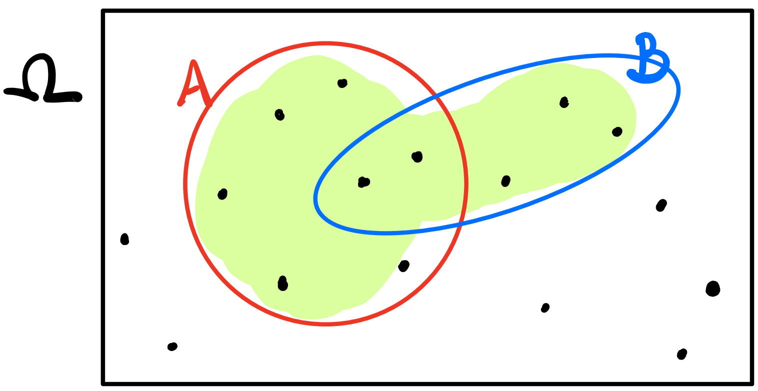
§ 4.5 Event Relations & Probability Rules

$\cup$ "cup"

Def: Given two events $A \subseteq \Omega$ & $B \subseteq \Omega$, the union of A and B, $A \cup B$, is the event that either A occurs or B occurs (or both occur!).

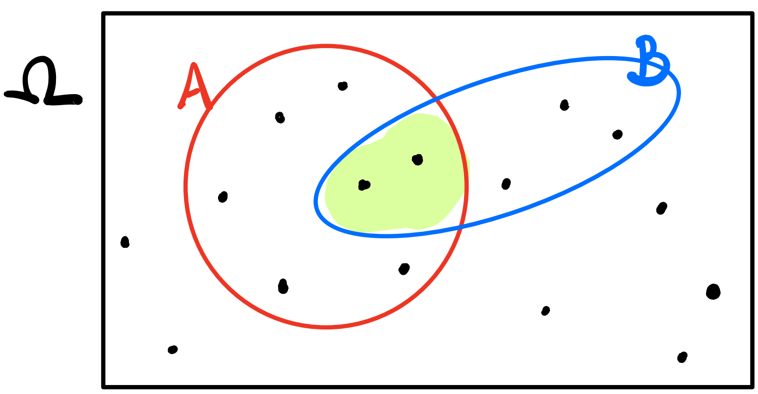
↳ not exclusive



$A \cup B$

$\cap$ "cap"

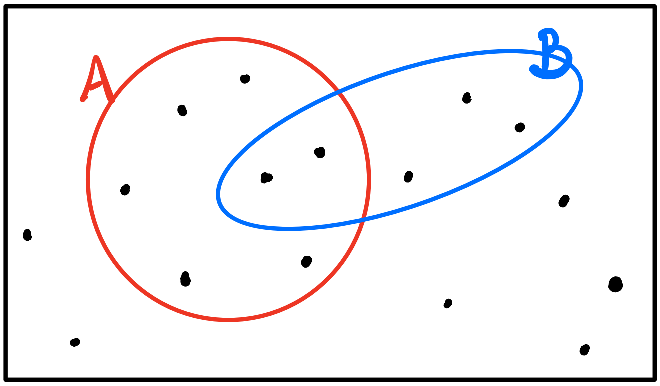
Def: Given two events $A \subseteq \Omega$ & $B \subseteq \Omega$, the intersection of A and B, $A \cap B$, is the event that both A and B occur.



$A \cap B$

Counting Simple Events:

Ω



$$|A \cup B| = |A| + |B| - |A \cap B|$$

Assume finite number of simple events, all equally likely

$$\frac{|A \cup B|}{|\Omega|} = \frac{|A|}{|\Omega|} + \frac{|B|}{|\Omega|} - \frac{|A \cap B|}{|\Omega|}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

ADDITION RULE

ex. Suppose a restaurant sells 140 pizzas:

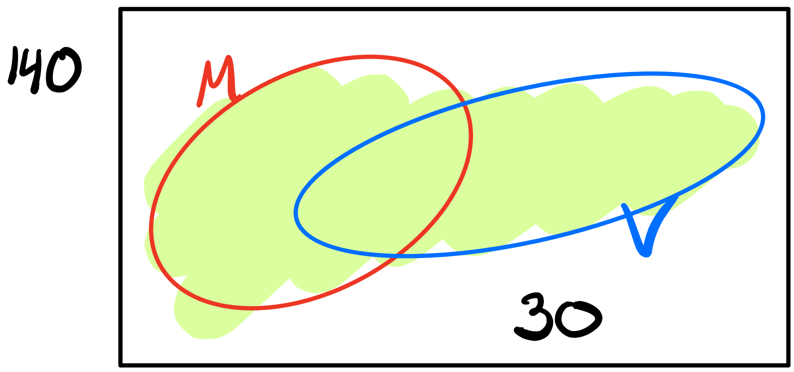
80 pizzas with meat

90 pizzas with vegetables

30 pizzas with neither meat nor vegetables.

How many pizzas did they serve with both

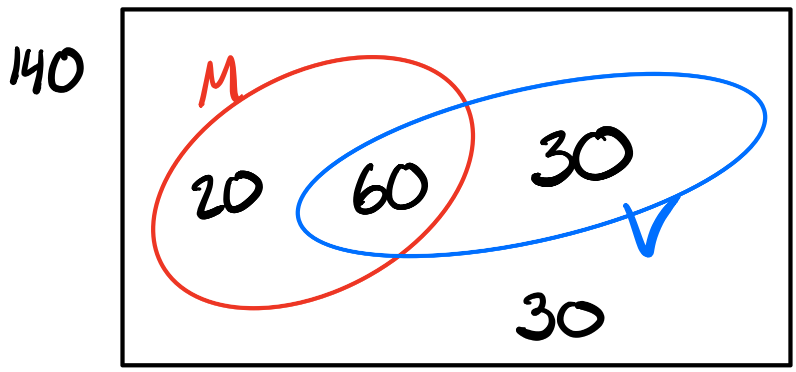
MEAT AND VEGETABLES



$$|M \cup V| = |M| + |V| - |M \cap V|$$

$$110 = 80 + 90 - |M \cap V|$$

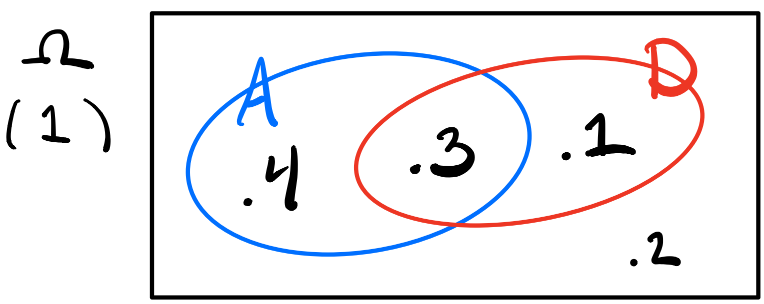
$$|M \cap V| = 80 + 90 - 110 = \boxed{60}$$



$|M| = 80$

$|V| = 90$

ex. SUPPOSE 70% OF CUSTOMERS ORDER APPETIZERS, 40% OF CUSTOMERS ORDER DESSERT, AND 20% ORDER NEITHER. FIND THE PROBABILITY THAT A RANDOMLY SELECTED CUSTOMER ORDERS BOTH APP & DESSERT.



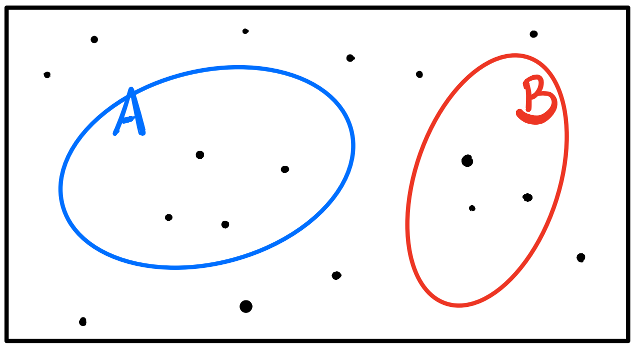
$P(A) = .7$

$P(D) = .4$

$$P(A \cup D) = P(A) + P(D) - P(A \cap D)$$
$$.8 = .7 + .4 - P(A \cap D)$$
$$P(A \cap D) = .7 + .4 - .8 = .3 \text{ or } 30\%$$

DEF: TWO EVENTS $A \subseteq \Omega$ AND $B \subseteq \Omega$ ARE MUTUALLY EXCLUSIVE IF $A \cap B = \emptyset$ i.e. $P(A \cap B) = 0$.

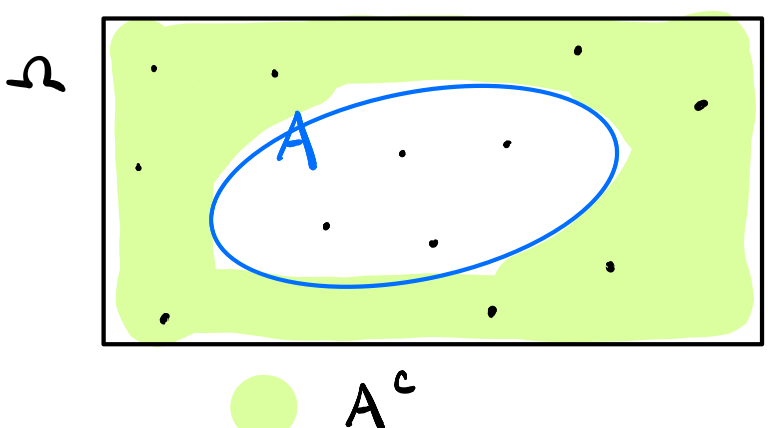
NO OVERLAP



IN THIS CASE: $P(A \cup B) = P(A) + P(B) - \underbrace{P(A \cap B)}_{0}$

$$P(A \cup B) = P(A) + P(B)$$

Def: GIVEN AN EVENT $A \subseteq \Omega$, THE COMPLEMENT OF A, A^c OR A', IS THE EVENT THAT A DOES NOT OCCUR, i.e. IT CONTAINS ALL SIMPLE EVENTS NOT IN A.



NOTE $A \cap A^c = \emptyset$

A, A^c ARE MUTUALLY EXCLUSIVE.

NOTE: $(A^c)^c = A$

SINCE $|\Omega| = |A| + |A^c|$

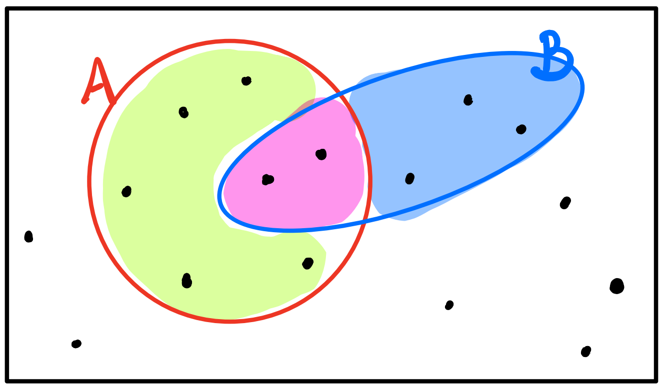
$$\Rightarrow \frac{|\Omega|}{|\Omega|} = \frac{|A|}{|\Omega|} + \frac{|A^c|}{|\Omega|}$$

$$1 = P(A) + P(A^c)$$

$$P(A^c) = 1 - P(A)$$

RULE OF COMPLEMENTS

Ω



NOTE:

$$A \cup B = \underbrace{(A \cap B^c) \cup (A \cap B) \cup (A^c \cap B)}_{\text{MUTUALLY EXCLUSIVE!}}$$

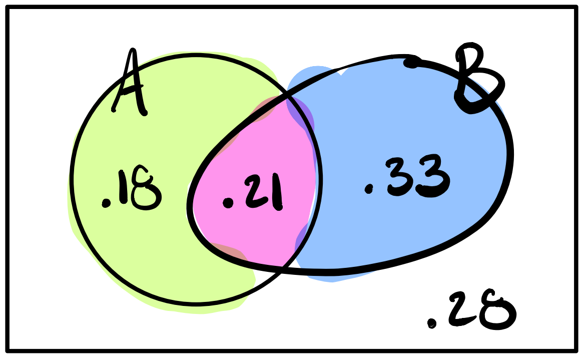
$$\Rightarrow P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B)$$

IN GENERAL, WHEN $A_1, A_2, \ldots, A_n$ ARE MUTUALLY EXCLUSIVE EVENTS, WE HAVE

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \ldots + P(A_n)$$

ex. AN EXPERIMENT CAN RESULT IN EVENTS A & B, WITH THE FOLLOWING PROBABILITIES:

	A	A^c
B	.21	.33
B^c	.18	.28



FIND $P(A)$. $P(A) = P(A \cap B^c) + P(A \cap B)$

IN GENERAL, GIVEN TWO EVENTS $A \subseteq \Omega$ & $B \subseteq \Omega$,

$A = (A \cap B) \cup (A \cap B^c)$

MUTUALLY EXCLUSIVE.

$\Rightarrow$ $P(A) = P(A \cap B) + P(A \cap B^c)$

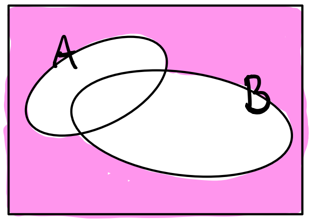
FIND $P(A \cup B)$. $P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B)$

$= .18 + .21 + .33 = .72$

DE MORGAN'S LAW:

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$



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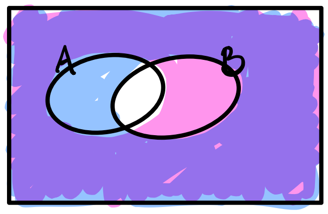
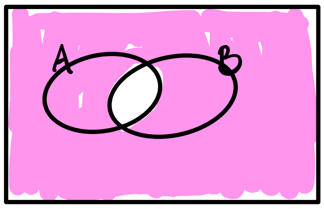


INTERSECTION

$$(A \cup B)^c = A^c \cap B^c$$



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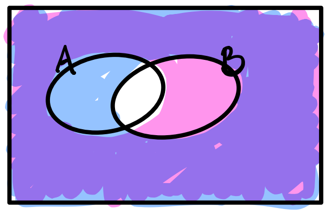
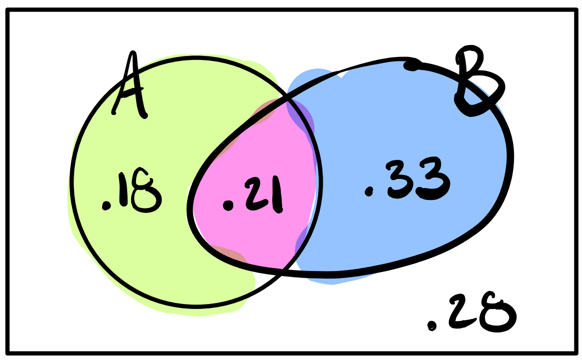


UNION

$$(A \cap B)^c = A^c \cup B^c$$

ex. AN EXPERIMENT CAN RESULT IN EVENTS A & B, WITH THE FOLLOWING PROBABILITIES:

	A	A^c
B	.21	.33
B^c	.18	.28



FIND $P(A \cup B) = 1 - P((A \cup B)^c)$

$= 1 - P(A^c \cap B^c)$

$= 1 - .28$

$= .72$

$P(A \cup B) = P(A \cap B^c) + P(A \cap B) + P(A^c \cap B)$

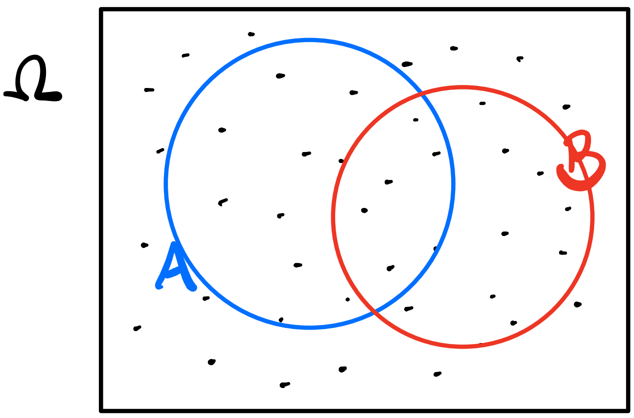
$= .18 + .21 + .33$

$= .72$

§4.6 Conditional Probability, Independence, & the Multiplication Rule

Assume $|\Omega| < \infty$, all equally likely.

Conditional Probability:



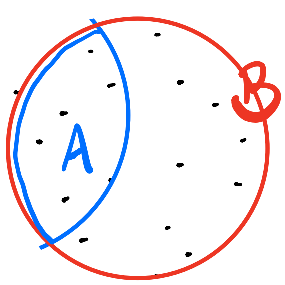
$P(A) = \dfrac{|A|}{|\Omega|}$, $P(B) = \dfrac{|B|}{|\Omega|}$

Now, what is the probability that A occurs, given that B occurs?

i.e. If B occurs, what is the probability that A occurs?

"Conditional clause"

Now we need to update our set of all possible outcomes to include <u>only</u> simple events in B.



= New Ω

PROBABILITY THAT A OCCURS, GIVEN THAT B OCCURS

$$= P(A \mid B) = \frac{|A \cap B|}{|B|}$$

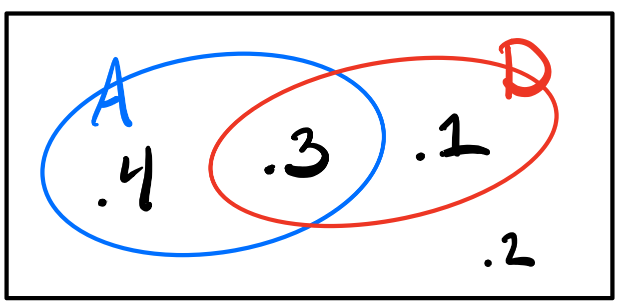
$$= \frac{|A \cap B| / |\Omega|}{|B| / |\Omega|}$$

$$\boxed{P(A \mid B) = \frac{P(A \cap B)}{P(B)}}$$

ex. SUPPOSE 70% OF CUSTOMERS ORDER APPETIZERS, 40% OF CUSTOMERS ORDER DESSERT, AND 20% ORDER NEITHER. FIND THE PROBABILITY THAT A FIND THE PROBABILITY THAT A CUSTOMER ORDERS DESSERT GIVEN THAT THEY ORDER AN APPETIZER.

Ω

(1)



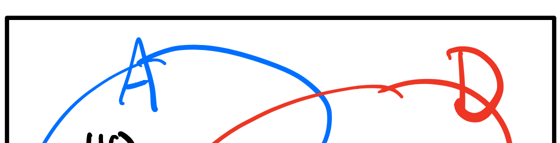
$P(A) = .7$

$P(D) = .4$

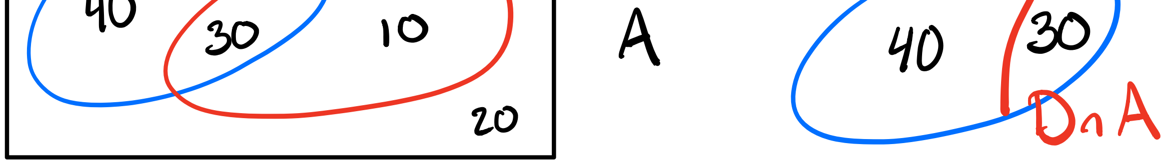
(1) INTRODUCE NOTATION: A, D

(2) WHAT'S BEING ASKED? $P(D \mid A)$

100



GIVEN $\longrightarrow$



$\frac{30}{70}$ ORDER DESSERT

$$P(D \mid A) = \frac{P(D \cap A)}{P(A)} = \frac{.3}{.7} = .4286$$

42.86%

ex.

	A	A^c
B	.1	.2
B^c	.3	.4

FIND $P(A) = P(A \cap B) + P(A \cap B^c) = .1 + .3 = .4$

FIND $P(B) = P(B \cap A) + P(B \cap A^c) = .1 + .2 = .3$

FIND $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{.1}{.3} = .3333$

FIND $P(B \mid A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{.1}{.4} = .25$

Def: TWO EVENTS $A \subseteq \Omega$ & $B \subseteq \Omega$ ARE CALLED **INDEPENDENT** IF

$P(A \mid B) = P(A)$, OR EQUIVALENTLY, IF

$P(B \mid A) = P(B)$.

OTHERWISE THEY ARE CALLED NOT INDEPENDENT.

IN THE PREVIOUS EXAMPLE, A & B ARE <u>NOT</u> INDEPENDENT.