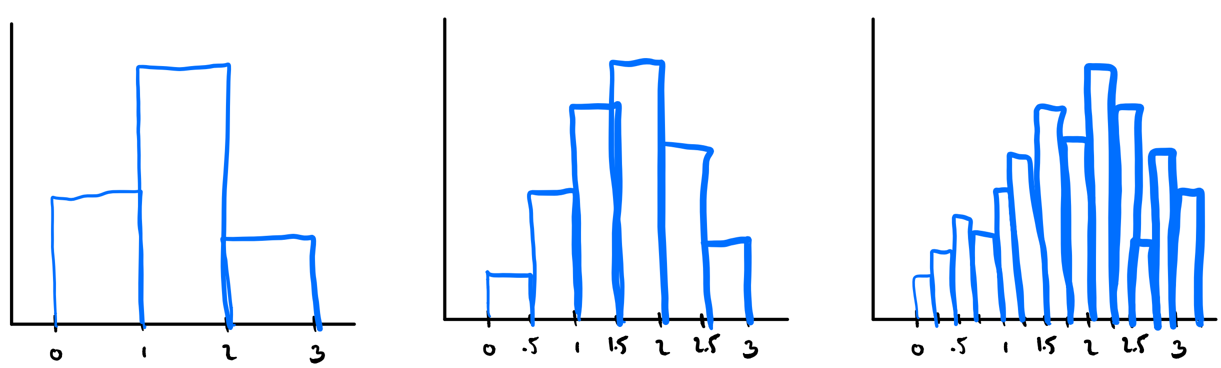
$$\underbrace{P(x=0) + P(x=0.12) + P(x=0.1227) + \dots}$$

∞'LY MANY PROBS. SUM TO ∞

IF THEY WERE POSITIVE !

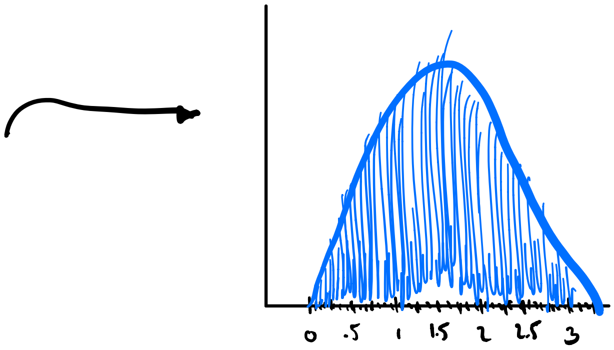
NEW APPROACH:

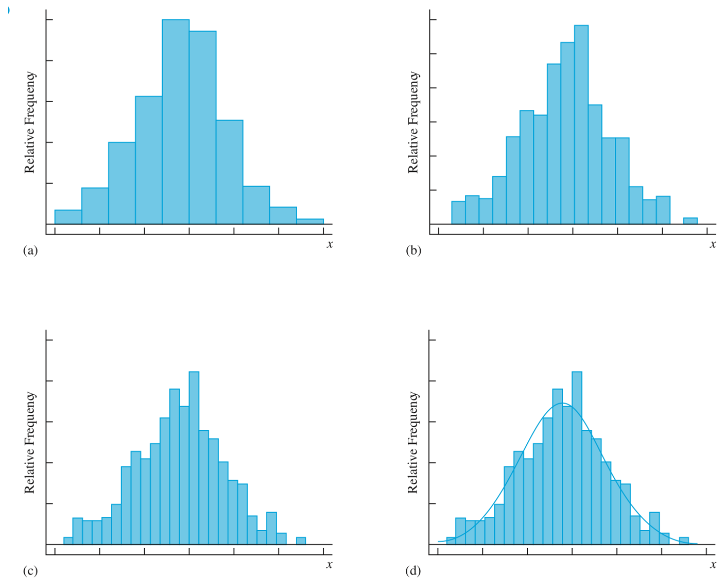
MAKE A HISTOGRAM FOR CONTINUOUS RANDOM VARIABLE X ...



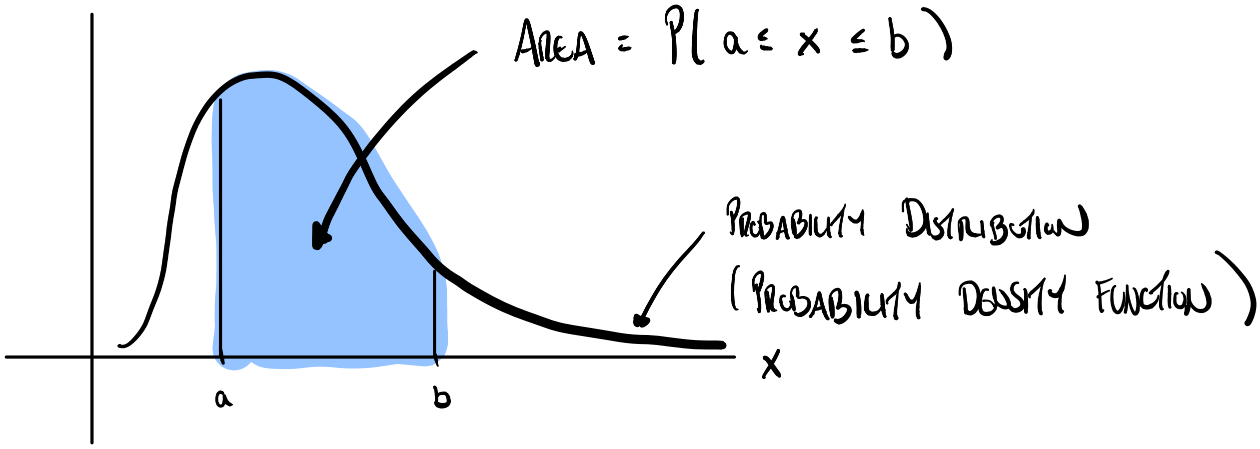
[0,1) [1,2) [2,3)

... WITH MORE & MORE (NARROWER & NARROWER) CLASSES.





IN GENERAL, AS CLASSES GET NARROWER, HISTOGRAM/GRAPH GETS SMOOTHER

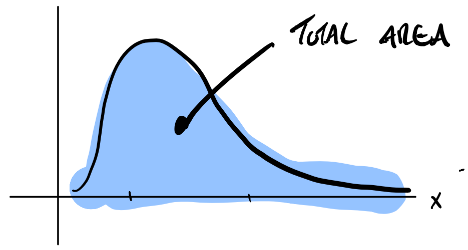


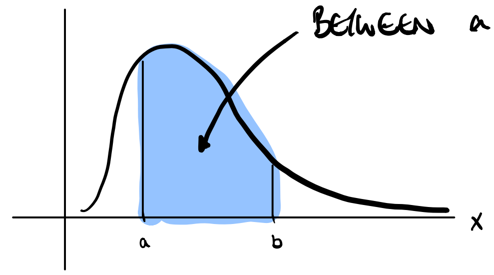
THE PROBABILITY DISTRIBUTION IS CREATED BY DISTRIBUTING 1 UNIT OF PROBABILITY ALONG THE x-AXIS.

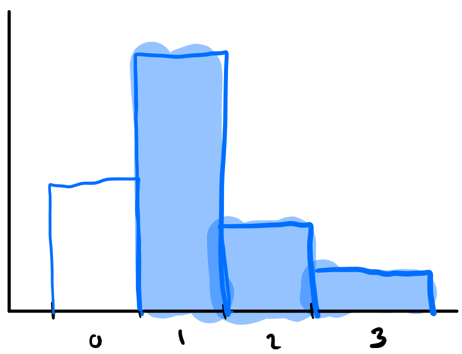
DENSITY OF PROBABILITY VARIES WITH x.

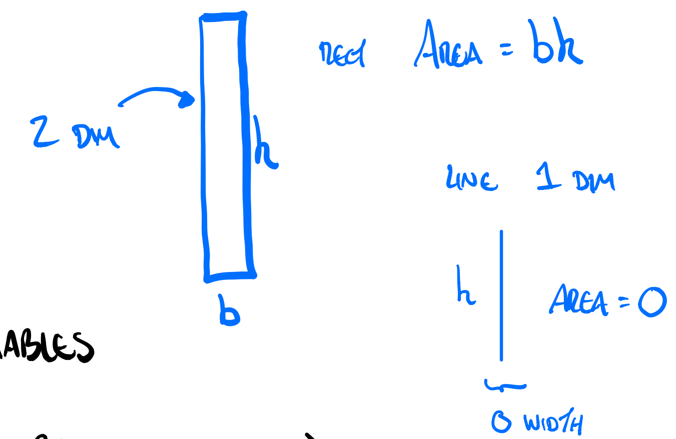
THIS MAY BE DESCRIBED AS A PROBABILITY DENSITY FUNCTION. $f(x)$

DISCRETE	CONTINUOUS
• SUM OF ALL PROBABILITIES = 1 $P(x=0) + P(x=1) + \dots + P(x=n) = 1$	• AREA UNDER PROBABILITY DISTRIBUTION = 1
• $P(1 \le x \le 3) = P(x=1) + P(x=2) + P(x=3)$ SUM OF PROB IN THAT INTERVAL	• $P(a \le x \le b) =$ AREA UNDER PROB DISTRIBUTION BETWEEN a & b.





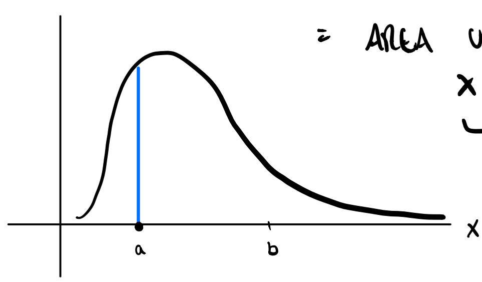




NOTE: FOR CONTINUOUS RANDOM VARIABLES

(i) $P(X=a) = P(a \le X \le a)$

= AREA UNDER PROB. DISTR. BETWEEN $\underbrace{x=a \text{ \& } x=a}_{0 \text{ WIDTH}}$



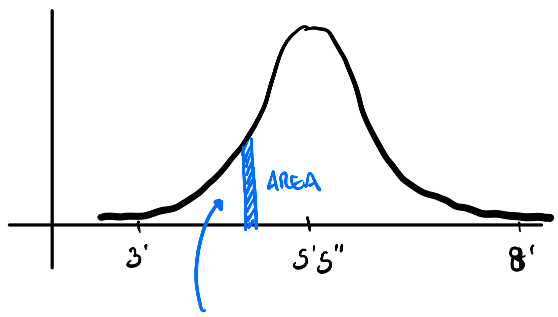
a = 1.21716320018...

0 AREA UNDER CURVE ABOVE POINT $x=a$

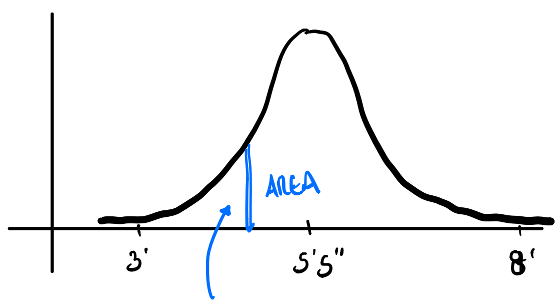
(ii) $P(X \le a) = P(X < a) + \underbrace{P(X = a)}_{0}$

$P(X \ge a) = P(X > a) + \underbrace{P(X = a)}_{0}$

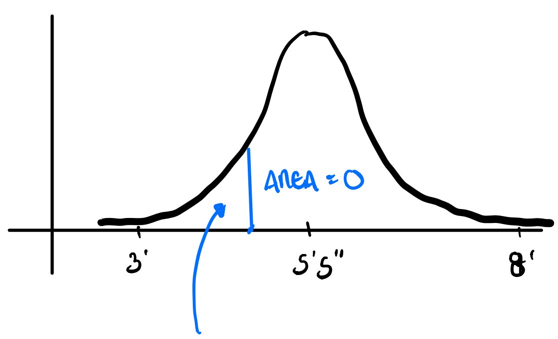
e.g. EXP: SELECT A RANDOM HUMAN BEING
CONTINUOUS RANDOM VARIABLE X = HEIGHT.



$P(5'0" \le X \le 5'1")$



$P(5'0" \le X \le 5'.25")$



NO ROUNDING

$P(X = 5') = 0$

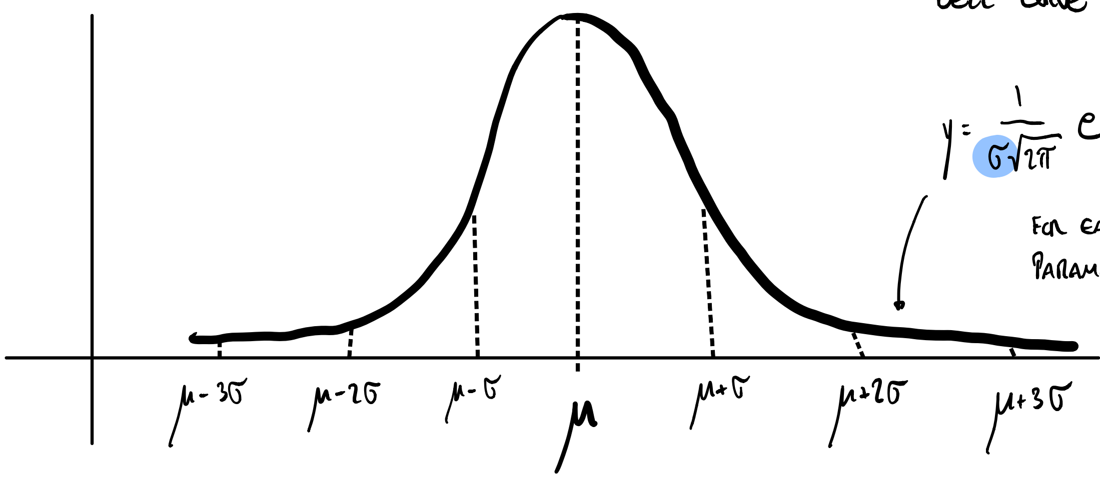
5.000000000000000000 ...

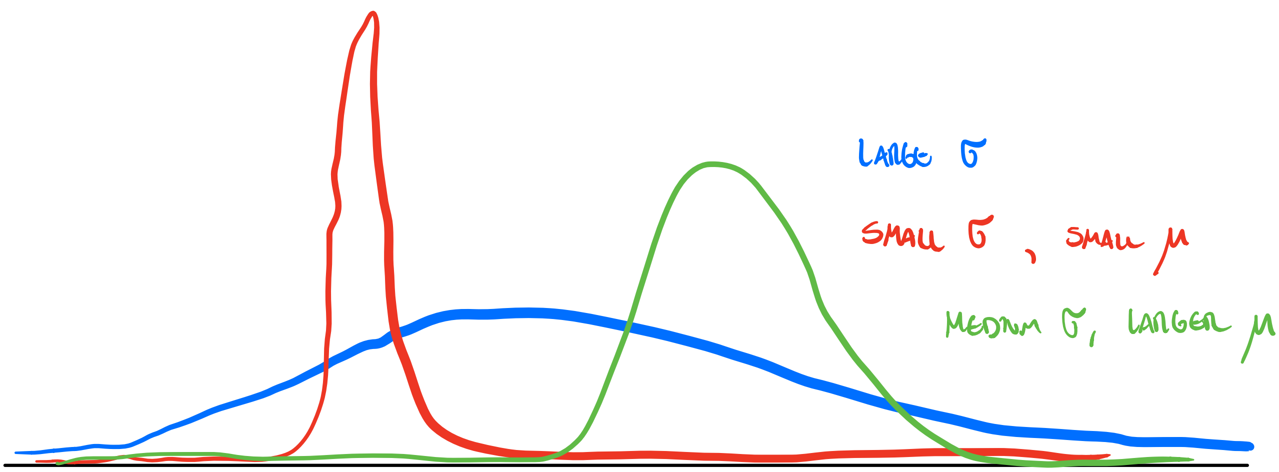
§6.2 The Normal Probability Distribution

Bell Curve

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

For each value of parameters σ, μ we get a different distribution





Recall: Emperical rule.

Almost all observations lie within 3 standard deviations of the mean (99.7%)

$$P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx .997$$

SKETCH NORMAL DISTRIBUTION WITH $\mu = 25$

$\sigma = 5$