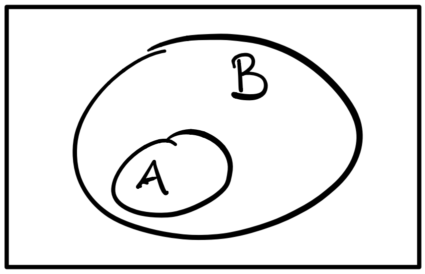
$A \subseteq B$

IS A SUBSET OF



§ 4.7 Bayes' Rule

Law of Total Prob.:

If $S_1, S_2, \dots, S_k$ are subsets of the sample space Ω, such that

- $S_1 \cup S_2 \cup \dots \cup S = \Omega$ (exhaustive)
- $S_i \cap S_j = \emptyset$ for any $i \neq j$ (mutually exclusive)

Then for any event $A \subseteq \Omega$, we have

$$P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2) + \dots + P(S_k)P(A|S_k)$$

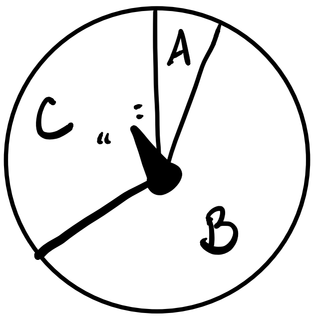
Bayes' Rule:

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)}$$

ex.

Suppose an experiment is performed as follows. You spin a spinner that can land on A, B, or C with probabilities .1, .6, and .3, respectively. Then you draw one marble from the box with that label (A, B, or C). Box A contains 10 red and 13 blue marbles, box B contain 32 red and 12 blue marbles, and box C contains 99 red and 1 blue marble.

If you perform this experiment and select a red marble, what is the probability that your marble cam from box C?



$P(A) = .1$

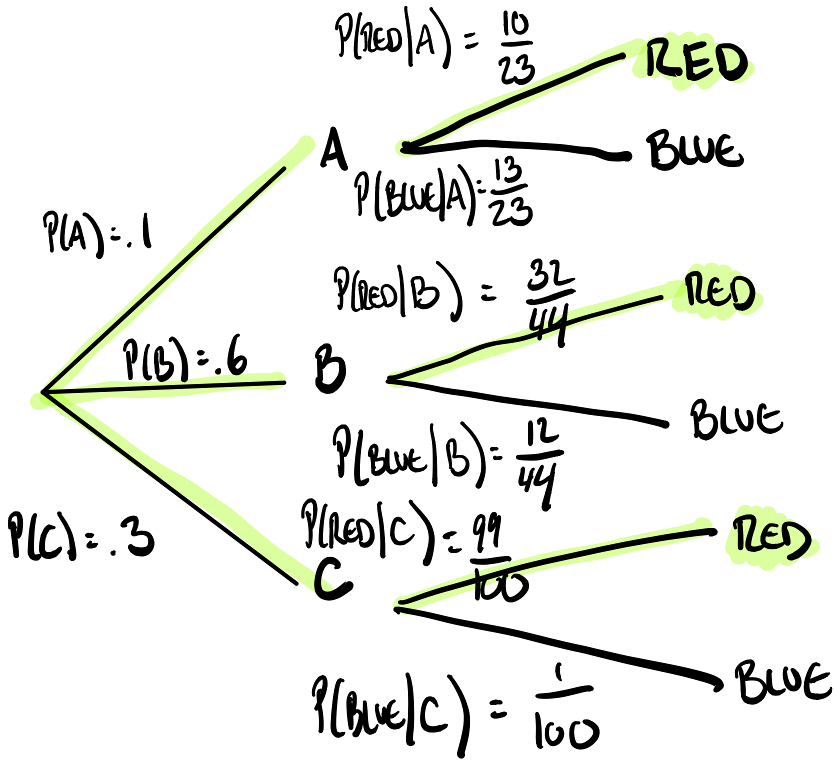
$P(B) = .6$

$P(C) = .3$

$P(\text{RED}|A) = \frac{10}{23}$

$P(\text{RED}|B) = \frac{32}{44}$

$P(\text{RED}|C) = \frac{99}{100}$



FIND $P(C|\text{RED})$.

BAYES' RULE:

$$P(C|RED) = \frac{P(C)P(RED|C)}{P(RED)} = \frac{(.3)\left(\frac{99}{100}\right)}{?}$$

$$P(RED) = P\Big((A \cap RED) \cup (B \cap RED) \cup (C \cap RED)\Big)$$

LoTP

$$= P(A \cap RED) + P(B \cap RED) + P(C \cap RED)$$

$$= P(A)P(RED|A) + P(B)P(RED|B) + P(C)P(RED|C)$$

$$= (.1)\left(\frac{10}{23}\right) + (.6)\left(\frac{32}{44}\right) + (.3)\left(\frac{99}{100}\right)$$

$$P(C|RED) = \frac{(.3)\left(\frac{99}{100}\right)}{(.1)\left(\frac{10}{23}\right) + (.6)\left(\frac{32}{44}\right) + (.3)\left(\frac{99}{100}\right)}$$

$$= .3823 \quad \text{(4 DECIMALS)}$$

4.72 Violent Crime City crime records show that 20% of all crimes are violent and 80% are nonviolent, involving theft, forgery, and so on. Ninety percent of violent crimes are reported versus 70% of nonviolent crimes.

a. What is the overall reporting rate for crimes in the city?

b. If a crime in progress is reported to the police, what is the probability that the crime is violent? What is the probability that it is nonviolent?

c. Refer to part b. If a crime in progress is reported to the police, why is it more likely that it is a nonviolent crime? Wouldn't violent crimes be more likely to be reported? Can you explain these results?

LET S_1 V = CRIME IS VIOLENT

S_2 V^c = CRIME IS NON VIOLENT

} EXHAUSTIVE & MUT. EXCL. SUB-SPACES.

A R = CRIME IS REPORTED

GIVEN: $P(V) = .2$ $\quad P(R|V) = .9$

$P(V^c) = .8$ $\quad P(R|V^c) = .7$

(a) $P(A) = P(S_1)P(A|S_1) + P(S_2)P(A|S_2)$ *

$$P(R) = P(V)P(R|V) + P(V^c)P(R|V^c)$$
$$= (.2)(.9) + (.8)(.7)$$
$$= .18 + .56$$
$$= .74$$

(b) $$P(V|R) = \frac{P(V)P(R|V)}{P(R)}$$

$$= \frac{(.2)(.9)}{.74}$$

$$= .2432$$

§4.8 DISCRETE PROBABILITY DISTRIBUTIONS

Def: A VARIABLE X IS A RANDOM VARIABLE IF THE VALUE IT TAKES CORRESPONDS TO THE OUTCOME OF A RANDOM EVENT.

e.g. EXP. SELECT A RANDOM PERSON FROM A SPECIFIED POPULATION.

LET X = THAT PERSON'S ~~AGE~~

ZIP CODE

\# OF SIBLINGS

WHOLE

# of Fingers

Notebooks in Backpack

Letters in Name.

#'s DISCRETE

Let X = Person's Height, Age

Weight

Bank Account Balance

Not (nec.) whole #'s CONTINUOUS

Def: A DISCRETE RANDOM VARIABLE can only take on a finite number of values, and we assume the values are whole numbers.

A CONTINUOUS RANDOM VARIABLE can equal ∞ many values, e.g. all number in an interval.

Continuous = Not Discrete.

For Now We Focus on Discrete R.V.

Def: The Probability Distribution for a Discrete R.V. is a Formula, Table, or Graph that Gives the Possible values for the Discrete R.V. X, and the Probability p(x) associated with each possible value.

$p(x) = P\left(\text{UNION OF ALL SIMPLE EVENTS THAT CAUSE THE RANDOM VARIABLE TO EQUAL } X.\right)$

ex. EXP: ROLL A DIE.

IF YOU ROLL 1-4: YOU LOSE $1.

IF YOU ROLL 5: YOU WIN $2.

IF YOU ROLL 6: YOU WIN $5.

LET X = MONEY YOU WIN/LOSE PLAYING THE GAME.
(+/-)

DESCRIBE THE PROBABILITY DISTRIBUTION FOR X.

SIMPLE EVENTS	PROBABILITY	VALUE OF X
1	1/6	-1
2	1/6	-1
3	1/6	-1
4	1/6	-1
5	1/6	2
6	1/6	5

x	-1	2	5
$p(x)$	4/6	1/6	1/6

ALL POSSIBLE VALUES

BETWEEN 0 & 1,
ADD UP TO 1.

Properties of Prob. Distributions for Discrete Random Variables:

- $0 \leq p(x) \leq 1$
- $\underbrace{\sum p(x)}$ $= 1$

Sum of all $p(x)$, over all possible values of X

4.86 If you toss a pair of dice, the sum T of the numbers appearing on the upper faces of the dice can assume the value of an integer in the interval $2 \leq T \leq 12$.

a. Find the probability distribution for T. Display this probability distribution in a table.

b. Construct a probability histogram for $P(T)$. How would you describe the shape of this distribution?

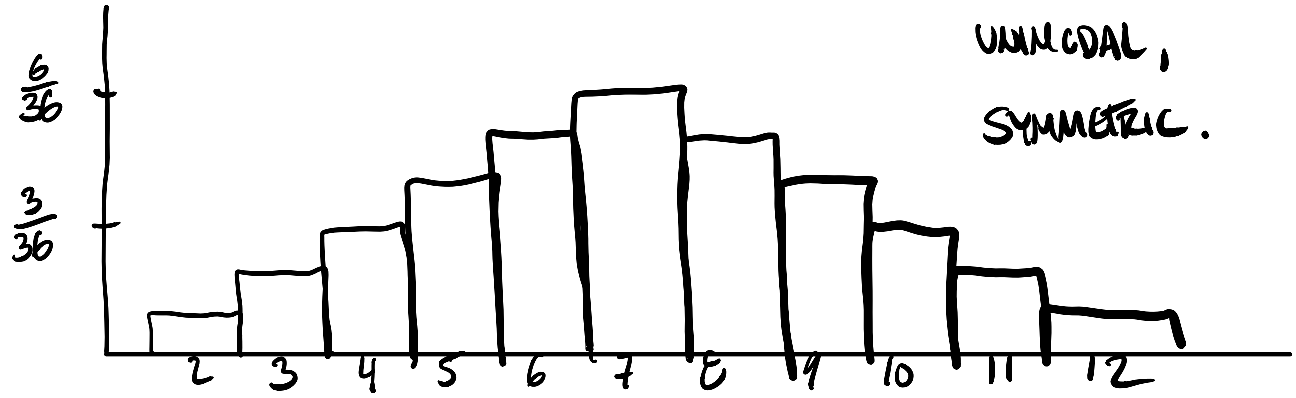
Sample space: Ω

1st die

2nd die

X	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

X	2	3	4	5	6	7	8	9	10	11	12
p(x)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$



UNIMODAL, SYMMETRIC.

MEAN & STANDARD DEVIATION FOR DISCRETE RANDOM VARIABLES

MEAN

EXP: ROLL A DIE.

IF YOU ROLL 1-4 : YOU LOSE \$1.

IF YOU ROLL 5 : YOU WIN \$2.

IF YOU ROLL 6 : YOU WIN \$5.

LET X = MONEY YOU WIN/LOSE PLAYING THE GAME.
(+ / -)

WHAT IS THE MEAN μ FOR THE RANDOM VARIABLE X ?

IMAGINE PERFORMING THE EXP. OVER & OVER, e.g. 36,000 TIMES.

SIMPLE EVENT	FREQUENCY	
1	~ 6,000	-1
2	~ 6,000	-1
3	~ 6,000	-1
4	~ 6,000	-1
5	~ 6,000	→ 2
6	~ 6,000	→ 5

X	FREQUENCY
-1	~ 24,000
2	~ 6,000
5	~ 6,000

COMPUTE THE MEAN FOR X

$$\mu = \frac{24000(-1) + 6000(2) + 6000(5)}{36000}$$

$$\mu = \frac{24000}{36000}(-1) + \frac{6000}{36000}(2) + \frac{6000}{36000}(5)$$

$$\mu = \left(\frac{4}{6}\right)(-1) + \left(\frac{1}{6}\right)(2) + \left(\frac{1}{6}\right)(5) = .5$$

THE AVERAGE VALUE OF X, WHEN IS PERFORMED REPEATEDLY, IS .5, i.e. $\mu = \$0.50$.

x	-1	2	5
$p(x)$	$4/6$	$1/6$	$1/6$

MEAN OF A DISCRETE RANDOM VARIABLE X IS

$$\mu = \sum x\,p(x)$$

ADD UP ALL OF THE PRODUCTS OF POSSIBLE VALUES OF X & CORRESPONDING PROBABILITIES $p(x)$.

NOTE: MEAN FOR RANDOM VARIABLE X, μ

SAME THING AS

EXPECTED VALUE OF R.V. X, $E[x]$

4.85 **Grocery Visits** Let x represent the number of times a customer visits a grocery store in a 1-week period. Assume this is the probability distribution of x:

x	0	1	2	3
$p(x)$	.1	.4	.4	.1

Find the expected value of x, the average number of times a customer visits the store.

$$\mu = E[x] = \sum x p(x) = 0(.1) + 1(.4) + 2(.4) + 3(.1)$$

$$= 1.5$$