§7.5 THE SAMPLING DISTRIBUTION OF THE SAMPLE MEAN

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15. (a) SAMPLE MEAN = POPULATION MEAN = $\boxed{10}$

$$S.E. = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{9}}{\sqrt{36}} = \frac{3}{6} = \boxed{\frac{1}{2}} \qquad \left(\sigma = \sqrt{\sigma^2} \right)$$

(b) $\mu = \boxed{5}$

$$S.E. = \frac{\sqrt{\sigma^2}}{\sqrt{n}} = \frac{\sqrt{4}}{\sqrt{100}} = \frac{2}{10} = \boxed{\frac{1}{5}}$$

(c) $\mu = \boxed{120}$

$$S.E. = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}} = \boxed{\frac{\sqrt{2}}{4}}$$

16. (a) NORMAL

(b) FOR PARTS a & b, THE DISTRIBUTION IS APPROXIMATELY NORMAL, SINCE $n \geq 30$

FOR PART c, $n < 30$ SO WE CAN'T SAY ANYTHING ABOUT THE DISTRIBUTION OF $\overline{X}$.

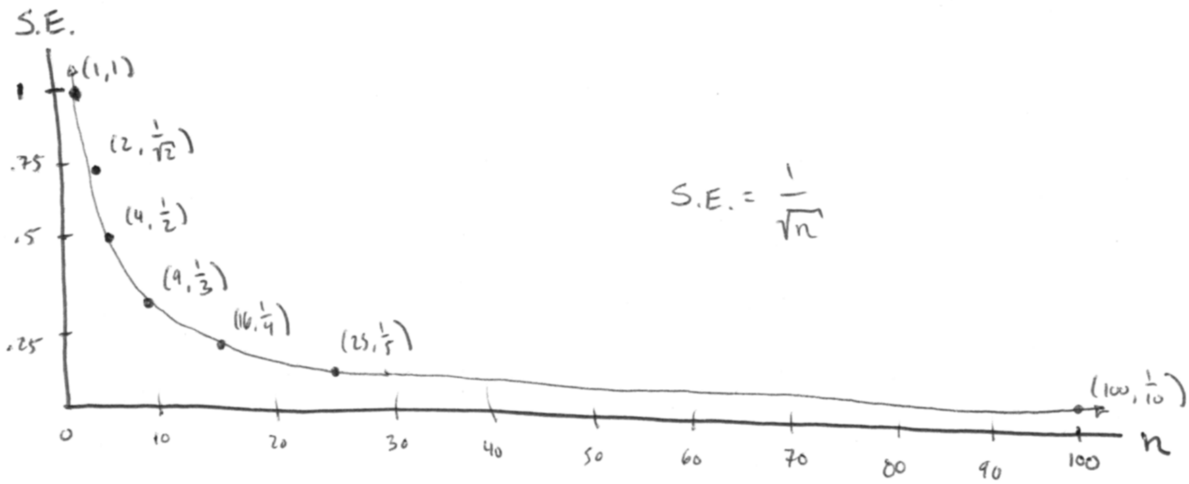
17. (a) $S.E. = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{1}} = \boxed{1}$

(b) $S.E. = \boxed{\frac{1}{\sqrt{2}}}$ OR $\frac{\sqrt{2}}{2}$

(c) $S.E. = \frac{1}{\sqrt{4}} = \boxed{\frac{1}{2}}$

(d) $S.E. = \frac{1}{3}$ (e) $S.E. = \frac{1}{4}$ (f) $S.E. = \frac{1}{5}$ (g) $S.E. = \frac{1}{10}$

20.



AS n INCREASES, S.E. DECREASES.

21. (a) NORMAL ($n \geq 30$)

(b) $\mu = 53$

$$S.E = \frac{\sigma}{\sqrt{n}} = \frac{21}{\sqrt{49}} = \frac{21}{7} = 3$$

22. $P(\bar{x} \geq 55) = P(z \geq .67) = 1 - P(x \leq .67) = 1 - .7486$

$$z = \frac{55 - \mu}{\sigma} = \frac{55 - 53}{3} = \frac{2}{3} \approx .67$$

$= \boxed{.2514}$

23. (a) NORMAL ($n \geq 30$)

(b) $\mu = 100$

$$S.E. = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{40}} = \frac{20}{2\sqrt{10}} = \cancel{\frac{\sqrt{10}}{2} \approx 1.5811} \quad \sqrt{10} \approx 3.1622$$

24. $P(105 \le \bar{x} \le 110) = P(1.58 \le z \le 3.16) = \boxed{.0563}$

$$z = \frac{105 - 100}{\cancel{1.5811} \; 3.1622} = \cancel{3.1624} \; 1.5811 \qquad z = \frac{110 - 100}{\cancel{1.5811} \; 3.1622} = \cancel{6.32} \; 3.1623$$

25. (a) $\boxed{\mu = 106}$

$$S.E. = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{25}} = \frac{12}{5} = \boxed{2.4}$$

(b) $P(\bar{x} \ge 110) = P(z \ge 1.67) = \boxed{.0475}$

(c) $P(102 \le \bar{x} \le 110) = P(-1.67 \le z \le 1.67) = \boxed{.9051}$

"MARGIN OF ERROR"

26. (a) NORMAL $(n \ge 30)$

(b) $\left[\mu - \overbrace{1.96\ S.E.}, \ \mu + \overbrace{1.96\ S.E.}\right]$

$$= \left[71802 - 1.96\left(\frac{4000}{\sqrt{60}}\right), \ 71802 + 1.96\left(\frac{4000}{\sqrt{60}}\right)\right]$$

$$= [70{,}789.86 \ , \ 72{,}814.14]$$

(c) $P(\bar{x} \ge 73{,}000) = P(z \ge 2.32) = \boxed{.0102}$

(d) YES, UNUSUAL

I'D CONCLUDE THAT MY SAMPLE WAS NOT RANDOM, OR THAT THE TRUE POPULATION MEAN IS PROBABLY HIGHER.

27. (a) TEMPERATURE, HUMIDITY, PRESSURE,

SO... WEATHER

CONTAMINANTS, VENTILATION, ETC.

(b) LARGE. S.E. DECREASES AS n GETS BIGGER.

(SEE QUESTION 20)

28. ASSUMING THE WEIGHTS OF TOMATOES IS NORMALLY DISTRIBUTED, SUMS OF 12 WEIGHTS WILL ALSO BE NORMALLY DISTRIBUTED. THIS FOLLOWS FROM THE CENTRAL LIMIT THEOREM.

WITHOUT THIS ASSUMPTION, WE CAN'T SAY ANYTHING SINCE

$n = 12$ IS NOT ≥ 30.

(IF $n \geq 30$, THEN SUMS OF n WEIGHTS WILL BE APPROXIMATELY NORMALLY DISTRIBUTED REGARDLESS OF THE DISTRIBUTION OF INDIVIDUAL WEIGHTS.)

29. $\# \text{ BACTERIA IN } 1 \text{ ft}^3 \text{ OF } H_2O = \sum_{i=1}^{1728} \left(\# \text{ BACTERIA IN } i^{TH} \text{ in}^3 \text{ OF } H_2O \right)$

BY CENTRAL LIMIT THEOREM, SUMS OF ~~[illegible]~~ 30 OR MORE RANDOM VARIABLES ARE APPROXIMATELY NORMAL. μ MUST BE LARGE SO THAT ~~θ IS MORE THAN 3~~ THE ORIGINAL DISTRIBUTION IS NOT STRONGLY SKEWED.

(DON'T WORRY ABOUT #29, SINCE WE DIDN'T COVER POISSON RAN. VAR.'S)

30. (a) NORMAL SINCE ORIGINAL DISTRIBUTION IS NORMAL

(b) $\mu = 21$

$S.E. = \frac{2}{\sqrt{10}} \approx .6325$

$P(\bar{x} < 20) = P(z < \frac{20-21}{.6325}) = P(z < -1.58)$

$= \boxed{.0571}$

(c) $P(z < z_0) = .001$

$\Rightarrow z_0 = \underbrace{-3.08 \text{ TO } -3.10}$

ALL THESE HAVE $P(z \le z_0) \approx .001$

I'LL TAKE $z_0 = -3.09$.

~~THEN FIND~~ CORRESPONDING $x_0 = 20$

$z_0 = \frac{x_0 - \mu}{S.E.} \Rightarrow -3.09 = \frac{20-\mu}{.6325}$

$-1.9544 = 20 - \mu \Rightarrow \boxed{\mu = 21.9544}$

31. (a) FOR SUMS (NOT AVERAGES) (SEE p. 253)

$$\text{MEAN} = n\mu = (3)(422) = \boxed{1266}$$

$$\text{S.E.} = \sqrt{n}\,\sigma = \sqrt{3}\,(13) \approx \boxed{22.5167}$$

(b) $$P(x \geq 1300) = P\left(z \geq \frac{1300 - 1266}{22.5167}\right)$$

$$= P(z \geq 1.51) = \boxed{.0655}$$

32. (a) APPROXIMATELY NORMAL

ASSUMING THE NUMBER OF CUSTOMERS (WHICH IS FIXED) IS AT LEAST 30, THE SUME OF THESE INDIVIDUAL SALES WILL BE APPROX. NORMALLY DISTRIBUTED BY THE CENTRAL LIMIT THEOREM.

(b) $$\text{MEAN} = n\mu = (30)(8.50) = \boxed{\$255}$$

$$\text{S.E.} = \sqrt{n}\,\sigma = \sqrt{30}\,(2.50) \approx \boxed{\$13.69}$$

33. (a) $\mu = 98.6$, $\text{S.E.} = \dfrac{.8}{\sqrt{130}} \approx .0702$

$$P(\bar{x} \leq 98.25) = P(z \leq -4.99) \approx 0.$$

(b) YES, CLEARLY.

34. (a) $\mu = 5.97$

$$S.E. = \frac{1.95}{\sqrt{31}} \approx .3502$$

$$P(\bar{x} \le 6.5) = P(z \le 1.51) = \boxed{.9345}$$

(b) $P(\bar{x} \ge 9.8) = P(z \ge 10.94) \approx 0$

(c) AVERAGE TENDON DIAMETER IN POPULATION OF PATIENTS WITH A.T. IS DIFFERENT (HIGHER) THAN AVE. DIAM. OF NON-AFFECTED TENDONS.