Name: * ANSWER KEY * 5/22/2017

Math 173 Introduction to Probability and Statistics **Final Exam**

Please box your final answers. Calculators are allowed, but not required. Answers may be left as fractions and/or expressions that may contain square-root ($\sqrt{\cdot}$), factorial (!), permutation (P_r^n), and combination (C_r^n) notation.

1. You are given a sample of $n = 12$ measurements: $\{5, 7, 2, 7, 2, 3, 6, 9, 3, 4, 5, 7\}$.

   (a) (4 points) What is the median m?

   MIDDLE

   2 2 3 3 4 $\boxed{5 \quad 5}$ 6 7 7 7 9

   $$m = \frac{5+5}{2} = \boxed{5}$$

   (b) (4 points) What is the mean $\bar{x}$?

   $$\frac{1}{12}\sum_{i=1}^{12} x_i = \frac{60}{12} = \boxed{5}$$

   (c) (4 points) What is the mode M?

   $$\boxed{M = 7}$$

   (d) (4 points) What is the variance s^2?

x	$x - \bar{x}$	$(x-\bar{x})^2$
5	0	0
7	2	4
2	-3	9
7	2	4
2	-3	9
3	-2	4
6	1	1
9	4	16
3	-2	4
4	-1	1
5	0	0
7	2	4

   $$s^2 = \frac{1}{12-1}\sum_{i=1}^{12}(x_i - \bar{x})^2$$

   $$= \boxed{\frac{56}{11} \approx 5.0909}$$

   (e) (4 points) What is the standard deviation s?

   $$s = \sqrt{s^2} = \boxed{\sqrt{\frac{56}{11}} \approx 2.2563}$$

2. An experiment can result in none, one, or both of the events A and B with the probabilities shown in the following table.

	A	A^c	
B	.16	.04	.2
B^c	.64	.16	.8
	.8	.2	

(a) (4 points) Find $P(A \cup B)$.

$= 1 - P(A^c \cap B^c) = 1 - .16 = \boxed{.84}$

(b) (4 points) Find $P(B|A^c)$

$= \dfrac{P(B \cap A^c)}{P(A^c)} = \dfrac{.04}{.04 + .16} = \dfrac{.04}{.2} = \dfrac{1}{5} = \boxed{.2}$

(c) (4 points) Are A and B independent? Why?

Yes, $P(A \cap B) = P(A)P(B)$, $P(A|B) = P(A)$, $P(B|A) = P(B)$, ETC...

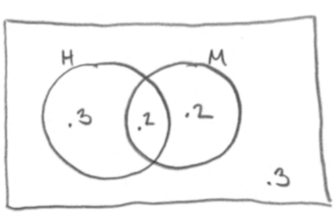
$.16 = (.8)(.2)$

(d) (4 points) Are A and B mutually exclusive? Why?

No, $P(A \cap B) \neq 0$

3. Suppose that on a cold day 50% of children wear a hat, 40% of children wear mittens, and 30% of children wear neither a hat nor mittens. One child is selected randomly.

(a) (4 points) What is the probability that this child wears a hat <u>and</u> mittens?



$P(H \cup M) = 1 - P(H^c \cap M^c) = 1 - .3 = .7$

$P(H \cup M) = P(H) + P(M) - P(H \cap M)$

$.7 = .5 + .4 - P(H \cap M)$

$\Rightarrow P(H \cap M) = \boxed{.2}$

(b) (4 points) What is the probability that this child wears mittens, <u>given that</u> this child wears a hat?

$P(M|H) = \dfrac{P(M \cap H)}{P(H)} = \dfrac{.2}{.5} = \boxed{\dfrac{2}{5} \approx .4}$

4. An urn contains 4 red marbles, 5 white marbles, and 6 blue marbles. 2 marbles are randomly selected from the urn, without replacement.

   (a) (4 points) Find the probability that the two marbles selected are the same color.

$$P(\text{SAME COLOR}) = P(2\text{ RED}) + P(2\text{ WHITE}) + P(2\text{ BLUE})$$

$$= \frac{C^4_2 C^5_0 C^6_0}{C^{15}_2} + \frac{C^4_0 C^5_2 C^6_0}{C^{15}_2} + \frac{C^4_0 C^5_0 C^6_2}{C^{15}_2} = \frac{6+10+15}{105}$$

$$= \boxed{\frac{31}{105} \approx .2952}$$

   (b) (4 points) Find the probability that the two marbles selected are different colors.

$$P(\text{DIFF. COLORS}) = 1 - P(\text{SAME COLOR})$$

$$= \boxed{\frac{74}{105} \approx .7048}$$

5. (8 points) An insurance company believes that people can be divided into two classes: those who are accident prone and those who are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability .4, whereas this probability decreases to .2 for a non-accident-prone person. If we assume that 30% of the population is accident prone, what is the probability that a new policyholder will have an accent within a year of purchasing a policy.

ACCIDENT

LET S_1 = ACCIDENT PRONE
S_2 = NOT ACCIDENT PRONE $\Big\}$ → MUTUALLY DISJOINT → EXHAUSTIVE

A = EVENT THAT PERSON HAS AN ACCIDENT

GIVEN: $P(A|S_1) = .4$, $P(A|S_2) = .2$, $P(S_1) = .3$

THEN : $P(S_2) = 1 - P(S_1) = .7$

LAW OF TOTAL PROB: $P(A) = P(A|S_1)P(S_1) + P(A|S_2)P(S_2)$

$$= (.4)(.3) + (.2)(.7)$$

$$= .12 + .14 = \boxed{.26}$$

6. (8 points) A laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1% of the healthy persons tested. (That is, if a healthy person is tested, then, with probability .01, the result will imply he or she has the disease.) If 0.5% of the population actually has the disease, what is the probability a person has the disease given that the test result is positive?

LET P = EVENT THAT PERSON TESTS POSITIVE

D : EVENT THAT PERSON HAS DISEASE

$$P(D \mid P) = \frac{P(P \mid D)P(D)}{P(P)} \quad \text{(BAYE'S RULE)}$$

$$= \frac{P(P \mid D)P(D)}{P(P \mid D)P(D) + P(P \mid D^c)P(D^c)} \quad \text{(LAW OF TOTAL PROB.)}$$

$$= \frac{(.95)(.005)}{(.95)(.005) + (.01)(.995)} = \frac{.00475}{.00475 + .00995} \approx \boxed{.3231}$$

7. A home security system is designed to have a 99% reliability rate. Suppose that 9 homes equipped with this system experience an attempted burglary. Find the probabilities of the following events.

(a) (4 points) More than seven of the alarms are triggered.

BINOMIAL DISTRIBUTION : $P(x=k) = C^n_k p^k q^{n-k}$

$$P(x>7) = P(x=8) + P(x=9) = C^9_8 (.99)^8 (.01)^1 + C^9_9 (.99)^9 (.01)^0$$

$$= 9(.99)^8(.01)^1 + 1(.99)^9(.01)^0$$

$$\approx .0830 + .9135$$

$$\approx \boxed{.9965}$$

(b) (4 points) Eight or fewer of the alarms are triggered.

$$P(x \leq 8) = 1 - P(x=9) = 1 - C^9_1 (.99)^9 (.01)^0$$

$$\approx 1 - .9135 = \boxed{.0865}$$

8. (8 points) 3 animals are to be randomly selected from a group of 3 goats and 4 sheep. Let x be the number of goats selected. Fill in the following chart with all possible values of the random variable x, along with the corresponding probabilities $p(x)$.

x	0	1	2	3
$p(x)$	$\frac{C^3_0 C^4_3}{C^7_3} \downarrow \frac{1\cdot 4}{35} \downarrow \boxed{\frac{4}{35}}$	$\frac{C^3_1 C^4_2}{C^7_3} \downarrow \frac{3\cdot 6}{35} \downarrow \boxed{\frac{18}{35}}$	$\frac{C^3_2 C^4_1}{C^7_3} \downarrow \frac{3\cdot 4}{35} \downarrow \boxed{\frac{12}{35}}$	$\frac{C^3_3 C^4_0}{C^7_3} \downarrow \frac{1\cdot 1}{35} \downarrow \boxed{\frac{1}{35}}$
	.1143	.5143	.3429	.0286

9. (8 points) From experience, a shipping company knows that the cost of delivering a small package within 24 hours is \$14. The company charges \$16 for shipment but guarantees to refund the charge if delivery is not made within 24 hours. If the company fails to deliver only 2% of its packages within the 24-hour period, what is the expected gain/profit per package?

REVENUE	-	COST	=	PROFIT x	$p(x)$
16	-	14	=	2	.98
0	-	14	=	-14	.02

$$E[x] = \sum x_i p(x_i) = (2)(.98) + (-14)(.02)$$
$$= 1.96 - .28$$
$$= \boxed{1.68}$$

10. Suppose a fair die with faces labeled 1-6 is rolled n times. Let x be the number of times that a 6 is rolled.

(a) (4 points) When $n = 8$, find $P(x \leq 1)$ exactly.

$$P(x \leq 1) = P(x=0) + P(x=1) = C^8_0 \left(\tfrac{1}{6}\right)^0 \left(\tfrac{5}{6}\right)^8 + C^8_1 \left(\tfrac{1}{6}\right)^1 \left(\tfrac{5}{6}\right)^7$$

$$= .2326 + .3721 = \boxed{.6047}$$

(b) (4 points) When $n = 180$, use a normal distribution to approximate $P(x \geq 38)$.

$$\mu = np = (180)\left(\tfrac{1}{6}\right) = 30$$

$$\sigma = \sqrt{npq} = \sqrt{(180)\left(\tfrac{1}{6}\right)\left(\tfrac{5}{6}\right)}$$

$$= \sqrt{25} = 5$$

$$P(x \geq 38) \approx P_{NORM}(x \geq 37.5)$$

$$z = \frac{x-\mu}{\sigma} = \frac{37.5 - 30}{5} = 1.5$$

$$\approx P(z \geq 1.5) = 1 - P(z \leq 1.5) = \boxed{.0668}$$

11. (8 points) A random sample of 130 human body temperatures, provided by Allen Shoemaker in the Journal of Statistical Education, had a mean of 98.25° and a standard deviation of 0.73°. Construct a 99% confidence interval for the average body temperature of healthy people, and briefly explain what a 99% confidence interval is.

$$S.E. = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}} = \frac{.73}{\sqrt{130}} \approx .0640$$

$$99\% \text{ CONFIDENCE INTERVAL} = \bar{x} \pm 2.58\,(S.E.) = 98.25 \pm 2.58\,(.0640)$$

$$= 98.25 \pm .1651 = \boxed{[\,98.0849,\ 98.4151\,]}$$

* IF RANDOM SAMPLES WERE REPEATEDLY TAKEN AND CONFIDENCE INTERVALS WERE REPEATEDLY CONSTRUCTED IN THIS WAY, 99% OF THOSE INTERVALS WOULD CONTAIN THE TRUE POPULATION MEAN.

12. (8 points) The first day of baseball comes in late March, ending in October with the World Series. Does fan support grow as the season goes on? Two CNN/USA Today/Gallup polls, one conducted in March and one in November, both involved random samples of 1001 adults aged 18 and older. In the March sample, 45% of the adults claimed to be fans of professional baseball, while 51% of the adults in the November sample claimed to be fans. Construct a 99% confidence interval for the difference in the proportion of adults who claim to be fans in March versus November.

$$S.E. = \sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}} \approx \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = \sqrt{\frac{(.45)(.55)}{1001} + \frac{(.51)(.49)}{1001}} \approx .0223$$

$$99\%\ \text{CONF. INT.} = \hat{p}_1 - \hat{p}_2 \pm 2.58\,(S.E.)$$

$$= .45 - .51 \pm 2.58\,(.0223) = \boxed{[-.1175\ ,\ -.0025]}$$

13. Suppose a scheduled airline flight must average at least 60% occupancy in order to be profitable to the airline. An examination of the occupancy rate for 120 10:00 A.M. flights from Atlanta to Dallas showed a mean occupancy per flight of 58% and a standard deviation of 11%.

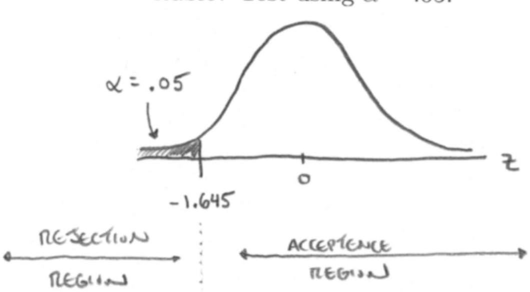
(a) (2 points) If μ is the mean occupancy per flight and if the company wishes to determine whether or not this scheduled flight is unprofitable, give the alternative and the null hypotheses for the test.

$$\boxed{\begin{array}{l} H_0: \mu = .6 \\ H_a: \mu < .6 \end{array}}$$

(b) (2 points) Does the alternative hypothesis in part a imply a one- or two-tailed test? Explain.

$\boxed{\text{ONE-TAILED.}}$ THE AIRLINE SHOULD ONLY CANCEL THE FLIGHT IF $\mu < .6$, NOT IF $\mu > .6$.

(c) (4 points) Do the occupancy data for the 120 flights suggest that this scheduled flight is unprofitable? Test using $\alpha = .05$.



$$z = \frac{\bar{x} - \mu}{S.E.} = \frac{.58 - .6}{.0100} = -2$$

$$-2 < -1.645$$

IN THE REJECTION REGION.

$$S.E. = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}} = \frac{.11}{\sqrt{120}} \approx .0100$$

$\boxed{\text{YES,}}$ THE FLIGHT IS PROBABLY NOT PROFITABLE AND SHOULD PROBABLY BE CANCELLED.