Name: *ANSWER KEY* 4/5/2017

Math 173 Introduction to Probability and Statistics **Midterm Exam**

Please box your final answers. Calculators are allowed, but not required. Answers may be left as fractions and/or expressions that may contain square-root ($\sqrt{\cdot}$), factorial (!), permutation (P^n_r), and combination (C^n_r) notation.

1. You are given a sample of $n = 8$ measurements: 5, 1, 5, 6, 4, 4, 2, 5.

   (a) (4 points) What is the median m?

   ORDER LEAST TO GREATEST: $1, 2, 4, \underbrace{4, 5}, 5, 5, 6$

   $m = \boxed{4.5}$

   (b) (4 points) What is the mean $\bar{x}$?

   $$\bar{x} = \frac{1}{n}\sum x_i = \frac{5+1+5+6+4+4+2+5}{8} = \frac{32}{8} = \boxed{4}$$

   (c) (4 points) What is the mode M?

   MOST FREQUENT MEASUREMENT: $\boxed{5}$

   (d) (4 points) What is the variance s^2?

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
1	-3	9
2	-2	4
4	0	0
4	0	0
5	1	1
5	1	1
5	1	1
6	2	4

$$s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2 = \frac{9+4+0+0+1+1+1+4}{7} = \boxed{\frac{20}{7} \approx 2.8571}$$

   (e) (4 points) What is the standard deviation s?

   $$s = \sqrt{s^2} = \boxed{\sqrt{\frac{20}{7}} \approx 1.6903}$$

2. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c	TOTAL
B	.18	.42	.6
B^c	.12	.28	.4
TOTAL	.3	.7	1

(a) (4 points) Find $P(A^c|B^c)$.

$$P(A^c|B^c) = \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{.28}{.4} = \boxed{.7}$$

(b) (4 points) Are A and B independent? Why?

$\boxed{\text{Yes.}}$ ANY OF THESE REASONS:

$P(A \cap B) = P(A) \cdot P(B)$,

$P(A^c \cap B^c) = P(A^c) \cdot P(B^c)$,

$P(A|B) = P(A|B^c) = P(A)$,

$P(B|A) = P(B|A^c) = P(B)$,

$P(A^c|B) = P(A^c|B^c) = P(A^c)$,

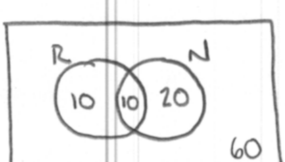
$P(B^c|A) = P(B^c|A^c) = P(B^c)$, ETC.

(c) (4 points) Are A and B mutually exclusive? Why?

$\boxed{\text{No, } P(A \cap B) \neq 0.}$

3. Sixty percent of the students at a school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing

(a) (4 points) a ring <u>or</u> a necklace?



$$P(R \cup N) = P(\overbrace{(R^c \cap N^c)}^{\text{WEAR NEITHER}}{}^c)$$

$$= 1 - P(R^c \cap N^c)$$

$$= 1 - .60 = \boxed{.40}$$

(b) (4 points) a ring <u>and</u> a necklace?

$$P(R \cup N) = P(R) + P(N) - P(R \cap N)$$

$$.4 = .2 + .3 - P(R \cap N)$$

$$P(R \cap N) = \boxed{.1}$$

4. An urn contains 3 red, 3 white, and 3 blue marbles. 3 marbles are randomly selected without replacement.

(a) (6 points) Find the probability that all balls are the same color.

$P(\text{ALL BALLS SAME COLOR}) = P(RRR) + P(WWW) + P(BBB)$

$$= \frac{C^3_3 C^3_0 C^3_0}{C^9_3} + \frac{C^3_0 C^3_3 C^3_0}{C^9_3} + \frac{C^3_0 C^3_0 C^3_3}{C^9_3}$$

$$= 3 \cdot \frac{C^3_3}{C^9_3} = 3 \cdot \frac{1}{84} = \boxed{\frac{1}{28} \approx .0357}$$

(b) (6 points) Find the probability that each ball is a different color.

$$P(\text{1 OF EACH COLOR}) = \frac{C^3_1 C^3_1 C^3_1}{C^9_3} = \frac{3 \cdot 3 \cdot 3}{84}$$

$$= \boxed{\frac{9}{28} \approx .3214}$$

5. (8 points) Under the "no pass, no play" rule for athletes, an athlete who fails a course is disqualified from participating in sports activities during the next grading period. Suppose the probability that an athlete who has not previously been disqualified will be disqualified is .15 and the probability that an athlete who has been disqualified will be disqualified again in the next time period is .5. If 30% of the athletes have been disqualified before, what is the unconditional probability that an athlete will be disqualified during the next grading period?

LET DQ = EVENT THAT ATHLETE IS DISQUALIFIED

PDQ = EVENT THAT ATHLETE WAS PREVIOUSLY DISQUALIFIED.

GIVEN: $P(DQ \mid PDQ^c) = .15$, $P(DQ \mid PDQ) = .5$, $P(PDQ) = .3$

$$P(DQ) = P(DQ \cap PDQ) + P(DQ \cap PDQ^c)$$

$$= P(PDQ)P(DQ \mid PDQ) + P(PDQ^c)P(DQ \mid PDQ^c)$$

$$= (.3)(.5) + (.7)(.15) = .15 + .105 = \boxed{.255}$$

6. (8 points) A student taking a multiple-choice test knows the answer to 80% of the questions, and always marks the correct answer to these questions. On questions that she does not know the answer to, she guesses and marks the correct answer 25% of the time. What is the conditional probability that this student knows the answer to a question, given that she answers it correctly?

LET K = EVENT THAT STUDENT KNOWS THE ANSWER

C = EVENT THAT STUDENT MARKS THE CORRECT ANSWER

$$P(K|C) = \frac{P(K)P(C|K)}{P(C)} = \frac{P(K)P(C|K)}{P(K)P(C|K) + P(K^c)P(C|K^c)}$$

$$= \frac{(.8)(1)}{(.8)(1) + (.2)(.25)} = \frac{.8}{.8 + .05} = \boxed{\frac{16}{17} \approx .9412}$$

7. (8 points) A certain lottery ticket is sold for \$2. Fifteen percent of tickets contain a \$2 prize, 10% of tickets contain a \$5 prize, and 5% of tickets contain a jackpot prize. How big can the jackpot prize be in order for the lottery company to break even, in the long run?

LET X = MONEY GAINED BY LOTTERY COMPANY ON SALE OF 1 TICKET

J = JACKPOT PRIZE.

x	$p(x)$
$2-2=0$	$.15$
$2-5=-3$	$.10$
$2-J$	$.05$
2	$.70$

WE WANT $E[X] = \mu = 0$.

THAT IS, $\sum x\, p(x) = 0$

$$(0)(.15) + (-3)(.10) + (2-J)(.05) + (2)(.7) = 0$$

$$-.3 + .1 - .05J + 1.4 = 0$$

$$.05J = 1.2$$

$$\boxed{J = 24}$$

8. Suppose you have a weighted coin that lands "heads" with probability .6 and lands "tails" with probability .4. You flip this coin five times and count the number of heads that appear, x.

(a) (4 points) Find $P(x = 2)$.

NOTE THAT x IS A BINOMIAL RANDOM VARIABLE; $n = 5$, $p = .6$, $q = .4$.

$$P(x=2) = C^5_2 \, p^2 q^{5-2}$$

$$= 10\,(.6)^2(.4)^3 = 10\,(.36)(.064) = \boxed{.2304}$$

(b) (4 points) Find the mean μ and the standard deviation σ for x.

$$\mu = np = 5(.6) = \boxed{3}$$

$$\sigma = \sqrt{npq} = \sqrt{5(.6)(.4)} = \boxed{\sqrt{1.2} \approx 1.0954}$$

(c) (4 points) Use Tchebycheff's Theorem to provide a lower bound for the probability that x is in the interval $\mu \pm 2\sigma$?

TCHEBYCHEFF'S THM: $P(x$ IS WITHIN k STND. DEV.'S OF $\mu) \geq 1 - \frac{1}{k^2}$

HERE $k = 2$, SO PROBABILITY $\geq 1 - \frac{1}{2^2} = \boxed{.75}$

9. (8 points) A piece of electronic equipment contains six computer chips, two of which are defective. Three computer chips are randomly chosen for inspection, and the number of defective chips x is recorded. Fill in the following chart with all possible values of the random variable x, along with the corresponding probabilities $p(x)$.

x	0	1	2
$p(x)$	.2	.6	.2

NOTE THAT X IS A HYPERGEOMETRIC RANDOM VARIABLE.

$$P(X=0) = \frac{C^2_0\, C^4_3}{C^6_3} = \frac{1 \cdot 4}{20} = \frac{4}{20} = \frac{1}{5} = .2$$

$$P(X=1) = \frac{C^2_1\, C^4_2}{C^6_3} = \frac{2 \cdot 6}{20} = \frac{12}{20} = \frac{3}{5} = .6$$

$$P(X=2) = \frac{C^2_2\, C^4_1}{C^6_3} = \frac{1 \cdot 4}{20} = \frac{4}{20} = \frac{1}{5} = .2$$

$P(X \geq 3) = 0$ (IMPOSSIBLE)