

§ 8.6 ESTIMATING THE DIFFERENCE BETWEEN 2
POPULATION MEANS

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$$\underline{41.} \quad (\mu_1 - \mu_2) \approx (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} (\text{S.E.})$$
$$\uparrow$$
$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= (12.7 - 7.4) \pm 1.96 \sqrt{\frac{1.30}{35} + \frac{4.14}{49}}$$

$$= 5.3 \pm .6900 = [4.61, 5.99]$$

(b) YES, SINCE THE CONFIDENCE INTERVAL DOES NOT
CONTAIN 0, WE CAN BE REASONABLY SURE THERE
IS A DIFFERENCE.

$$\underline{42.} \quad (a) \quad (\mu_1 - \mu_2) \approx (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= (2.9 - 5.1) \pm 1.645 \sqrt{\frac{.83}{64} + \frac{1.67}{64}}$$

$$= -2.2 \pm .3251 = [-2.5251, -1.8749]$$

IF RANDOM SAMPLES OF SIZE n_1 & n_2 ARE REPEATEDLY TAKEN
FROM POPULATIONS 1 & 2 RESPECTIVELY & CONFIDENCE
INTERVALS ARE CONSTRUCTED IN THIS WAY, 90% OF THESE
INTERVALS WILL CONTAIN THE TRUE DIFFERENCE BETWEEN
POPULATION MEANS $\mu_1 - \mu_2$.

$$(b) (\mu_1 - \mu_2) \approx (2.9 - 5.1) \pm 2.58 \sqrt{\frac{.83}{64} + \frac{1.67}{64}}$$

$$= -2.2 \pm .51 = [-2.71, -1.69]$$

YES BECAUSE THE INTERVAL DOES NOT CONTAIN 0.

43. (a) $(\bar{x}_1 - \bar{x}_2) \pm 2.58 \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

$$= (125.2 - 123.7) \pm 2.58 \sqrt{\frac{5.6^2}{100} + \frac{6.8^2}{100}}$$

$$= 1.5 \pm 2.2727 = [-.7727, 3.7727]$$

(b) No, BECAUSE THE INTERVAL CONTAINS 0 (ZERO DIFFERENCE).

44. (a) POINT ESTIMATE: $\bar{x}_1 - \bar{x}_2 = 125.2 - 123.7 = 1.5$

MARGIN OF ERROR $1.96 \sqrt{\frac{5.6^2}{500} + \frac{6.8^2}{500}} = .7722$

(b) YES, 0 IS OUTSIDE THE MARGIN OF ERROR

NOT IN THE INTERVAL $1.5 \pm .7722$

(IN COMPARISON TO QUESTION 43, THE SAMPLE SIZES
HERE ARE MUCH LARGER.)

$$\underline{45.} \quad (\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= (167.1 - 140.9) \pm 1.96 \sqrt{\frac{24.3^2}{30} + \frac{17.6^2}{30}}$$

$$= 26.2 \pm 10.74 = [15.46, 36.94]$$

IF RANDOM SAMPLES AND CONFIDENCE INTERVALS WERE REPEATEDLY CONSTRUCTED IN THIS WAY, 95% OF THESE INTERVALS WOULD CONTAIN THE TRUE DIFFERENCE OF POPULATION MEANS.

$$\underline{46.} \quad (2.4 - 3.1) \pm 1.645 \sqrt{\frac{1.44}{100} + \frac{2.64}{100}}$$

$$= -.7 \pm .3323 = [-1.0323, -.3677]$$

WE CAN BE 90% CONFIDENT THAT REGION 2 RECEIVES BETWEEN .3677 AND 1.0323 MORE CALLS PER HOUR THAN REGION 1.

OR: REGION 1 RECEIVES BETWEEN .3677 AND 1.0323

LESS CALLS PER HOUR THAN REGION 2. (EQUIVALENT)

47.

$$(a) \bar{x} \pm 1.96 \frac{s}{\sqrt{n}} = 18.5 \pm 1.96 \left(\frac{8.03}{\sqrt{365}} \right) =$$

$$= 18.5 \pm .8238 = [17.68, 19.32]$$

$$(b) 16.5 \pm 1.96 \left(\frac{6.96}{\sqrt{298}} \right) = 16.5 \pm .79 = [15.71, 17.29]$$

$$(c) (18.5 - 16.5) \pm 1.96 \sqrt{\frac{8.03^2}{365} + \frac{6.96^2}{298}}$$

$$= 2 \pm 1.14 = [.86, 3.14]$$

(d) YES, SINCE THE INTERVAL DOES NOT CONTAIN 0.

48.

$$(\bar{x}_A - \bar{x}_B) \pm 1.96 \sqrt{\frac{s_A^2}{30} + \frac{s_B^2}{30}}$$

$$= (21.3 - 13.4) \pm 1.96 \sqrt{\frac{2.6^2}{30} + \frac{1.9^2}{30}}$$

$$= 7.9 \pm 1.15 = [6.75, 9.05]$$

ON AVERAGE PEOPLE ON DIET A PROBABLY LOSE BETWEEN 6.75 & 9.05 LBS. MORE THAN PEOPLE ON DIET B OVER A 30 DAY PERIOD.

49. (a) $56202 - 50657 = \boxed{\$5545}$

MARGIN OF ERROR: $1.96 \sqrt{\frac{2225^2}{50} + \frac{2375^2}{50}}$

$= \boxed{\$902.08}$

(b) YES, ENGINEERS ON AVERAGE PROBABLY MAKE BETWEEN \$4642.92 AND \$6447.08 MORE THAN COMPUTER SCIENTISTS (FRESH OUT OF COLLEGE).

50. (a) $14.06 \pm 1.96 \left(\frac{5.65}{\sqrt{376}} \right) = 14.06 \pm .57 = \boxed{[13.49, 14.63]}$

(b) $12.96 \pm 1.96 \left(\frac{5.93}{\sqrt{308}} \right) = 12.96 \pm .66 = \boxed{[12.30, 13.62]}$

(c) $(14.06 - 12.96) \pm 1.96 \sqrt{\frac{5.65^2}{376} + \frac{5.93^2}{308}}$
 $= 1.10 \pm .87 = \boxed{[.23, 1.97]}$

(d) YES, SINCE IT DOES NOT CONTAIN 0.

51. (a) $(150 - 165) \pm 1.96 \sqrt{\frac{17.2^2}{50} + \frac{22.5^2}{50}}$
 $= -15 \pm 7.85 = \boxed{[-22.85, -7.15]}$

$$(b) \quad 165 - 125 \pm 2.58 \sqrt{\frac{22.5^2}{50} + \frac{12.0^2}{50}}$$

$$= 40 \pm 9.44 = [30.56, 49.44]$$

(c) No. There is likely a time difference in average prices at these hotels.

(d) Yes, yes.

52. (a) $15 - 23 \pm 2.58 \sqrt{\frac{4^2}{30} + \frac{10^2}{40}}$

$$= -8 \pm 4.49 = [-12.49, -3.51]$$

(b) Yes, interval does not contain 0.

Control took less time. on average, between 3.51 & 12.49 minutes less time.

53. $98.11 - 98.39 \pm 1.96 \sqrt{\frac{.70^2}{65} + \frac{.74^2}{65}}$

$$= -.28 \pm .2476 = [-.5276, -.0324]$$

Yes (0 is not in the interval).

Body temp of men is probably between .0324 & .5276 degrees F less than body temp of women.