•) REL FREQ. HISTOGRAMS

- READ INFO FROM HISTOGRAM

•) GIVE YOU DATA, COMPUTE

DATA : $x_1, x_2, \dots, x_n$

- MEAN — SAMPLE $\bar{x}$, POPULATION μ $\Big\}$ MEAN $= \dfrac{\sum x_i}{n} = \dfrac{\text{SUM OF ALL DATA}}{\text{\# OF PIECES OF DATA}}$

- MEDIAN : MIDDLE VALUE WHEN LINED UP IN ORDER
- MODE : MOST FREQUENTLY APPEARING VALUE(S)

- RANGE : MAX - MIN
- VARIANCE

POPULATION VARIANCE $\sigma^2 = \dfrac{\sum (x_i - \mu)^2}{N}$

SAMPLE VARIANCE $s^2 = \dfrac{\sum_i (x_i - \bar{x})^2}{n-1}$

DATA : 2 5 6 7 (POPULATION)

$$\mu = \frac{2+5+6+7}{4} = \frac{20}{4} = 5$$

x_i	$x_i - \mu$	$(x_i - \mu)^2$
2	-3	9
5	0	0
6	1	1
7	2	4

$$\sigma^2 = \frac{\sum_i (x_i - \mu)^2}{N}$$

$$= \frac{9+0+1+4}{4}$$

$$\sigma^2 = \frac{14}{4} = 3.5$$

STAN. DEV. $\sigma = \sqrt{3.5}$

DATA: 2 5 6 7 (SAMPLE)

$$\bar{x} = \frac{2+5+6+7}{4} = \frac{20}{4} = 5$$

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
2	-3	9
5	0	0
6	1	1
7	2	4

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

$$= \frac{9+0+1+4}{3}$$

$$s^2 = \frac{14}{3} = 4.6666$$

STAND. DEV. $s = \sqrt{4.6666}$

STANDARD DEV. FOR POPULATION

$$\sigma = \sqrt{\sigma^2}$$

STANDARD DEV. FOR SAMPLE

$$s = \sqrt{s^2}$$

·) CHEVYCHEV'S RULE :

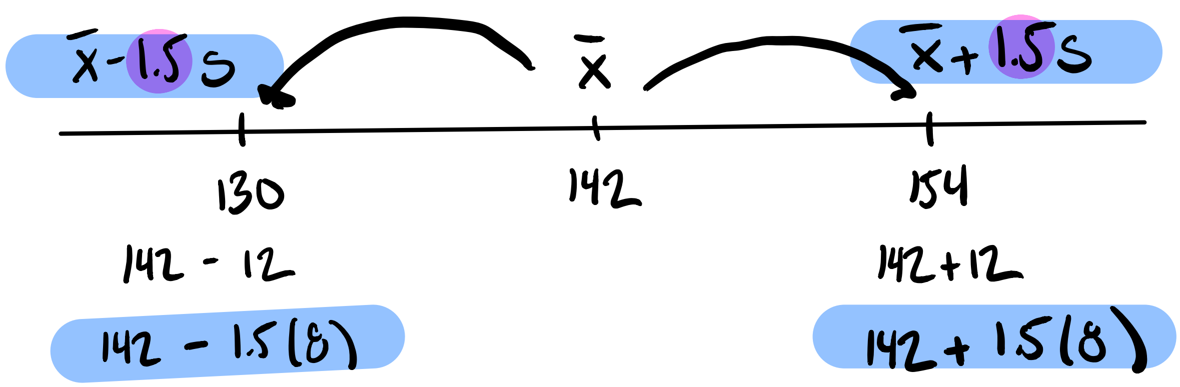
Given a population/sample of measurements (with any distribution at all!), the proportion of measurements that lie within k standard evictions of the mean is at least

$$1 - \frac{1}{k^2}$$

ex. MEAN $\bar{x} = 142$

STAND DEV. $s = 8$

PROPORTION OF MEASUREMENTS BETWEEN 130 & 154 IS AT LEAST $1 - \frac{1}{1.5^2}$



PROPORTION OF DATA THAT LIES WITHIN 1.5 STAND. DEV. OF MEAN

IS AT LEAST $1 - \frac{1}{1.5^2} = 1 - \frac{1}{2.25}$

$= .5555$

i.e. AT LEAST 55.55 % OF DATA IS BETWEEN 130 & 154.

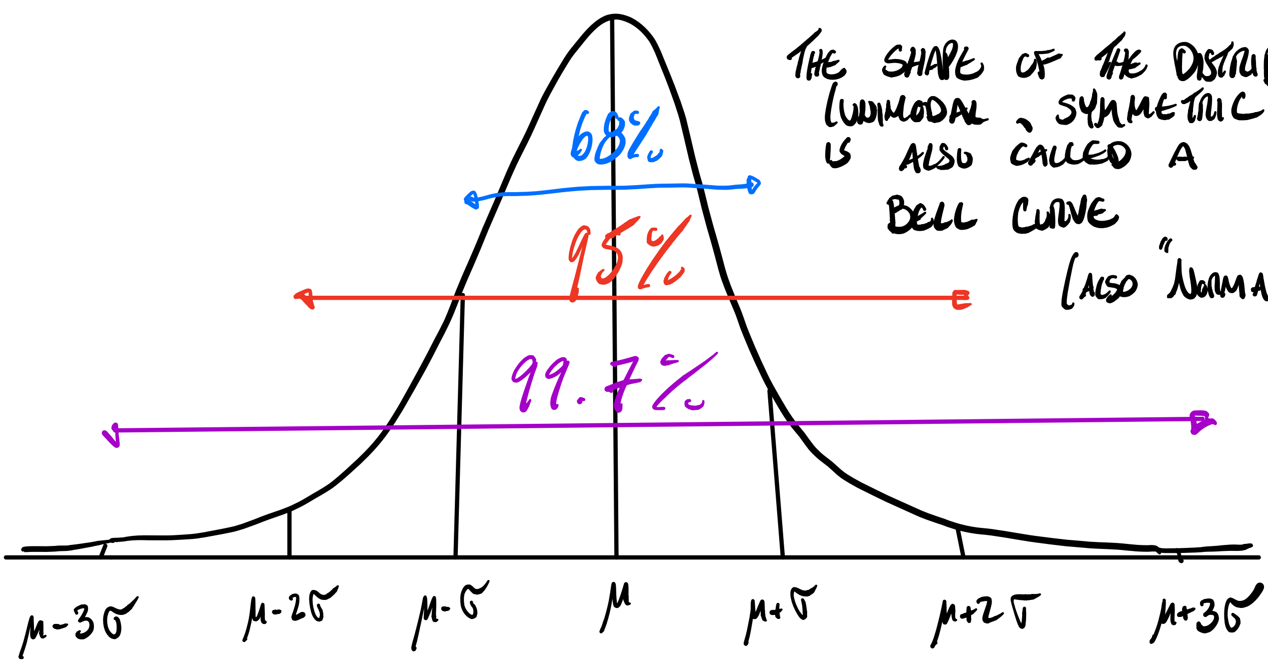
- EMPERICAL RULE : APPLIES TO DATA WITH DISTRIBUTION THAT IS UNIMODAL & SYMMETRIC.

THE EMPERICAL RULE :

APPROX. 68% OF DATA LIES WITHIN 1 STAND. DEV. OF MEAN

APPROX. 95 % OF DATA LIES WITHIN 2 STAND. DEV. OF MEAN

APPROX. 99.7 % OF DATA LIES WITHIN 3 STAND. DEV. OF MEAN



THE SHAPE OF THE DISTRIBUTION (UNIMODAL, SYMMETRIC) IS ALSO CALLED A BELL CURVE (ALSO "NORMAL")

·) BASICS OF PROBABILITY:

SETS, INTERSECTIONS $\cap$, UNIONS $\cup$,

COMPLEMENTS, $|A| = \#$ OF ELEMENT IN SET A

SAMPLE SPACE Ω

EVENTS $A \subseteq \Omega$ (SUBSETS)

$$\Omega = \{ \underbrace{E_1, E_2, \dots, E_n}_{\text{SIMPLE EVENTS (INDIVIDUAL OUTCOMES)}} \}$$

$$\sum P(E_i) = 1 \qquad 0 \leq P(E_i) \leq 1$$

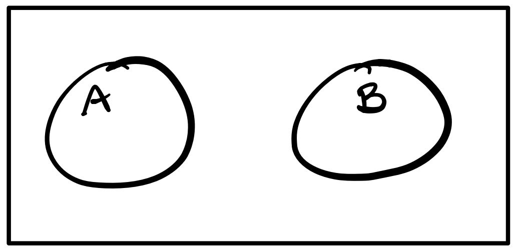
ADDITION RULE: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

· CONDITIONAL PROBABILITY:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad P(A \cap B) = P(B)\,P(A|B)$$
$$= P(A)\,P(B|A)$$

- A, B MUTUALLY EXCLUSIVE IF $P(A \cap B) = 0$.



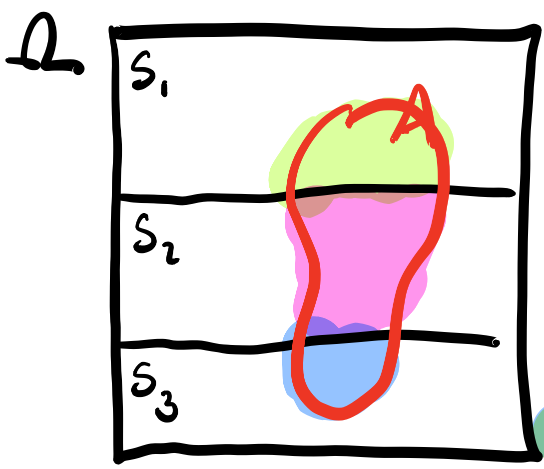
- A, B INDEPENDENT IF ANY OF THE FOLLOWING ARE TRUE

$$P(A \cap B) = P(A)P(B)$$

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

- LAW OF TOTAL PROB.



$S_1 \cup S_2 \cup S_3 = \Omega$ EXHAUSTIVE

$S_1 \cap S_2 = \phi$

$S_1 \cap S_3 = \phi$

$S_2 \cap S_3 = \phi$

MUTUALLY EXCLUSIVE

$$P(A) = P(A \cap S_1) + P(A \cap S_2) + P(A \cap S_3)$$

$$P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3)$$

USE L.T.P IF CONDITIONAL PROB ARE KNOWN

& YOU WANT TO KNOW AN UNCONDITIONAL PROB

BAYES' RULE:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$ DEF. OF COND. PROB

$$= \frac{P(A)P(B|A)}{P(B)}$$ MULTIPLICATION RULE

$$\boxed{P(A|B) = \frac{P(A)P(B|A)}{P(B)}}$$

USE BAYES' RULE IF YOU KNOW ONE CONDITIONAL PROB (e.g. $P(B|A)$) & YOU WANT THE OPPOSITE CONDITIONAL PROB. $P(A|B)$

ex.

Suppose 30% of customers leave a tip greater than 25%, 50% of customers leave a tip between 20% and 25%, and 20% of customers leave a tip less than 20%.
80% of customers who tip greater than 25% are repeat customers.
40% of customers who tip 20-25% are repeat customers.
10% of customers who tip less than 20% are repeat customers.
Find the probability that a customer tips greater than 25% if they are a repeat customer.

LET S_1: CUSTOMER TIPS $> 25\%$

S_2 = CUSTOMER TIPS $20-25\%$

S_3 = CUSTOMER TIPS $< 20\%$

A = CUSTOMER IS REPEAT CUSTOMER.

GIVEN $P(S_1) = .3$ $P(A|S_1) = .8$

$P(S_2) = .5$ $P(A|S_2) = .4$

$P(S_3) = .2$ $P(A|S_3) = .1$

FIND $P(S_1|A)$

RELATED!

BAYES' RULE $$P(S_1|A) = \frac{P(S_1)P(A|S_1)}{P(A)}$$

* BAYES' RULE PRODUCES THE FRACTION

LOTP GIVES THE DENOMINATOR. *

$$P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + P(S_3)P(A|S_3)$$

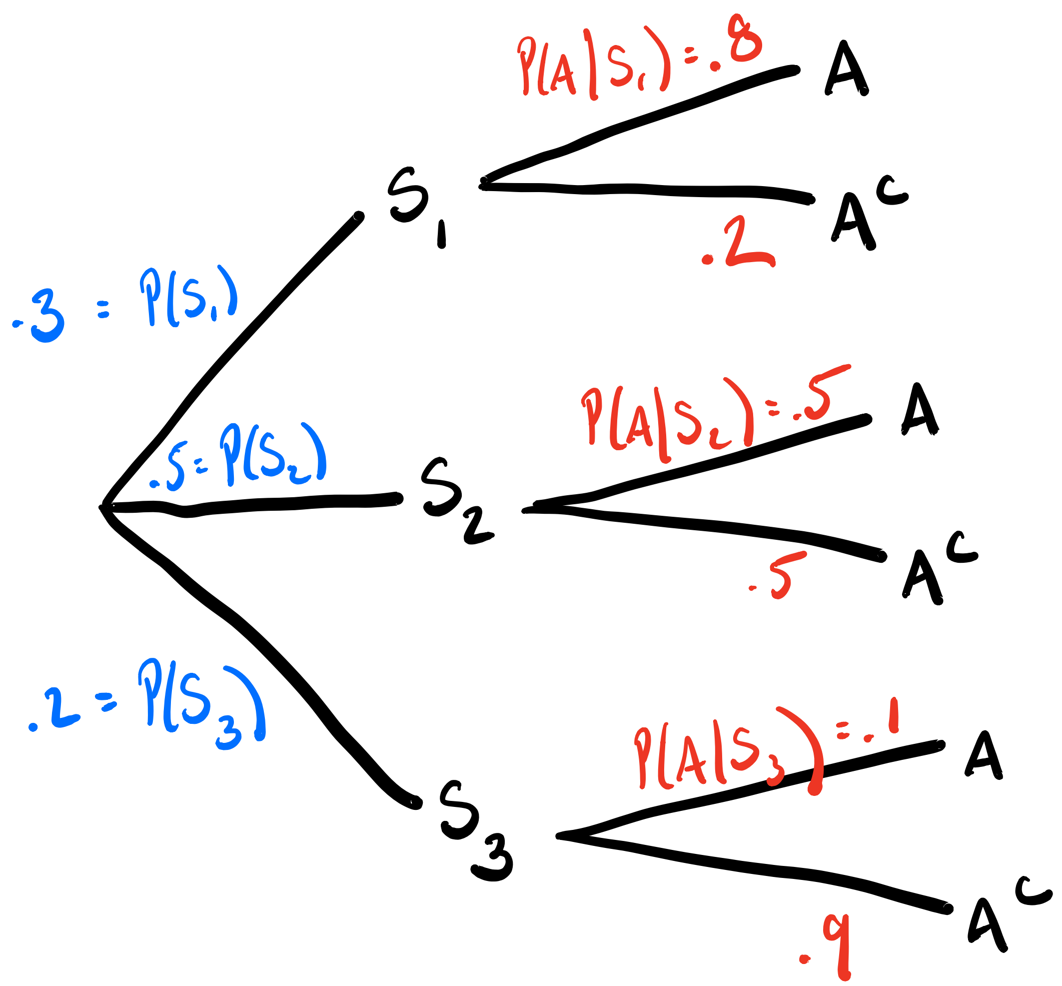
$$P(A) = (.3)(.8) + (.5)(.4) + (.2)(.1)$$

$$= .24 + .20 + .02$$

$$= .46$$

$$P(S_1|A) = \frac{P(S_1)P(A|S_1)}{P(A)} = \frac{(.3)(.8)}{(.46)}$$

$$= \boxed{.5217}$$



$$P(S_1|A) + P(S_2|A) + P(S_3|A)$$

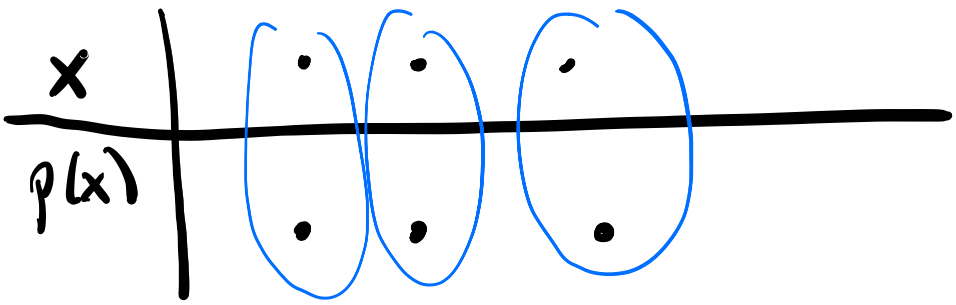
$$= \frac{P(S_1 \cap A)}{P(A)} + \frac{P(S_2 \cap A)}{P(A)} + \frac{P(S_3 \cap A)}{P(A)}$$

$$= \frac{P(S_1 \cap A) + P(S_2 \cap A) + P(S_3 \cap A)}{P(A)}$$

$$= \frac{P\left((S_1 \cap A) \cup (S_2 \cap A) \cup (S_3 \cap A) \right)}{P(A)}$$

$$= \frac{P(A)}{P(A)} = 1.$$

- DISCRETE R.V. & $\underbrace{\text{THEIR PROB. DISTR.}}$



EXPECTED VALUE $\mu = E[x] = \sum x\,p(x)$

STANDARD DEV. $\sigma = \sum (x-\mu)^2 p(x)$

Name:____ **3/27/2019**

Math 17300-CD Introduction to Probability and Statistics **Midterm Exam**

1. You are given a population of $n = 5$ measurements: $1, 5, 7, 1, 6$.

   (a) (4 points) What is the median, m?

   (b) (4 points) What is the mean, μ?

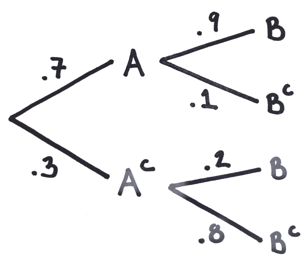
   (c) (4 points) What is/are the mode/modes, M?

   (d) (4 points) What is the population variance, σ^2?

   (e) (4 points) What is the sample standard deviation, σ?

2. You have been kidnapped by an angry dragon. To escape, you must sneak past the dragon, who is blocking the exit. Luckily, dragons sleep a lot and there is a 70% chance that the dragon is sleeping. If the dragon is sleeping, there is a 90% chance that you will escape. But if the dragon is not sleeping, there is a only a 20% chance that you will escape. Let

$$A = \text{the event that the dragon is sleeping}$$
$$B = \text{the event that you escape.}$$



(a) (8 points) Use the given probabilities to fill in the chart below.

	A	A^C
B		
B^C		

(b) (4 points) Are A and B independent? Why?

(c) (4 points) Are A and B mutually exclusive? Why?

(d) (4 points) What is the probability that you escape, i.e. what is $P(B)$?

(e) (4 points) If you do escape, what is probability that the dragon is sleeping, i.e. what is $P(B|A)$?

3. On a particular day, a company recorded how its employees commuted to work and whether or not the arrived on time. It found that 55% of employees took the subway to work, 20% drove, 15% walked, and 10% biked. It also found that 80% of people who took the subway arrived on time, 85% of people who drove arrived on time, 98% of people who walked arrived on time, and 90% of people who biked arrived on time.

   (a) (8 points) What percentage of employees arrived on time?

   (b) (8 points) If an employee arrived on time, what is the probability that she took the subway?

4. (4 points) You are helping a friend choose which classes to take next semester. There are 3 math classes, 5 science classes, and 6 humanities classes available. She must register for 1 math class, 2 science classes, and 2 humanities classes. How many possible ways are there for your friend to choose her classes?

5. A certain lottery ticket costs \$5 to buy, and each ticket has a 10% chance of winning \$5 (winning your money back), a 2% chance of winning \$50, and a 1% chance of winning \$250. Let x be the net gain (profit) from buying one lottery ticket.

   (a) (8 points) Describe the probability distribution for x by filling out the following chart.

x	
$p(x)$	

   (b) (4 points) What is the expected value $E[x]$ (i.e. μ) for x?

6. You are building a device that requires 5 working transistors. Company A will sell you a pack of 7 transistors, each of which has an 80% chance of working. Company B will sell you a pack of 8 transistors, each of which has a 70% chance of working.

$n = 7$

	p													
k	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	k
0	.932	.698	.478	.210	.082	.028	.008	.002	.000	.000	.000	.000	.000	0
1	.998	.956	.850	.577	.329	.159	.062	.019	.004	.000	.000	.000	.000	1
2	1.000	.996	.974	.852	.647	.420	.227	.096	.029	.005	.000	.000	.000	2
3	1.000	1.000	.997	.967	.874	.710	.500	.290	.126	.033	.003	.000	.000	3
4	1.000	1.000	1.000	.995	.971	.904	.773	.580	.353	.148	.026	.004	.000	4
5	1.000	1.000	1.000	1.000	.996	.981	.938	.841	.671	.423	.150	.044	.002	5
6	1.000	1.000	1.000	1.000	1.000	.998	.992	.972	.918	.790	.522	.302	.068	6
7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	7

$n = 8$

	p													
k	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	k
0	.923	.663	.430	.168	.058	.017	.004	.001	.000	.000	.000	.000	.000	0
1	.997	.943	.813	.503	.255	.106	.035	.009	.001	.000	.000	.000	.000	1
2	1.000	.994	.962	.797	.552	.315	.145	.050	.011	.001	.000	.000	.000	2
3	1.000	1.000	.995	.944	.806	.594	.363	.174	.058	.010	.000	.000	.000	3
4	1.000	1.000	1.000	.990	.942	.826	.637	.406	.194	.056	.005	.000	.000	4
5	1.000	1.000	1.000	.999	.989	.950	.855	.685	.448	.203	.038	.006	.000	5
6	1.000	1.000	1.000	1.000	.999	.991	.965	.894	.745	.497	.187	.057	.003	6
7	1.000	1.000	1.000	1.000	1.000	.999	.996	.983	.942	.832	.570	.337	.077	7
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	8

(a) (6 points) If you buy a pack of 7 transistors from company A, what is the probability that 5 or more of the transistors work?

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7. (4 points) A group of 8 friends are out celebrating for the night. They decide to choose one person to pay for dinner, a second person to pay for concert tickets, and a third person to pay for drinks. How many different ways are there for them to do this?

$$\underset{\text{DINNER}}{\underline{8}} \times \underset{\text{CONCERT}}{\underline{7}} \times \underset{\text{DRINKS}}{\underline{6}} = P^8_3$$

$$= 336$$

A B C
C B A

8. (8 points) Suppose a group of 15 friends gather to watch a basketball game. Nine of them are cheering for team A, and six of them are cheering for team B. Three friends are randomly chosen to go pick up some pizza. What is the probability that exactly two of the three friends are cheering for the same team? Hint: there are two types of ways this can happen, and we must add the corresponding probabilities.

P(2 CHEERING FOR SAME TEAM)

$$= P(2A1B \cup 1A2B) = P(2A1B) + P(1A2B)$$

\# WAYS TO CHOOSE 3 FRIENDS: $C^{15}_3 = 455$

\# WAYS TO CHOOSE 2A & 1B: $C^9_2 \times C^6_1 = 216$

\# WAYS TO CHOOSE 1A & 2B: $C^9_1 \times C^6_2 = 135$

$$P(2A1B) + P(1A2B) = \frac{\#2A1B}{\text{TOTAL }\#} + \frac{\#1A2B}{\text{TOTAL }\#}$$

$$= \frac{216}{455} + \frac{135}{455} = \frac{352}{455} = \boxed{.7736}$$

Name:____ **3/27/2019**

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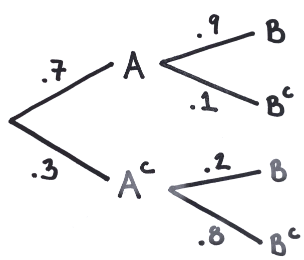

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0	.932	.698	.478	.210	.082	.028	.008	.002	.000	.000	.000	.000	.000	0
1	.998	.956	.850	.577	.329	.159	.062	.019	.004	.000	.000	.000	.000	1
2	1.000	.996	.974	.852	.647	.420	.227	.096	.029	.005	.000	.000	.000	2
3	1.000	1.000	.997	.967	.874	.710	.500	.290	.126	.033	.003	.000	.000	3
4	1.000	1.000	1.000	.995	.971	.904	.773	.580	.353	.148	.026	.004	.000	4
5	1.000	1.000	1.000	1.000	.996	.981	.938	.841	.671	.423	.150	.044	.002	5
6	1.000	1.000	1.000	1.000	1.000	.998	.992	.972	.918	.790	.522	.302	.068	6
7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	7

$n = 8$

	p													
k	.01	.05	.10	.20	.30	.40	.50	.60	.70	.80	.90	.95	.99	k
0	.923	.663	.430	.168	.058	.017	.004	.001	.000	.000	.000	.000	.000	0
1	.997	.943	.813	.503	.255	.106	.035	.009	.001	.000	.000	.000	.000	1
2	1.000	.994	.962	.797	.552	.315	.145	.050	.011	.001	.000	.000	.000	2
3	1.000	1.000	.995	.944	.806	.594	.363	.174	.058	.010	.000	.000	.000	3
4	1.000	1.000	1.000	.990	.942	.826	.637	.406	.194	.056	.005	.000	.000	4
5	1.000	1.000	1.000	.999	.989	.950	.855	.685	.448	.203	.038	.006	.000	5
6	1.000	1.000	1.000	1.000	.999	.991	.965	.894	.745	.497	.187	.057	.003	6
7	1.000	1.000	1.000	1.000	1.000	.999	.996	.983	.942	.832	.570	.337	.077	7
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	8

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