SUMMARY OF PROBABILITY SO FAR:

SAMPLE SPACE (S OR Ω)

SET OF ALL POSSIBLE OUTCOMES OF AN EXPERIMENT.

$$\Omega = \{ E_1, E_2, \ldots, E_n \}$$

n POSSIBLE OUTCOMES
(SIMPLE EVENTS)

$$0 \leq P(E_1), P(E_2), \ldots, P(E_n) \leq 1$$

$$P(\Omega) = P(E_1) + P(E_2) + \ldots + P(E_n) = 1$$

FOR ANY EVENT $A \subseteq \Omega$

SOME POSSIBLE OUTCOMES.

e.g. $A = \{ E_2, E_5, E_7 \}$

IF ANY OF THESE SIMPLE ARE OBSERVED THEN WE SAY EVENT A OCCURS / IS OBSERVED

$$P(A) = P(E_2) + P(E_5) + P(E_7)$$

EXAMPLE: EXP. ROLL 1 DIE

$$\Omega = \underbrace{\{1, 2, 3, 4, 5, 6\}}_{\text{SIMPLE EVENTS}}$$

EVENT A = ROLL A 4 OR A 5

$= \{4, 5\}$

B = ROLL A 3 OR HIGHER

$= \{3, 4, 5, 6\}$

IF I PERFORM THE EXPERIMENT 100 TIME &

OBSERVE EVENT A 31 TIMES,

THEN $P(A) \approx \frac{31}{100}$

IF I PERFORM THE EXPERIMENT 10,000 TIME &

OBSERVE EVENT A 3,322 TIMES,

THEN $P(A) \approx \frac{3322}{10{,}000}$

IF BB PLAYER GOES TO BAT 350 TIMES IN A SEASON & THEY GET A HIT 100 TIMES, THEN "BATTING AVERAGE" IS $\frac{100}{350} = .286$

$P(\text{THIS PLAYER GETS A HIT}) = .286$

EXAMPLE: EXP. ROLL 1 DIE

$$\Omega = \underbrace{\{1, 2, 3, 4, 5, 6\}}_{\text{SIMPLE EVENTS}}$$

EVENT A = ROLL A 4 OR A 5

$= \{4, 5\}$

B = ROLL A 3 OR HIGHER

$= \{3, 4, 5, 6\}$

$$P(A) = P(\{4, 5\})$$

$$= P(\{4\}) + P(\{5\})$$

$$= \frac{1}{6} + \frac{1}{6} = \boxed{\frac{1}{3}}$$

4.2 A sample space S consists of five simple events with these probabilities:

$$P(E_1) = P(E_2) = .15 \qquad P(E_3) = .4$$
$$P(E_4) = 2P(E_5)$$

a. Find the probabilities for simple events E_4 and E_5.

b. Find the probabilities for these two events:

$A = \{E_1, E_3, E_4\}$

$B = \{E_2, E_3\}$

c. List the simple events that are either in event A or event B or both.

d. List the simple events that are in both event A and event B.

$P(E_4) . 2$

$P(E_5) = .1$

$$S = \{E_1, E_2, E_3, E_4, E_5\}$$

$$P(S) = P(\{E_1, E_2, E_3, E_4, E_5\}) = 1$$

$$= P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) = 1$$

$$.15 + .15 + .4 + 2P(E_5) + P(E_5) = 1$$

$$.7 + 3P(E_5) = 1$$

$$\left(3P(E_5) = .3 \right) \div 3$$

$$P(E_5) = .1$$

$$P(E_4) = 2P(E_5) = 2(.1) = .2$$

$$P(A) = P(\{E_1, E_3, E_4\})$$

$$= P(E_1) + P(E_3) + P(E_4)$$

$$= .15 + .4 + .2$$

$$= .75$$

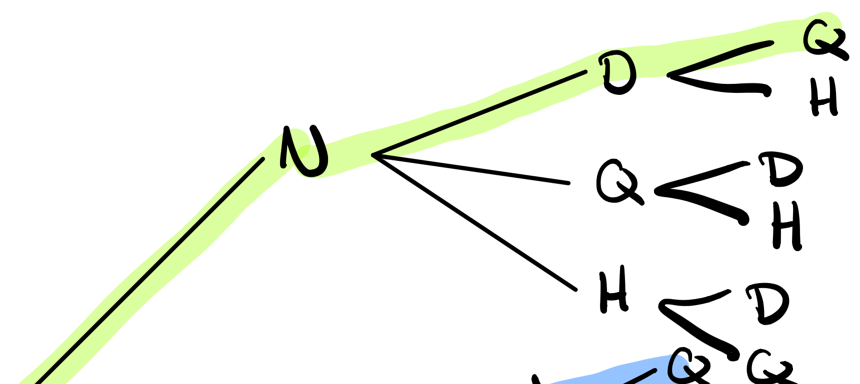
4.5 **Four Coins** A jar contains four coins: a nickel, a dime, a quarter, and a half-dollar. Three coins are randomly selected from the jar.

a. List the simple events in S.

b. What is the probability that the selection will contain the half-dollar?

c. What is the probability that the total amount drawn will equal 60¢ or more?

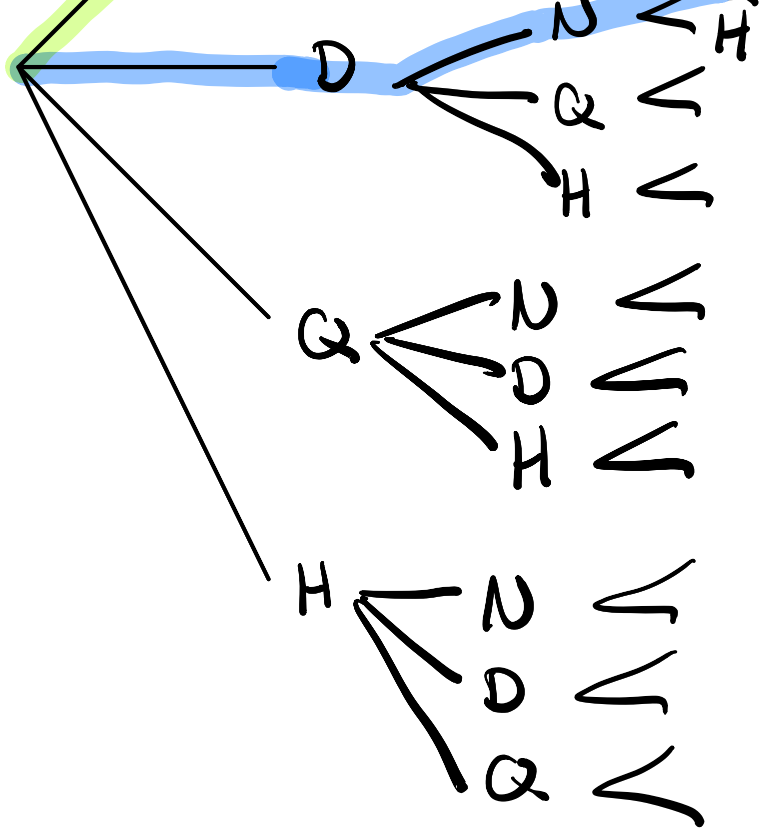
S = {NDQ, DQH, NQH, NDH}



NDQ

DO YOU WANT TO DISTINGUISH BETWEEN THESE OUTCOMES?

DNQ



$4 \cdot 3 \cdot 2 = 24$

SIMPLE EVENTS

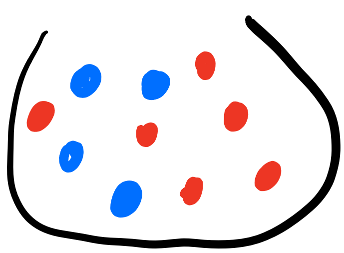
§ 4.4 USEFUL COUNTING RULES

SUPPOSE THE SAMPLE SPACE Ω FOR AN EXPERIMENT CONTAINS FINITELY MANY POSSIBLE OUTCOMES, ALL EQUALLY LIKELY.

THEN GIVEN ANY EVENT $A \subseteq \Omega$

$$P(A) = \frac{|A|}{|\Omega|}.$$

ex.



EXP: SELECT 1 MARBLE.

EVENT B : SELECT BLUE MARBLE

NOTE $|\Omega|$ = # POSSIBLE OUTCOMES = 10

& ALL 10 POSS. OUTCOMES ARE EQUALLY LIKELY.

$$P(B) = \frac{|B|}{|\Omega|} = \frac{4}{10} = \boxed{.4}$$

COUNT # OF SIMPLE EVENTS IN B

COUNT THE # OF SIMPLE EVENTS IN Ω

SINCE COUNTING CAN BE SURPRISINGLY TRICKY, WE NOW DEVELOP SOME TOOLS TO HELP

(COMBINATORICS)

↑

THE STUDY OF COUNTING.

THE mn RULE

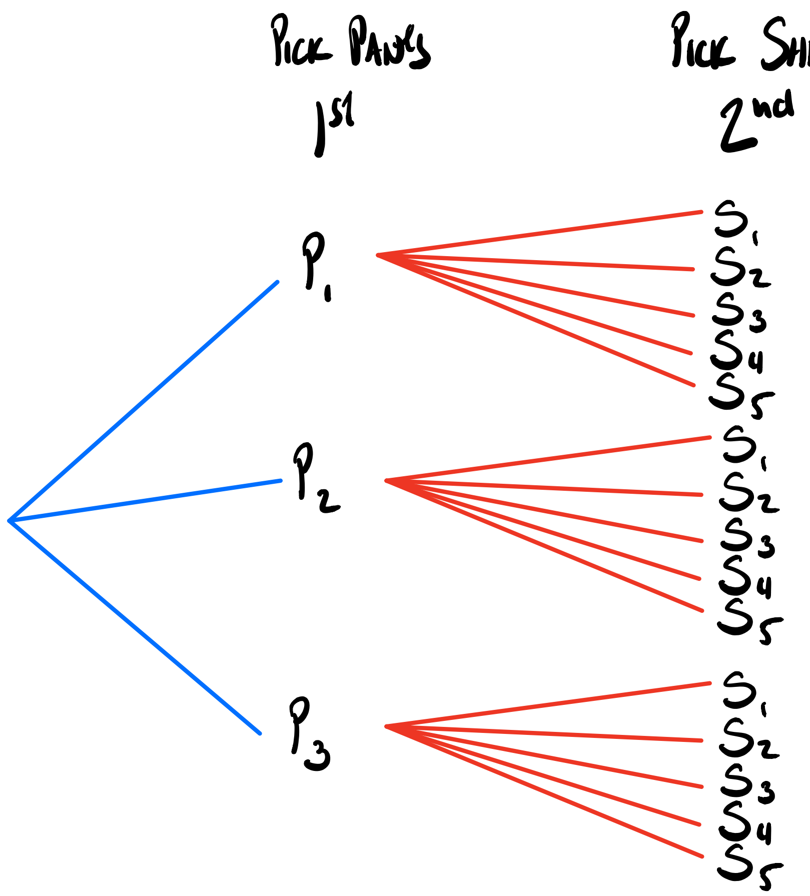
SUPPOSE AN EXPERIMENT IS PERFORMED IN 2 STAGES:

THE FIRST STAGE HAS m POSSIBLE OUTCOMES

THE SECOND STAGE HAS n POSSIBLE OUTCOMES

THEN THE EXPERIMENT HAS mn POSSIBLE OUTCOMES

ex. YOU OWN 3 PAIRS OF PANTS, & 5 SHIRTS. HOW MANY OUTFITS CAN YOU MAKE?



\# POSSIBLE OUTCOMES

= # PATHS FROM LEFT TO RIGHT

= 3 GROUPS OF 5

$= 3 \cdot 5 = 15$

THE EXTENDED mn RULE

SUPPOSE AN EXPERIMENT IS PERFORMED IN k STAGES:

THE FIRST STAGE HAS n_1 POSSIBLE OUTCOMES

THE SECOND STAGE HAS n_2 POSSIBLE OUTCOMES

$\vdots$

THE k^{TH} STAGE HAS n_k POSSIBLE OUTCOMES

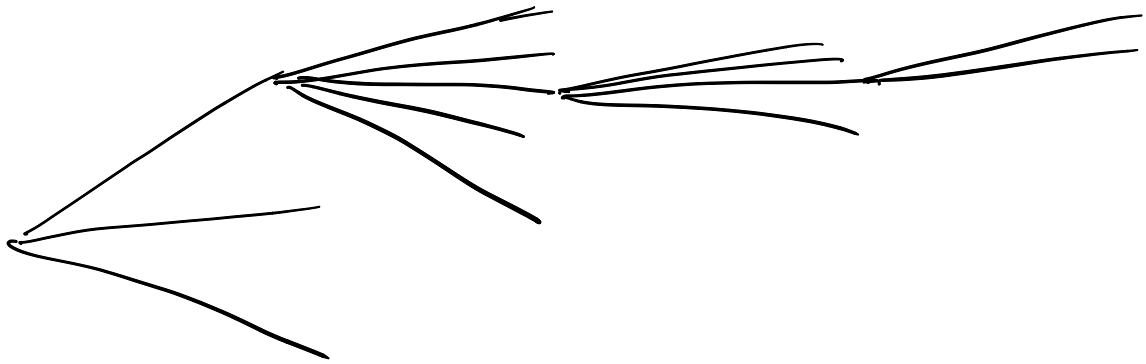
THEN THE EXPERIMENT HAS $n_1 n_2 \cdots n_k$ POSSIBLE OUTCOMES.

ex. A RESTAURANT SERVES 3 APPETIZERS, 5 ENTREES, & 4 DESSERTS,

2 BEVERAGES.

ASSUMING A MEAL CONSISTS OF 1 APP, 1 ENTREE, 1 DESSERT, & 1 BEVERAGE, HOW MANY MEALS DOES THIS RESTAURANT SERVE?

$$3 \times 5 \times 4 \times 2 = 120$$



ex. SUPPOSE YOU ARE GIVEN 10 CARDS AND ASKED TO SHUFFLE THEM. HOW POSSIBLE ARRANGEMENTS / ORDERINGS / "PERMUTATIONS" OF THE 10 CARDS ARE POSSIBLE?

$$\underset{1}{\underline{10}} \times \underset{2}{\underline{9}} \times \underset{3}{\underline{8}} \times \underset{4}{\underline{7}} \times \underset{5}{\underline{6}} \times \underset{6}{\underline{5}} \times \underset{7}{\underline{4}} \times \underset{8}{\underline{3}} \times \underset{9}{\underline{2}} \times \underset{10}{\underline{1}} =$$

$$\boxed{3{,}628{,}800}$$

10 STAGE EVENT

10 CHOICES FOR 1^{st} CARD

9 CHOICES FOR 2^{nd} CARD

⋮

2 9^{TH}

1 10^{TH}

Def: GIVEN n OBJECTS, THE NUMBER OF WAYS TO ARRANGE/ORDER/PERMUTE ALL n OBJECTS IS

$$\underbrace{n(n-1)(n-2)\cdots(3)(2)(1)}_{n!}$$

"n-FACTORIAL"

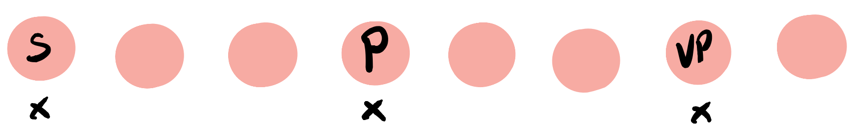
General Permutations

GIVEN n OBJECTS, THE NUMBER OF WAYS TO CHOOSE & ARRANGE/ORDER/PERMUTE r OBJECTS IS

$$P^n_r = {}_nP_r = \frac{n!}{(n-r)!}$$

"n - PERMUTE - r"

ex. A SCHOOL CLUB WITH 8 MEMEBERS NEEDS TO PICK A PRESIDENT, VICE-PRESIDENT, SECRETARY. HOW MANY WAYS CAN THEY DO IT?



$$\underset{P}{\underline{8}} \times \underset{VP}{\underline{7}} \times \underset{S}{\underline{6}} = \boxed{336}$$

ALICE	BOB	CINDY
CINDY	ALICE	BOB

$$P^8_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!}$$

$$= \frac{8\cdot 7\cdot 6\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 8\cdot 7\cdot 6 = \underline{\underline{336}}$$

PRIME #: ONLY FACTORS/DIVISORS ARE 1 & ITSELF.

6 1×6
2×3

5 1×5

COMBINATIONS

GIVEN n OBJECTS, THE NUMBER OF WAYS TO CHOOSE r OBJECTS (ORDER DOESN'T MATTER) IS

$$C^n_r = {}_nC_r = \frac{n!}{(n-r)!\,r!}$$

"n - CHOOSE - r"

ex. GIVEN 4 OBJECTS A, B, C, D

(a) · HOW MANY WAYS ARE THERE TO CHOOSE 3 OBJECTS IF ORDER MATTERS?

(b) · HOW MANY WAYS ARE THERE TO CHOOSE 3 OBJECTS IF ORDER DOES NOT MATTER?

(a) $\underset{1^{st}}{\underline{4}} \times \underset{2^{nd}}{\underline{3}} \times \underset{3^{rd}}{\underline{2}} = 24$

$$\left(P^4_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = \frac{4\cdot 3\cdot 2\cdot 1}{1} = 24 \right)$$

(b)

ABC	ACB	BAC	BCA	CAB	CBA
ABD	ADB	BAD	BDA	DAB	DBA
ACD	ADC	CAD	CDA	DAC	DCA
BCD	BDC	CBD	CDB	DBC	DCB

GIVEN 1 WAY OF CHOOSING 3 OBJECTS FROM 4, THERE ARE $3! = 6$ DIFFERENT PERMUTATIONS (ARRANGEMENTS) OF THOSE SAME 3 OBJECTS

BUT IF ORDER DOESN'T MATTER, THEN WE ONLY WANT TO COUNT THESE AS A SINGLE COMBINATION.

$$\Rightarrow \quad \frac{24}{6} = 4$$

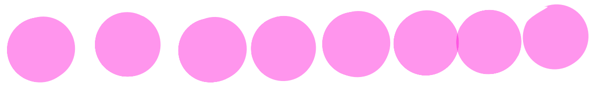
WAYS TO CHOOSE 3 OBJECTS FROM 4 OBJECTS IF ORDER DOES NOT MATTER.

$$\left(C^4_3 = \frac{4!}{1!\,3!} = 4 \right)$$

SINCE EACH COMBINATION OF r OBJECTS HAS r! PERMUTATIONS, WE DIVIDE # OF PERMUTATIONS P^n_r BY r! TO GET # OF COMBINATIONS (ORDER DOESN'T MATTER)

$$C^n_r = \frac{P^n_r}{r!} = \frac{n!}{(n-r)!\,r!}$$

ex. 8 club members need to choose 3 members as representatives.



$$\underline{\;8\;} \times \underline{\;7\;} \times \underline{\;6\;} = P^8_3 \qquad \text{(order matters)}$$

A	B	C
B	A	C

$$\frac{P^8_3}{3!} = C^8_3 = \frac{8!}{(8-3)!\,3!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}$$

$$= \boxed{56}$$

§4.4 HW posted.