§ 4.2 Events & the Sample Space (continued)

RECALL: SAMPLE SPACE Ω IS THE SET OF ALL POSSIBLE OUTCOMES OF AN EXPERIMENT (ANY PROCESS BY WHICH AN OBSERVATION IS MADE).

AN EVENT IS ANY SUBSET OF Ω.

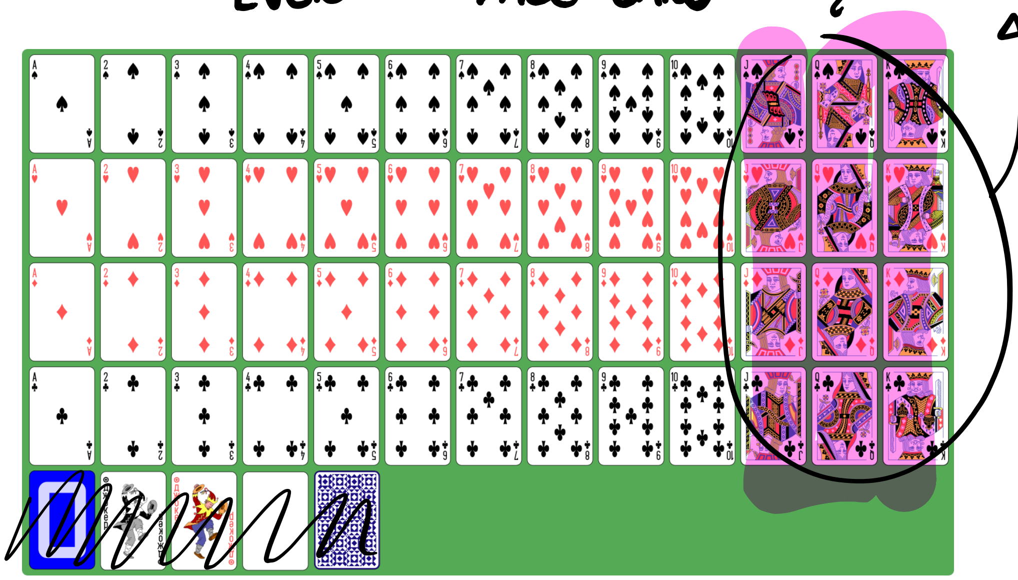
SIMPLE EVENT IS AN EVENT THAT CONTAINS ONLY 1 POSSIBLE OUTCOME.

__ex.__ EXPERIMENT: CHOOSE 1 CARD FROM A STANDARD DECK OF 52 CARDS

SAMPLE SPACE $\Omega = \{A♠, 8♣, 6♥, \ldots\}$

$$|\Omega| = 52$$

EVENT: "FACE CARD" = { }



EVENT: "TWO OF HEARTS" = $\{2♥\}$ (SIMPLE EVENT)

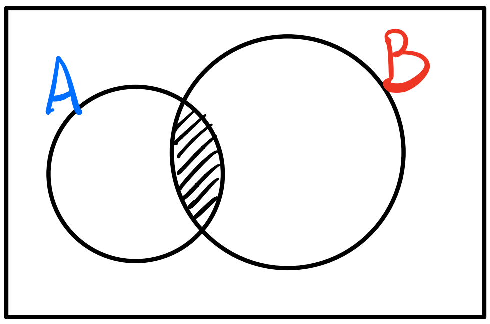
We say an event A occurs if any of the possible outcomes contained in A occur.

Def: Two events A, B are **mutually exclusive** if no possible outcome belongs to both A & B. (intersection is empty)

Def: Given two sets, A & B, the intersection is the set of all elements contained in both A & B. Notation:

$$A \cap B$$

"A intersect B"



$A \cap B$

Non-Empty Intersection.

e.g. Exp: Select 1 card from deck of 52.

Events: A = draw a heart.

B = draw a nine.

C = draw a jack.

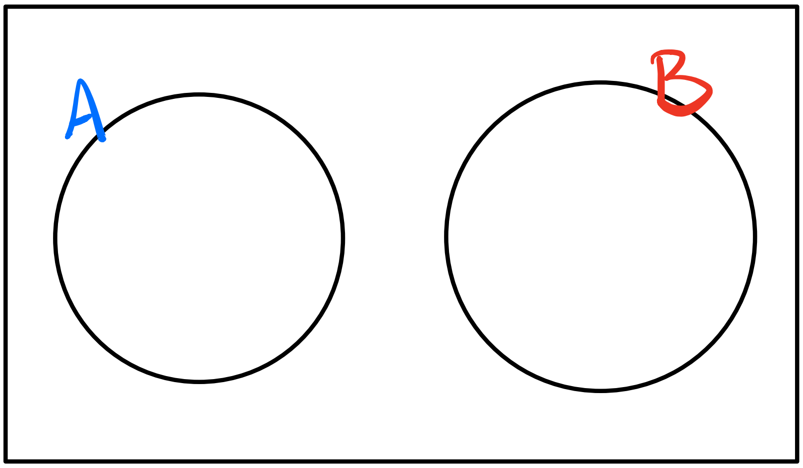
$A \cap B = \{9\heartsuit\}$

$A \cap C = \{J\heartsuit\}$

$B \cap C = \{\} = \emptyset$

WE SAY THAT EVENTS B & C ARE

MUTUALLY EXCLUSIVE.

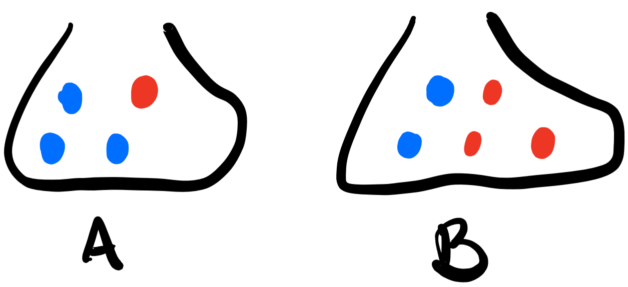


EMPTY INTERSECTION

$A \cap B = \emptyset$; A & B ARE MUTUALLY EXCLUSIVE.
"NO OVERLAP"

NOTE: DISTINCT SIMPLE EVENTS ARE MUTUALLY EXCLUSIVE.

Ex. EXPERIMENT: (1) FLIP A COIN
(2) IF H: DRAW A MARBLE FROM JAR A
IF T: DRAW A MARBLE FROM JAR B



DESCRIBE THE SAMPLE SPACE

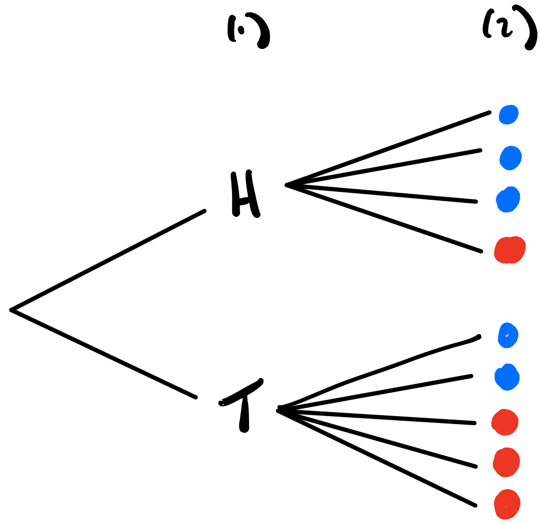
↕

DESCRIBE ALL POSSIBLE OUTCOMES

(LIST IF POSSIBLE)

TREE DIAGRAM:

EXPERIMENT IN 2 STAGES:



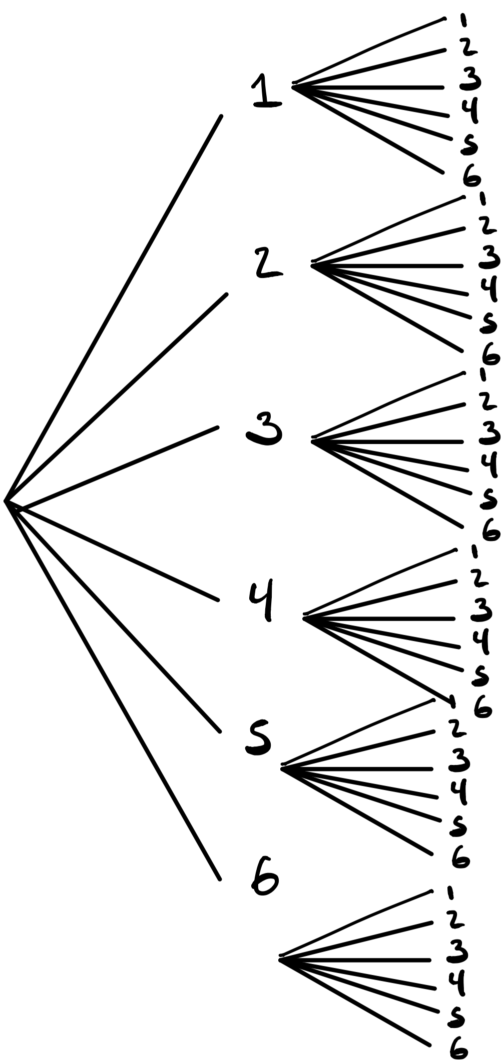
EACH PATH FROM LEFT TO RIGHT REPRESENTS A SIMPLE EVENT

(SINGLE POSSIBLE OUTCOME)

9 POSSIBLE OUTCOMES

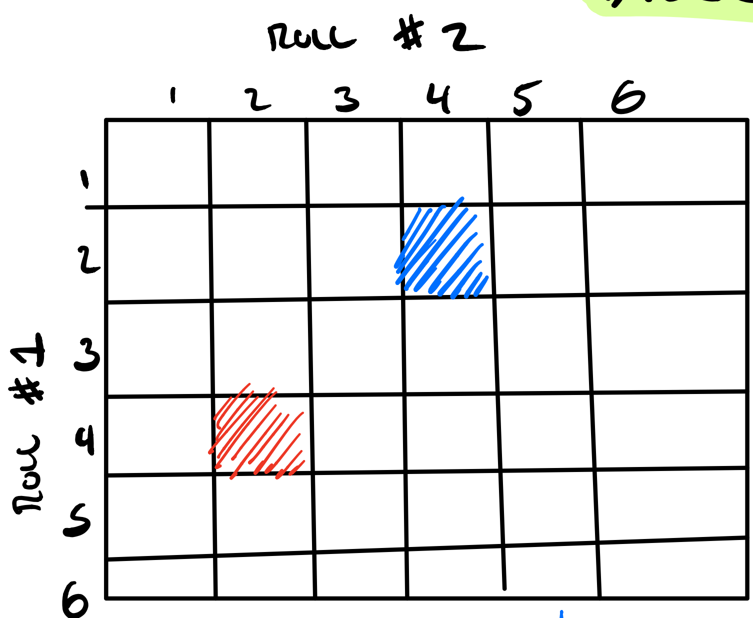
9 SIMPLE EVENTS.

e.g. EXP: ROLL 2 DICE. DESCRIBE SAMPLE SPACE Ω.



SAMPLE SPACE CONTAINS 36 POSSIBLE OUTCOMES.

TABLE



SIMPLE EVENT: 1^{st} 2, 2^{nd} 4

1^{st} 4, 2^{nd} 2

§4.3 Calculating Probabilities using Simple Events

Probability is a way of assigning a measure to events, i.e. subset of a sample space.
Probability measures the likelihood of events.

Finite scale:

For any event $A \subset \Omega$,

Guaranteed not to occur

$$0 \leq P(A) \leq 1$$

Guaranteed to occur

"The probability of A"

The Theory of Probability:

Given an event $A \subset \Omega$.

To calculate $P(A)$, in theory, we could repeat the experiment n times

Experiment #	Did A occur?
1	Yes
2	No
3	No
4	Yes

Frequency of A

5	Yes
6	No
$\vdots$	$\vdots$
n	No

$\Big\}$ (# OF TIMES THAT A OCCURRED)

$0 \leq \downarrow \leq n$

$$P(A) \approx \frac{\text{FREQUENCY OF } A}{n}$$

Def: $$P(A) = \lim_{n \to \infty} \frac{\text{FREQUENCY OF } A}{n}$$

$$\frac{7}{10}, \frac{71}{100}, \frac{714}{1000}, \frac{7148}{10000}, \ldots$$

BETTER & BETTER APPROXIMATIONS OF $P(A)$.

NOTE: THE SAMPLE SPACE ITSELF IS AN EVENT

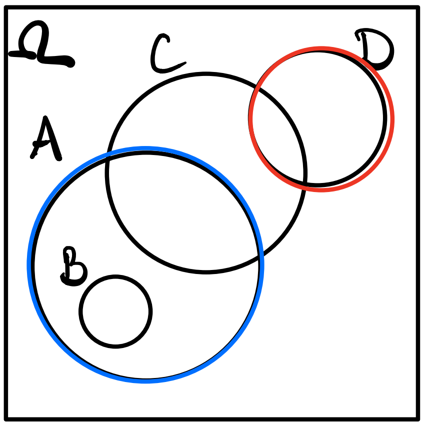
$\Omega \subset \Omega$ AND

$P(\Omega)$ = PROBABILITY OF OBSERVING A POSSIBLE OUTCOME IN Ω $= 1$

CONTAINS EVERY POSSIBLE OUTCOME!

e.g. FLIP 2 COINS. $\Omega = \{HH, HT, TH, TT\}$

VISUALIZATION: SAMPLE SPACE Ω ~ RECTANGLE.



$0 \leq P(A), P(B), P(C), P(D) \leq 1$

B = DEALT JACK

A = DEALT FACE CARD

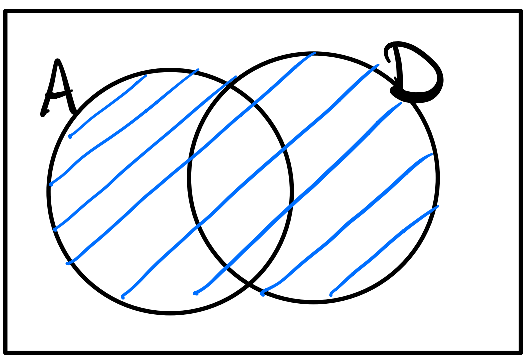
$B \subset A$

PROPERTIES: IF $B \subset A$

THEN $P(B) \leq P(A)$

IF $A \cap D = \phi$ (MUTUALLY EXCLUSIVE)

THEN $P(A \cup D) = P(A) + P(D)$



SUBSET $\subset$

$\cap$ INTERSECTION

$\cup$ UNION

▨ $A \cup D$ "A UNION D"

e.g. EXP. SELECT 1 CARD FROM STANDARD DECK

EVENTS A: EIGHT : $\{8\heartsuit, 8\diamondsuit, 8\clubsuit, 8\spadesuit\}$

B: SIX : $\{6\heartsuit, 6\diamondsuit, 6\clubsuit, 6\spadesuit\}$

SINCE $A \cap B = \emptyset$ MUTUALLY EXCL.

$$\Rightarrow \underbrace{P(A \cup B)}_{8 \text{ or } 6} = P(A) + P(B)$$

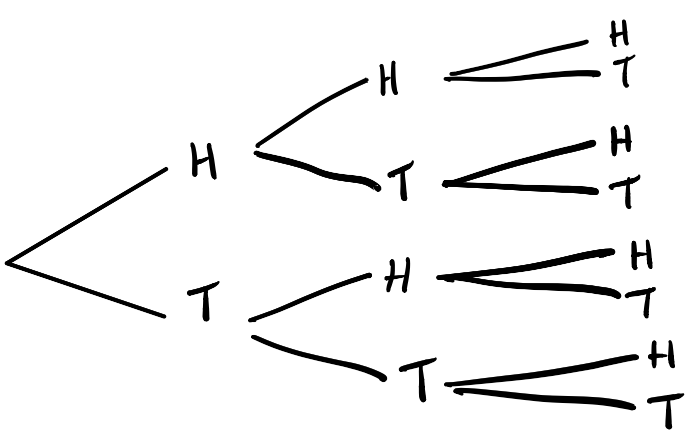
CALCULATING PROBABILITIES USING SIMPLE EVENTS

(ALL DISTINCT SIMPLE EVENTS ARE MUTUALLY EXCLUSIVE)

(PROB. OF EVERY SIMPLE EVENT IS BETWEEN 0 & 1)

<u>Def</u>: $P(A)$ = SUM OF THE PROBABILITIES OF ALL SIMPLE EVENTS CONTAINED IN A.

e.g. FIND THE PROBABILITY OF GETTING (EXACTLY) TWO HEADS IN 3 COIN TOSSES.



SIMPLE EVENTS

HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

8 SIMPLE EVENTS, ALL EQUALLY LIKELY.

$P(\Omega) = 1$

$P(\{HHH, HHT, \dots, TTT\}) = 1$

$\underset{(1)}{P(\{HHH\})} + \underset{(2)}{P(\{HHT\})} + \dots + \underset{(8)}{P(\{TTT\})} = 1$

ALL SIMPLE EVENTS HAVE PROBABILITY $\frac{1}{8}$.

So, $P(\text{TWO HEADS}) = P(\{HHT, HTH, THH\})$

$= P(\{HHT\}) + P(\{HTH\}) + P(\{THH\})$

$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \boxed{\frac{3}{8}}$

GENERAL METHOD:

(1) LIST ALL SIMPLE EVENTS IN SAMPLE SPACE
(KNOW WHAT THEY ARE: TREE DIAGRAM, TABLES, LISTS, ...)

(2) ASSIGN PROBABILITIES TO EACH SIMPLE EVENT.
SPECIAL CASE: n SIMPLE EVENTS,
ALL EQUALLY LIKELY

$\Rightarrow$ PROBABILITY OF ANY SIMPLE EVENT IS $\frac{1}{n}$.

(3) DETERMINE WHICH SIMPLE EVENTS RESULT IN
THE EVENT OF INTEREST

(4) SUM UP PROBABILITIES OF THESE SIMPLE EVENTS.

4.3 A sample space contains 10 simple events: E_1, $E_2, \ldots, E_{10}$. If $P(E_1) = 3P(E_2) = .45$ and the remaining simple events are equiprobable, find the probabilities of these remaining simple events.

$P(E_1) = .45 \qquad 3P(E_2) = .45$

$P(E_2) = .15$

WE KNOW: $P(\Omega) = 1$

$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_{10}) = 1$

$\cancel{.45} + \cancel{.15} + P(E_3) + \ldots + P(E_{10}) = 1$

$\cancel{-.45} \quad \cancel{-.15} \qquad -.45 \quad -.15$

$\underbrace{P(E_3) + \ldots + P(E_{10})}$ $= .4$

8 EVENTS, ALL EQUALLY LIKELY.

$\Rightarrow$ EACH PROB MUST BE $\frac{.4}{8} = \boxed{.05}$

4.5 Four Coins A jar contains four coins: a nickel, a dime, a quarter, and a half-dollar. Three coins are randomly selected from the jar.

a. List the simple events in S.

b. What is the probability that the selection will contain the half-dollar?

c. What is the probability that the total amount drawn will equal 60¢ or more?

85 ¢ 80 ¢ 65 ¢ 40 ¢

(a.) $S = \{DQH, NQH, NDH, NDQ\}$

(b.) ALL SIMPLE EVENTS ARE EQUALLY LIKELY. (PROB. $\frac{1}{4}$)

P(HALF DOLLAR) = P(DQH) + P(NQH) + P(NDH)

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \boxed{\frac{3}{4}}$

(c.) SAME AS PART (b.)

4.14 **100-Meter Run** Four equally qualified runners, John, Bill, Ed, and Dave, run a 100-meter sprint, and the order of finish is recorded.

a. How many simple events are in the sample space?

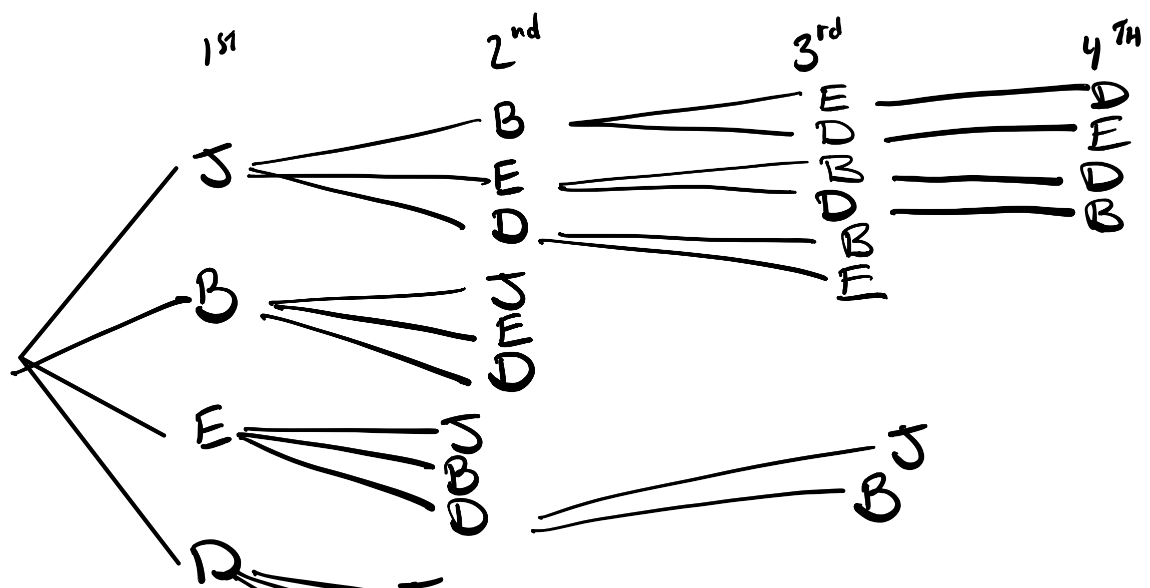
b. If the runners are equally qualified, what probability should you assign to each simple event?

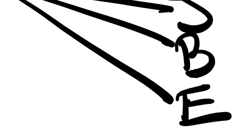
c. What is the probability that Dave wins the race?

d. What is the probability that Dave wins and John places second?

e. What is the probability that Ed finishes last?

(a.) $\Omega = \{JBED, JBDE, \ldots\}$





COUNT # OF PATHS FROM LEFT TO RIGHT:

4 GROUPS OF 3 GROUPS OF 2 (GROUPS OF 1)

$4 \times 3 \times 2 = \boxed{24 \text{ POSSIBLE OUTCOMES.}}$

§4.3 HW POSTED ONLINE