JOHN ADAMSKI
4/18/2017

§6.4 THE NORMAL APPROXIMATION TO THE BINOMIAL PROBABILITY DISTRIBUTION

LET x BE A BINOMIAL RANDOM VARIABLE WITH n TRIALS AND PROBABILITY OF SUCCESS p (AND PROB OF FAILURE $q = 1-p$).

WE KNOW $P(x=k) = C^n_k \, p^k q^{n-k}$.

IF WE NEED $P(x \le k)$ WE CAN LOOK THIS UP IN TABLES (APP. A) FOR CERTAIN VALUES OF n & p, BUT OTHERWISE WE'RE STUCK EVALUATING

$$P(x \le k) = \sum_{i=0}^{k} C^n_i \, p^i q^{n-i} \qquad (\text{LENGTHY!}).$$

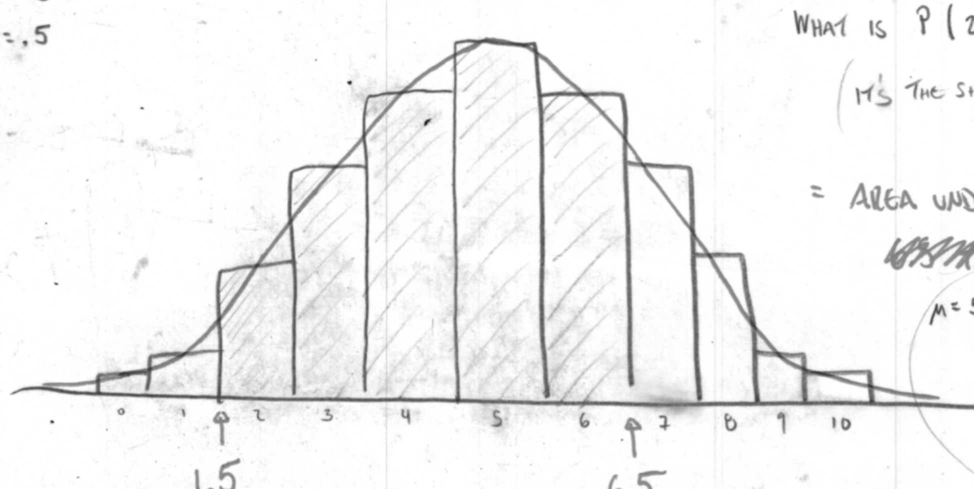
THE PROB DISTR. OF x IS APPROXIMATELY NORMAL WITH

$$\mu = np \quad \& \quad \sigma = \sqrt{npq}$$

AS LONG AS n IS LARGE AND NEITHER p OR q IS TOO CLOSE TO 0.

(RULE OF THUMB: $np > 5$, $nq > 5$)

e.g. $n = 10$
$p = .5$



CONTINUITY CORRECTION

WHAT IS $P(2 \le x \le 6)$?

(IT'S THE SHADED AREA.)

= AREA UNDER CURVE FROM 1.5 TO 6.5

$\mu = 5$, $\sigma = \sqrt{2.5} \approx 1.58$

$1.5 \mapsto \frac{1.5-5}{1.58} = -2.2136$

$6.5 \mapsto \frac{6.5-5}{1.58} = .9487$

GO SLOW! REVIEW!

$$P_{\text{BINOMIAL}}(2 \le x \le 6) \approx P_{\text{NORMAL}}(1.5 \le x \le 6.5)$$

$$= P(z \le .9487) - P(z \le -2.2136)$$

$.828 - .011 = .817$

$.8289 - .0136 = .8153$

PRETTY CLOSE !!

e.g. ASSUME 20% OF POPULATION DRINKS PEPSI, AND WE RANDOMLY SELECT 500 PEOPLE AND ASK IF THEY DRINK PEPSI.

$\mu = np = 100$

$\sigma = \sqrt{npq} = \sqrt{80} = 4\sqrt{5} \approx 9$

USING NORMAL APPROXIMATION,

(a) FIND PROB BETWEEN 105 & 120 PEOPLE DRINK PEPSI (INCLUSIVE).

(b) FEWER THAN 85 PEOPLE DRINK PEPSI.

(c) WHAT WOULD YOU CONCLUDE IF YOU FOUND 300 PEOPLE DRINK PEPSI?