4.54 Drug Testing Many companies are now testing prospective employees for drug use. However, opponents claim that this procedure is unfair because the tests themselves are not 100% reliable. Suppose a company uses a test that is 98% accurate—that is, it correctly identifies a person as a drug user or nonuser with probability .98—and to reduce the chance of error, each job applicant is required to take two tests. If the outcomes of the two tests on the same person are independent events, what are the probabilities of these events?

a. A nonuser fails both tests.

b. A drug user is detected (i.e., he or she fails at least one test).

c. A drug user passes both tests.

NOTATION:

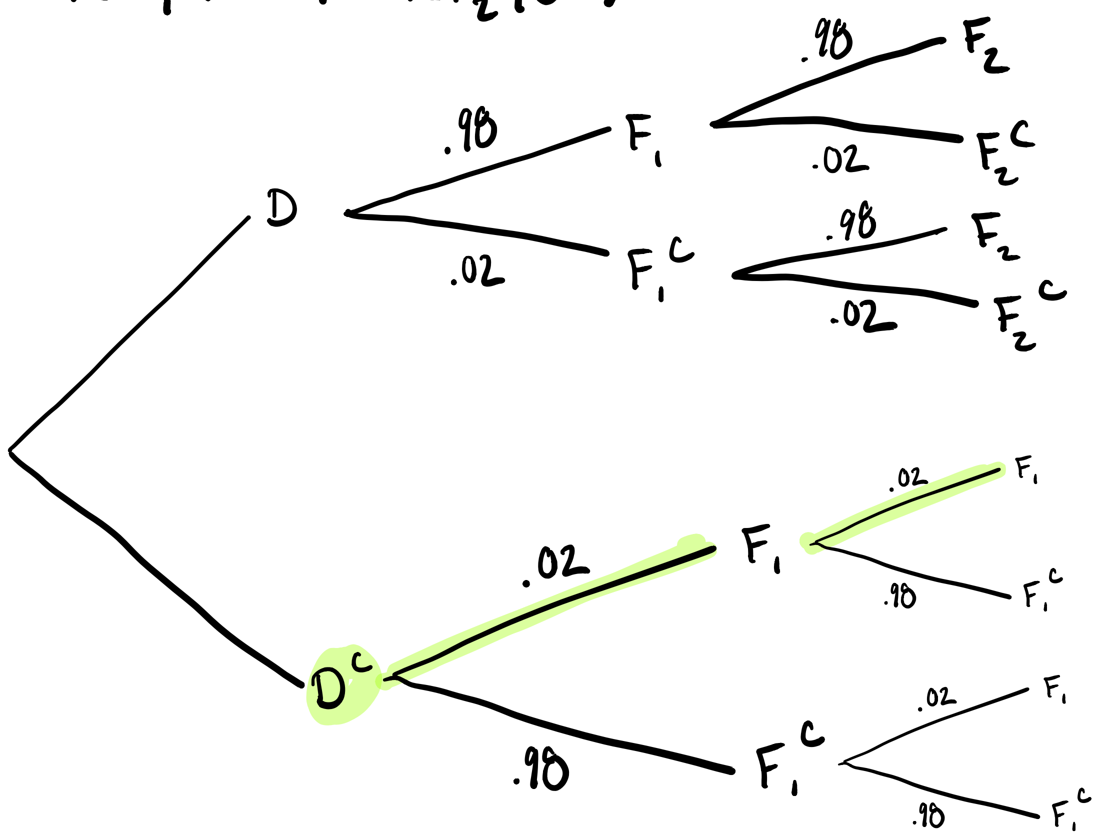
D = DRUG USER

F_1 = FAIL 1st TEST

F_2 = FAIL 2nd TEST

GIVEN:
$$P(F_1 \mid D) = P(F_2 \mid D) = .98$$
$$P(F_1^c \mid D) = P(F_2^c \mid D) = .02$$
$$P(F_1^c \mid D^c) = P(F_2^c \mid D^c) = .98$$
$$P(F_1 \mid D^c) = P(F_2 \mid D^c) = .02$$

(a)



$$P((F_1 \cap F_2) \mid D^c) = .02 \times .02 = \boxed{.0004}$$

$$P(F_1 \cap F_2) = P(F_1)P(F_2 | F_1)$$
$$= P(F_1)P(F_2)$$
$$= (.02)(.02) = .0004$$

(b) $P(\text{FAIL AT LEAST ONE TEST})$

$$= P\left(\left(F_1 \cap F_2^c\right) \cup \left(F_1^c \cap F_2\right) \cup \left(F_1 \cap F_2\right)\right)$$

MUTUALLY EXCLUSIVE

$$= P(F_1 \cap F_2^c) + P(F_1^c \cap F_2) + P(F_1 \cap F_2)$$

$$= P(F_1)P(F_2^c) + P(F_1^c)P(F_2) + P(F_1)P(F_2)$$

$$= (.98)(.02) + (.02)(.98) + (.98)(.98)$$

$$= .0196 + .0196 + .9604$$

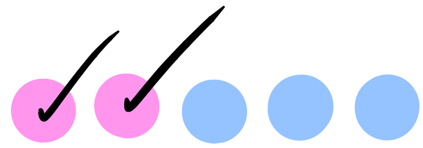
$$= .9996$$

§ 4.8 Discrete Random Variable & Probability Distributions

4.89 Gender Bias? A company has five applicants for two positions: two women and three men. Suppose that the five applicants are equally qualified and that no preference is given for choosing either gender. Let x equal the number of women chosen to fill the two positions.

a. Find $p(x)$.

b. Construct a probability histogram for x.



$x = 2$

EXPERIMENT: CHOOSE 2 PEOPLE TO HIRE FROM 5 PEOPLE : 2 WOMEN , 3 MEN.

(TOTAL # OF POSSIBLE OUTCOMES OF EXPERIMENT (SIZE OF SAMPLE SPACE) $|\Omega|$ } $C^5_2 = 10$)

PROBABILITY DISTRIBUTION FOR X

x	0	1	2
$p(x)$	.3	.6	.1

ALL POSSIBLE VALUE

CHECK : $\sum p(x) = 1$ ✓

WAYS TO CHOOSE p WOMEN OUT OF 2 , & q MEN OUT OF 3 IS

$$C^2_p \times C^3_q$$

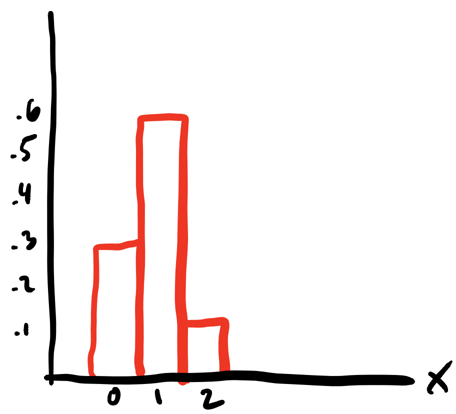
$$p(x=0) = P(0w, 2m) = \frac{\text{\# ways to choose } 0w, 2m}{\text{\# ways to choose 2 people}}$$

$$\frac{C^2_0 \, C^3_2}{C^5_2} = \frac{1 \times 3}{10} = .3$$

$$p(x=1) = P(1w, 1m) = \frac{C^2_1 \, C^3_1}{C^5_2} = \frac{2 \times 3}{10} = .6$$

$$p(x=2) = P(2w, 0m) = \frac{C^2_2 \, C^3_0}{C^5_2} = \frac{1 \times 1}{10}$$

$$= .1$$



FIND THE EXPECTED VALUE FOR x, i.e. $E[x]$, i.e. MEAN μ.

$$E[x] = \mu = \sum x p(x)$$

x	0	1	2
$p(x)$	.3	.6	.1

$$E[x] = \mu = 0(.3) + 1(.6) + 2(.1) = .8$$

IF WE WERE TO REPEAT THE EXPERIMENT MANY TIMES, & RECORD THE VALUES OF x, THE AVERAGE OF ALL x VALUES WOULD BE (APPROXIMATELY) .8.

ex. SUPPOSE A FARM HAS 6 CHICKENS

5 GOATS

8 COWS.

$\left.\begin{matrix} 5 \text{ GOATS} \\ 8 \text{ COWS} \end{matrix}\right\}$ 13 NON-CHICKENS

YOU SELECT 4 ANIMALS AT RANDOM.

LET X = # CHICKENS.

FIND PROBABILITY DISTRIBUTION FOR X & EXPECTED VALUE $E[X]$.

19 ANIMALS, CHOOSE 4. → TOTAL # POSSIBLE OUTCOMES = C^{19}_4

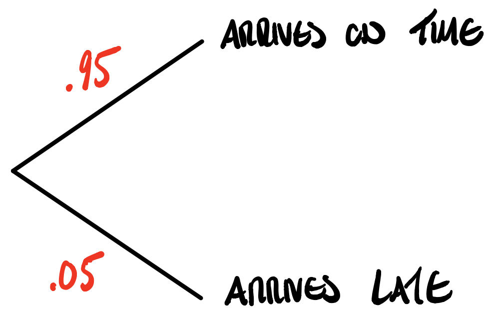
X	0	1	2	3	4
$p(x)$	$\frac{C^6_0 C^{13}_4}{C^{19}_4}$	$\frac{C^6_1 C^{13}_3}{C^{19}_4}$	$\frac{C^6_2 C^{13}_2}{C^{19}_4}$	$\frac{C^6_3 C^{13}_1}{C^{19}_4}$	$\frac{C^6_4 C^{13}_0}{C^{19}_4}$

THESE #'S ADD TO 4

$$E[X] = \mu = 0 \cdot \frac{C^6_0 C^{13}_4}{C^{19}_4} + 1 \cdot \frac{C^6_1 C^{13}_3}{C^{19}_4} + \dots + 4 \cdot \frac{C^6_4 C^{13}_0}{C^{19}_4}$$

Suppose a shipping company charge a flat rate of \$22.50 to ship a package. It costs the company \$11.75 to ship the package. If the package arrives late, the company guarantees a full refund to the customer. let x equal the net profit (gain/loss) experienced by the company for each package. If 95% of packages arrive on time, find the expected value for x.

EXP



Arrives on time:

$X = 22.50$ PAID
$- 11.75$ COST
$= 10.75$

Arrives late:

$X = 22.50$ PAID
$- 11.75$ COST
$- 22.50$ REFUND
$= - 11.75$

x	10.75	-11.75
$p(x)$	.95	.05

EXPECTED VALUE "AVERAGE" $E[x] = \mu = \sum x p(x)$

$$= (10.75)(.95) + (-11.75)(.05)$$

$$= \$ 9.625$$

AVERAGE (EXPECTED) PROFIT PER PACKAGE.

§5.2 THE BINOMIAL PROBABILITY DISTRIBUTION

"BERNOULLI EXPERIMENT"

Def: A BINOMIAL EXPERIMENT IS AN EXPERIMENT THAT SATISFIES THE FOLLOWING PROPERTIES:

(1) n IDENTICAL TRIALS

(2) EACH TRIAL RESULTS IN ONE OF TWO POSSIBLE OUTCOMES, BY CONVENTION WE LABEL THESE OUTCOMES

SUCCESS & FAILURE (COMPLIMENTS)

(3) THE n TRIALS ARE INDEPENDENT. THE RESULT OF ANY TRIAL IS NOT INFLUENCED BY THE RESULT OF ANY OTHER TRIAL.

(4) FOR EACH TRIAL,

$P(\text{SUCCESS}) = p$

$P(\text{FAILURE}) = 1 - p = q$

(5) RANDOM VARIABLE OF INTEREST:

$X = \#$ SUCCESSES IN n TRIALS.

$(x = 0, 1, 2, \ldots, n)$

e.g. FLIP A COIN 20 TIMES ($n = 20$ TRIALS).

EACH FLIP (TRIAL) RESULTS IN EITHER HEADS OR TAILS

(SUCCESS OR FAILURE).

X = # HEADS OBSERVED.

e.g. SHOOT 8 FREE THROWS ($n = 8$ TRIALS).

EACH SHOT YOU EITHER MAKE OR MISS.

FOR EACH TRIAL (SHOT) — MAKE: SUCCESS, MISS: FAILURE

$P(\text{SUCCESS}) = p$ IS THE SAME &

$P(\text{FAILURE}) = 1 - p = q$ IS THE SAME.

LET X = # SHOTS YOU MAKE (SUCCESSES).

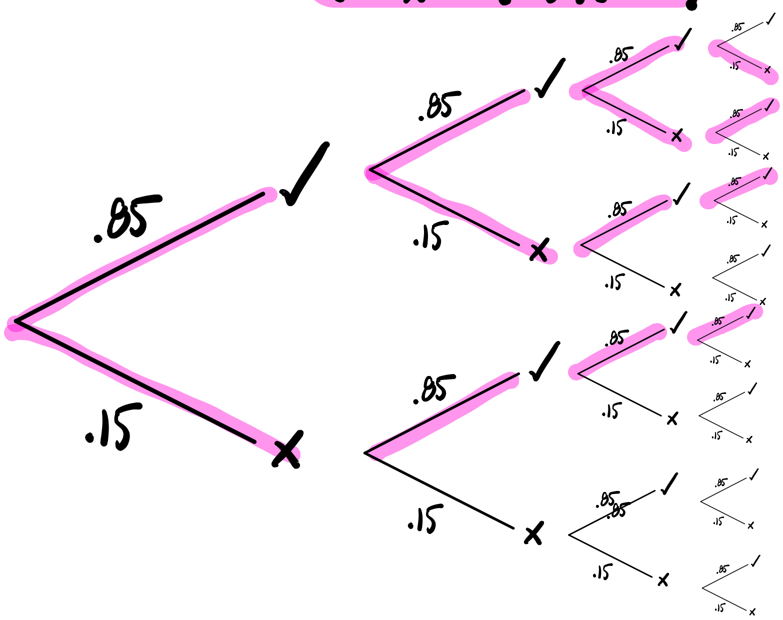
e.g. SUPPOSE A PLAYER MAKES 85% OF FREE THROWS.

IF THEY SHOOT 4 FREE THROWS, WHAT IS PROBABILITY THAT THEY MAKE EXACTLY 3 OF THEM?

$P(3 \text{ shots}) = P(✓✓✓×,\ ✓✓×✓,\ ✓×✓✓,\ ×✓✓✓)$

$= P(✓✓✓×) + P(✓✓×✓) + P(✓×✓✓) + P(×✓✓✓)$

$= (.85)(.85)(.85)(.15) + \ldots + (.15)(.85)(.85)(.85)$



$= 4(.85)^3(.15)$

$\uparrow$

C^4_3

FOR A BINOMIAL EXPERIMENT,

n TRIALS, EACH PROB. OF SUCCESS p

PROB. OF FAILURE $q = 1-p$

THE PROBABILITY OF GETTING EXACTLY k SUCCESSES IS

$$C^n_k \, p^k \, q^{n-k}$$

MIDTERM WILL COVER UP TO & INCLUDING §4.8 (NOT §5.2).

(MON 10/19)