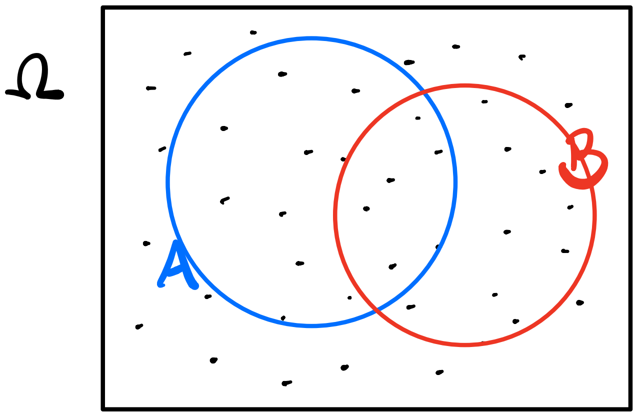
Review of Conditional Probability:



Assume $|\Omega| < \infty$, all equally likely.

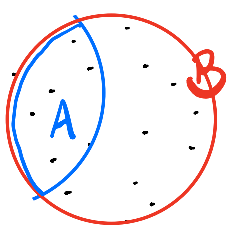
$$P(A) = \frac{|A|}{|\Omega|}, \quad P(B) = \frac{|B|}{|\Omega|}$$

i.e. If B occurs, what is the probability that A occurs?

"Conditional Clause"

Now we need to update our set of all possible outcomes to include only simple events in B.

(eliminate B^c)



= new Ω

Probability that A occurs, given that B occurs

$$= P(A|B) = \frac{|A \cap B| \,/\, |\Omega|}{|B| \,/\, |\Omega|}$$

$$\Rightarrow \boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$$

"The probability of A given B"

MANIPULATE FORMULA FOR CONDITIONAL PROBABILITY:

$$P(B)\cdot P(A|B) = \frac{P(A\cap B)}{P(B)}\cdot P(B)$$

$$P(A\cap B) = P(B)\,P(A|B)$$

SAME — SWITCH A & B

$$P(B\cap A) = P(A)\,P(B|A)$$

THE GENERAL MULTIPLICATION RULE

In a color preference experiment, eight toys are placed in a container. The toys are identical except for color—two are red, and six are green. A child is asked to choose two toys *at random*. What is the probability that the child chooses the two red toys?

OLD WAY TO ANSWER:

\# WAYS TO CHOOSE 2 TOYS FROM 8 TOYS IS

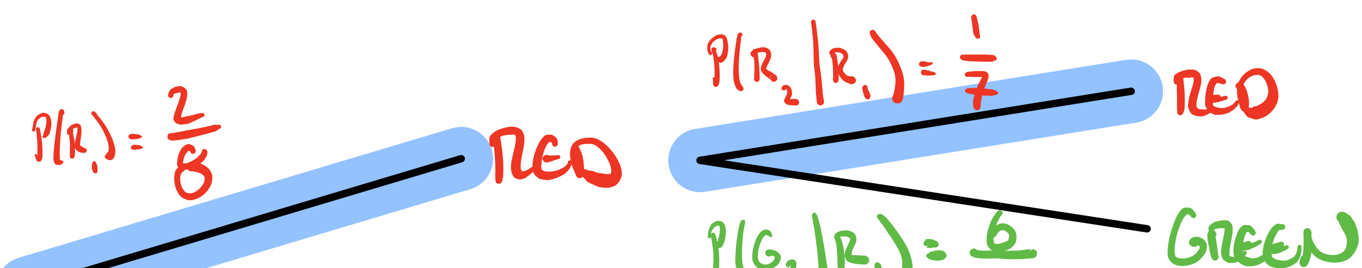
$$C^8_2 = \frac{8!}{2!\,6!} = \frac{8\cdot 7}{2} = 28$$

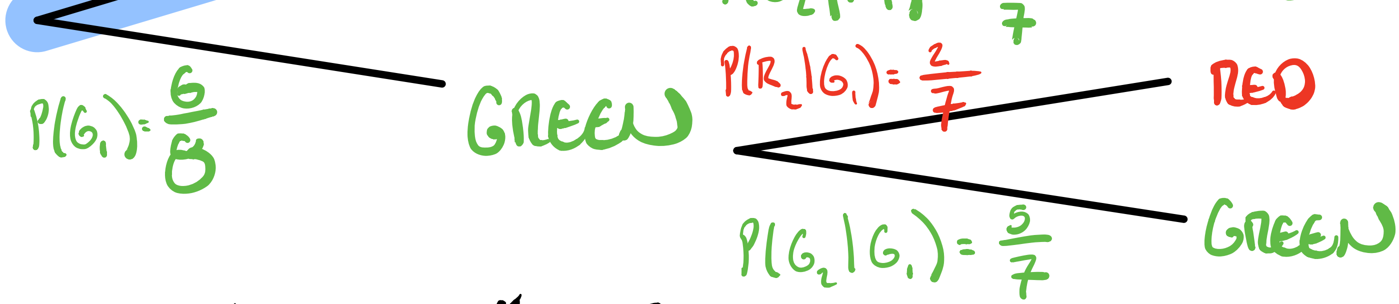
\# WAYS TO CHOOSE 2 RED TOYS : $C^2_2 = 1$

$$\therefore P(2 \text{ RED TOYS}) = \frac{C^2_2}{C^8_2} = \boxed{\frac{1}{28}}$$

NEW WAY: TREE DIAGRAMS W/ PROBABILITY

1ST TOY 2ND TOY





EVENTS

R_1 : 1ST TOY RED

R_2 : 2ND TOY RED

G_1 : 1ST TOY GREEN

G_2 : 2ND TOY GREEN

GENERAL MULTIPLICATION RULE: *

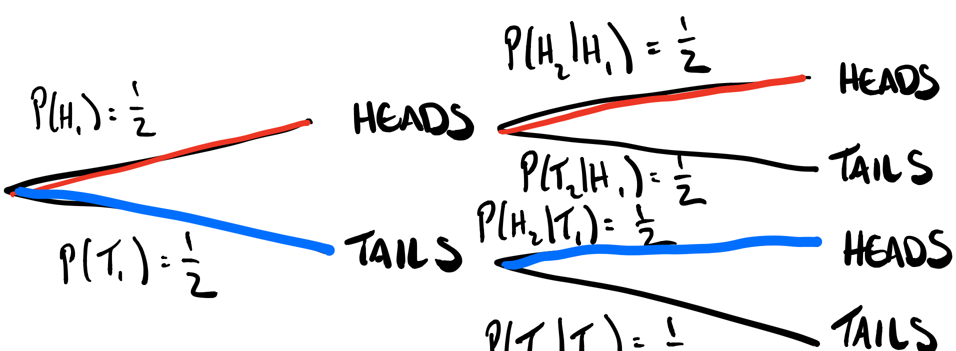
$$P(R_1 \cap R_2) = P(R_1)P(R_2|R_1) = \left(\frac{2}{8}\right)\left(\frac{1}{7}\right) = \frac{2}{56} = \boxed{\frac{1}{28}}$$

1ST TOY RED AND 2ND TOY RED

* WE MULTIPLY PROBABILITIES ALONG PATHS FROM LEFT TO RIGHT THROUGH THE TREE DIAGRAM

$$P(R_2 | R_1) = \frac{P(R_1 \cap R_2)}{P(R_1)}$$

ex. SUPPOSE WE FLIP A COIN TWICE.
FIND THE PROBABILITY THAT THE SECOND COIN IS HEADS.
FIND THE PROBABILITY THAT THE SECOND COIN IS HEADS GIVEN THAT THE FIRST COIN IS TAILS.



$$P(H_2) = P\Big(\underbrace{(H_2 \cap H_1) \cup (H_2 \cap T_1)}_{\text{MUTUALLY EXCLUSIVE}}\Big)$$

$$= P(H_2 \cap H_1) + P(H_2 \cap T_1) - \overbrace{P\big((H_2 \cap H_1) \cap (H_2 \cap T_1)\big)}^{0}$$

$$= P(H_1)P(H_2|H_1) + P(T_1)P(H_2|T_1)$$

$$= \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)$$

$$= \boxed{\tfrac{1}{2}}$$

$$P(H_2|T_1) = \boxed{\tfrac{1}{2}}$$

EXTRA INFO REGARDING T_1 DOESN'T HAVE ANY EFFECT ON PROBILITY H_2.

THIS MEANS H_2 & T_1 ARE

INDEPENDENT EVENTS.

Def: TWO EVENTS A & B ARE CALLED INDEPENDENT IF ANY OF THE FOLLOWING ARE TRUE:

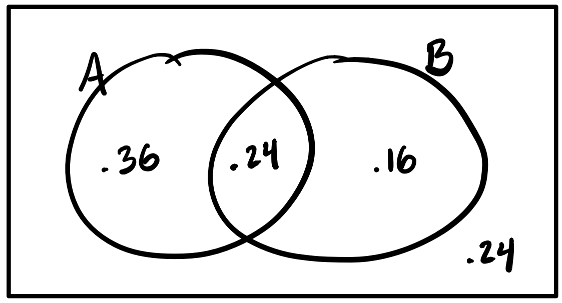
$$P(A \cap B) = P(A)P(B),$$

$$P(A) = P(A|B), \quad P(B) = P(B|A),$$

$$P(A) = P(A|B^c), \quad P(B) = P(B|A^c).$$

ex.

	A	A^c
B	.24	.16
B^c	.36	.24



$$P(A) = P(A \cap B) + P(A \cap B^c) = .24 + .36 = \boxed{.6}$$

$$P(B) = P(B \cap A) + P(B \cap A^c) = .24 + .16 = \boxed{.4}$$

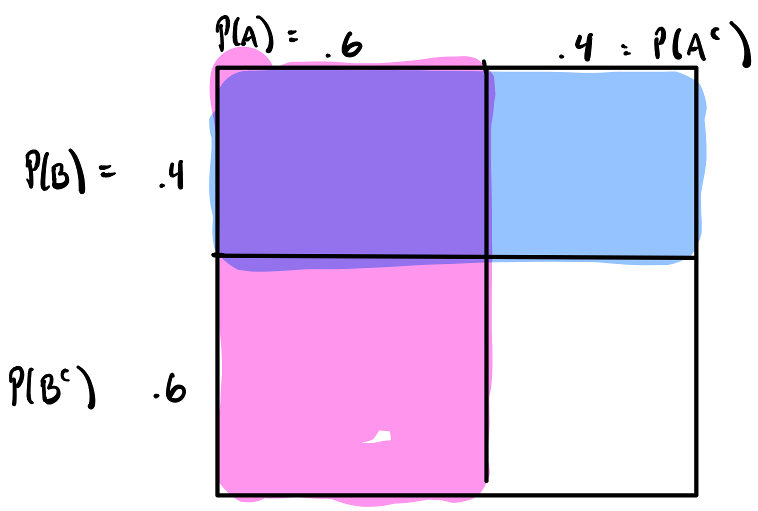
$$P(A \cap B) = .24$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.24}{.4} = \boxed{.6}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{.24}{.6} = \boxed{.4}$$

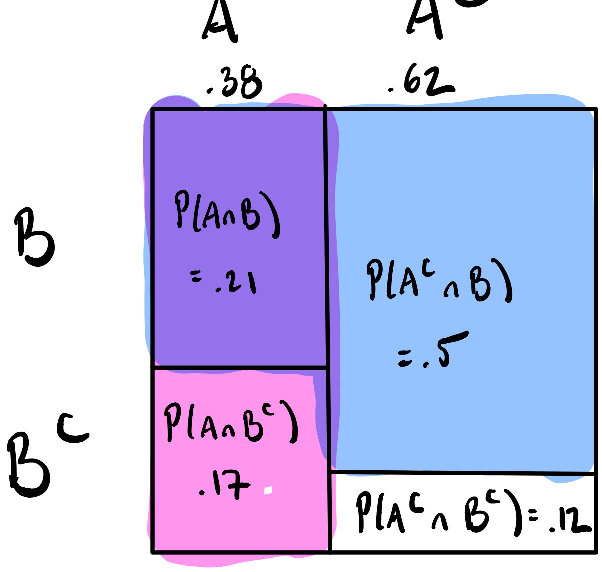
ARE A & B INDEPENDENT EVENTS? YES

$$\left(\begin{array}{c} P(A) = P(A|B), \quad P(B) = P(B|A) \\ P(A \cap B) = P(A)\,P(B) \\ .24 = .6 \times .4 = .24 \end{array} \right) \checkmark$$



	A	A^c
B	.24	.16
B^c	.36	.24

$\Omega = 1 \times 1$ SQUARE



	A	A^c
B	.21	.5
B^c	.17	.12

4.62 Smoking and Cancer A survey of people in a given region showed that 20% were smokers. The probability of death due to lung cancer, given that a person smoked, was roughly 10 times the probability of death due to lung cancer, given that a person did not smoke. If the probability of death due to lung cancer in the region is .006, what is the probability of death due to lung cancer given that a person is a smoker?

(1) INTRODUCE NOTATION: LET S = EVENT THAT A PERSON IS A SMOKER

D = DEATH DUE TO LUNG CANCER

(2) SUMMARIZE GIVEN INFO:

$P(S) = .2$ $\quad$ $P(D) = .006$

$P(S^c) = .8$

$$P(D|S) = 10\,P(D|S^c)$$

(3) WHAT IS BEING ASKED?

FIND $P(D|S)$.

(4) SOLVE.

$$P(D) = P(D \cap S) + P(D \cap S^c)$$

$$= P(S)P(D|S) + P(S^c)P(D|S^c)$$

$$.006 = (.2)10P(D|S^c) + (.8)P(D|S^c)$$

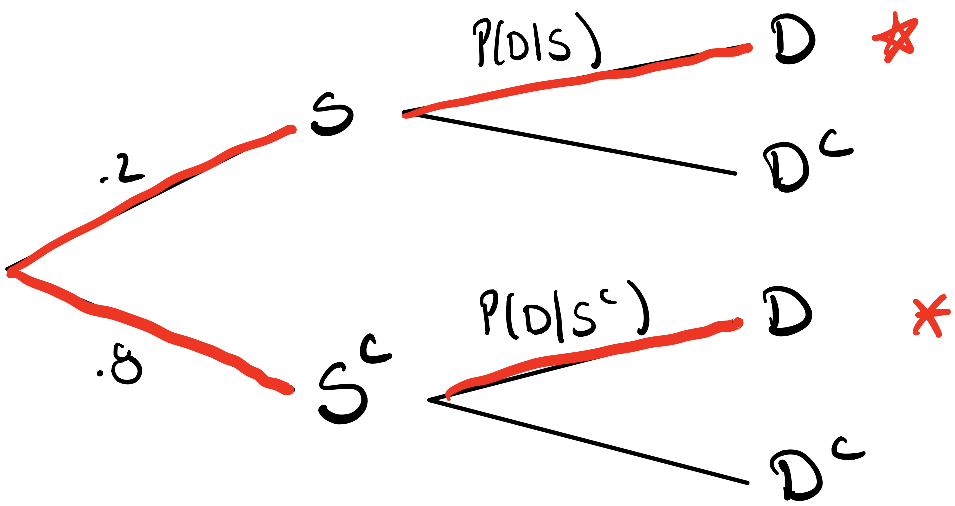
$$.006 = 2P(D|S^c) + .8P(D|S^c)$$

$$.006 = 2.8P(D|S^c)$$

$$.0021429 = P(D|S^c)$$

$$P(D|S) = 10P(D|S^c) = 10(.0021429)$$

$$= .02143$$



$$P(D) = \underset{*}{P(S \cap D)} + \underset{*}{P(S^c \cap D)} \ \ldots$$

4.67 Kobe and Lamar Two stars of the *LA Lakers* are very different when it comes to making free throws. ESPN.com reports that Kobe Bryant makes 85% of his free throw shots while Lamar Odum makes 62% of his free throws.[5] Assume that the free throws are independent and that each player shoots two free throws during a team practice.

a. What is the probability that Kobe makes both of his free throws?

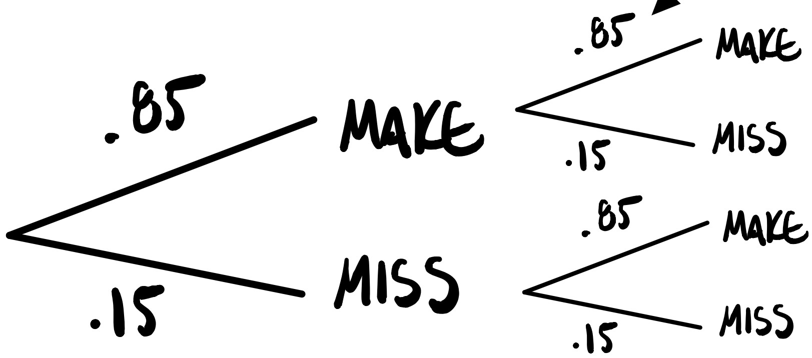
b. What is the probability that Lamar makes exactly one of his two free throws?

c. What is the probability that Kobe makes both of his free throws and Lamar makes neither of his?

$P(\text{MAKE } 1^{st} \cap \text{MAKES } 2^{nd})$
$= P(\text{MAKE } 1^{st}) P(\text{MAKE } 2^{nd})$
$= (.85)(.85)$
$= .7225$

$P(\text{MAKE } 2^{nd} \mid \text{MAKE } 1^{st}) = P(\text{MAKE } 2^{nd})$

(a)



(b) LET L_1 = LAMAR MAKES SHOT 1
L_2 = LAMAR MAKES SHOT 2

$P(L_1 \cap L_2) = P(L_1)P(L_2 \mid L_1)$ L_1 & L_2 INDEPENDENT
$= P(L_1)P(L_2)$
$= (.62)(.62)$
$= .3844$

(c) K_1 = KOBE MAKES 1^{st} SHOT
K_2 = KOBE MAKES 2^{nd} SHOT

$P(K_1 \cap K_2 \cap L_1^c \cap L_2^c) = P(K_1)P(K_2)P(L_1^c)P(L_2^c)$
$= (.85)(.85)(1-.62)(1-.62)$

$= .104329$